Why (don't) firms free ride on an intermediary's advice?*

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Abstract

When consumers rely on an intermediary's advice about which firm to buy from but can switch to buying directly after receiving advice, one might expect firms to discount their direct prices to encourage consumers to purchase directly after obtaining advice, thereby avoiding paying commissions. We provide a theory which can explain why firms often don't free ride in this way, as well as when they do. The theory can explain why online marketplaces and hotel booking platforms impose price-parity clauses to prevent such free riding, while insurance and financial advisors do not.

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1 Introduction

Firms that compete to attract uninformed consumers typically pay commissions or kickbacks to information intermediaries with the aim of influencing the intermediaries' advice to these consumers and so their chance of a sale. A broker for insurance or financial products may advise consumers which insurance or financial product they should purchase. A physician may advise consumers which drug to take. A retailer may provide advice to shoppers for experience or credence goods over which manufacturer's product they should purchase.

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An online platform may recommend which seller is best for the consumer. In all these cases, it is conceivable that after receiving the recommendation, consumers could bypass the intermediary and buy directly from the product provider if this allows them to purchase at a lower price. Given this possibility, one might expect firms to free ride on the intermediary's advice—discounting their direct prices to encourage consumers to purchase directly after obtaining advice, thereby avoiding paying commissions.

This type of free riding seems to arise in online platform settings, and is the main justification given by such platforms for their use of price-parity clauses to prevent firms charging less when selling directly (e.g. Amazon, Booking.com and Expedia, have all used such clauses as detailed in Edelman and Wright, 2015a and Hviid, 2015). However, in the case of insurance, financial and medical products, intermediaries do not impose any such contract restrictions, and yet firms typically do not discount prices for direct sales relative to intermediated sales (see Section 10 of Edelman and Wright, 2015a, for evidence on insurance and financial products).

This paper provides a theory to make sense of these observations. It explains why when firms set commissions, firms will set their direct and intermediated prices to be equal in equilibrium even when intermediaries do not restrict the firms' pricing in any way. It also explains why when commissions are set by intermediaries, an intermediary may instead need to rely on price-parity clauses to eliminate any discounts for direct sales. Thus, it can potentially reconcile the different observed outcomes across different industries. By doing so it provides policymakers with a more nuanced view of price parity—the phenomenon that direct and intermediated prices are the same. It shows that price parity is not necessarily caused by restrictive price-parity clauses.

The model we use is adapted from Inderst and Ottaviani (2012a). In their framework, firms compete by setting commissions which are paid to an intermediary. The intermediary has a better signal about which of the two firms' products is more suitable for consumers and can issue a recommendation to consumers about which product to purchase. The intermediary's advice is assumed crucial for trade—without advice, the expected match value is not sufficient, so consumers will never buy from one of the firms even if the firms price at marginal cost. The intermediary compares the commissions it obtains from each product but also takes into account which product is more suitable due to liability, ethical or reputational concerns. Firms set product prices, taking into account the advice consumers receive. In equilibrium, advice is informative and consumers rationally follow the intermediary's recommendation. We adapt this model by allowing that after obtaining advice, consumers can switch channels, purchasing directly from one of the firms potentially at a different price (e.g. at a discount). We assume consumers are heterogeneous in the cost they incur from switching channels so that the proportion of consumers switching to purchase directly is increasing in the discount offered.

To understand the logic for why firms don't offer discounts for direct sales in this setting, suppose to the contrary that initially they do. Lowering this discount increases the firm's margin on the inframarginal consumers who still continue to buy directly. This increase in the amount the firm collects from the inframarginal direct consumers has no bearing on the intermediary or its recommendation, and so is a pure gain for the firm. On the other hand, any loss in the firm's margin as a result of some additional consumers now purchasing through the intermediary at positive commission fees can be offset with a lower commission fee to leave the firm and the intermediary unaffected, as the expected commission revenue paid by the firm to the intermediary remains the same. Thus, on net the firm is better off reducing its discount on direct sales from any positive level, and lowering commissions by an offsetting amount.

This logic highlights that reducing commissions is better for firms than giving discounts, as a way to reduce the amount paid to the intermediary. This reflects that commissions are better targeted. Discounts involve payments to inframarginal consumers who would anyway buy directly. We show this argument, which we first develop assuming commissions are observable, continues to hold even when consumers do not observe commissions and so there is the potential for a firm to use a discount as a positive signal of the expected quality of its product. We establish this result under two extremes of beliefs: (i) consumers have naive beliefs, so they do not update their beliefs about the quality of the firm's product implied by the intermediary's recommendation based on out-of-equilibrium prices, and (ii) consumers are sophisticated in updating their beliefs (in a sense we will make clear later). We also show the argument holds when consumers are naive in that they always assume the recommended firm is more suitable regardless of what prices and/or commissions are observed.

The ability of firms to offset a lower discount with a lower commission, and do better, depends on the assumption that firms set commissions. If instead it is the intermediary that sets commissions, our equivalence argument between commissions and discounts breaks down, and we show firms will always want to offer positive discounts in response to high commissions. In this case, the intermediary will want to impose a price-parity restriction on firms to prevent them discounting for direct sales.[1] This allows the intermediary to shift surplus from firms to itself.

Taking these results together, our theory provides predictions on when free riding is more

[1]Hunold et al. (2017) provide evidence from Booking.com in Europe. They find the abolition of Booking.com's price-parity restrictions is associated with the hotels' direct channel having the strictly lowest price more often, and hotels promoting their direct online channel more actively.

likely to be a problem for advisors and when it is not. Similarly, it predicts when price-parity clauses will be used and when they are redundant. Specifically, to the extent that commissions seem to be set by firms in certain industries (such as the sale of traditional insurance, financial and medical products by brick-and-mortar advisors) and set by intermediaries in other industries (such as the online platforms like Amazon and Expedia)[2], our theory explains why price-party clauses are used in the latter online settings but not the former. At the end of the paper, we will conjecture a possible explanation for why commissions might tend to be set by intermediaries in online settings, but not when intermediaries need to provide advice by meeting consumers physically. In brief, the idea is that in contrast to an online setting, in an offline setting each intermediary may only be able to deal with a small number of consumers, in which case there may be many more intermediaries (advisors) than firms, and so it is more natural that the firms rather than the intermediaries set commissions. Combining this explanation with the predictions of our theory, we tie the incidence of free riding and price-parity clauses to the ability of intermediaries to use modern communication technologies to give advice on a mass scale.

The rest of the paper proceeds as follows. Section 2 briefly discusses related literature. Section 3 details the model, and provides some preliminary analysis. Section 4 establishes what happens when firms set commissions, while Section 5 analyzes the setting in which the intermediary sets commissions. Finally, Section 6 concludes.

2 Related literature

Our work relates to the rapidly growing literature that considers whether intermediaries bias their advice in favor of firms from which they derive larger revenues. Apart from Inderst and Ottaviani (2012a), whose setting we closely follow, other studies that address this issue include Armstrong and Zhou (2011), de Corniére and Taylor (2014, 2016), Hagiu and Jullien (2011), Inderst and Ottaviani (2012b) and Murooka (2015). For our purposes, a framework which links the intermediary's advice with its financial incentives is needed to address whether firms want to free ride on an intermediary's advice. If an intermediary's advice is not influenced by financial incentives, then firms would never pay commissions and firms would have no incentive to provide discounts to get consumers to buy directly. Such a framework is also necessary in order that an intermediary may sometimes want to steer consumers away from a firm that offers discounts, because other things equal, such a firm will

[2]See Section 10 of Edelman and Wright (2015a) for evidence consistent with this for insurance and financial products, as well as online marketplaces and online travel agents, and the references in Inderst and Ottaviani (2012a) for medical products.

generate less revenue for the intermediary. However, these works do not generally consider consumers choosing between buying through the intermediary and buying directly, and none considers the possibility of consumers switching to buy directly after obtaining advice. For this reason, they cannot address the puzzle we address—why firms apparently often do not discount for direct sales, even though the intermediary that provides advice to consumers does not impose any restriction on their pricing.

Some earlier works have considered intermediation when consumers can either buy through the intermediary or directly from firms. For example, Baye and Morgan (2001) and Galeotti and Moraga-González (2009) allow consumers to bypass the intermediary, in which case consumers are assumed to face a single (i.e. monopoly) seller. The intermediary's role is therefore to facilitate competition between listed firms. Consumers obtain full information about the firms once they are on board, and the intermediary plays no role in steering consumers to particular firms. Given firms are monopolists with respect to consumers who buy directly, if they are allowed to price differentially, they will actually set higher prices for direct sales.

Two recent papers (Edelman and Wright, 2015b, and Wang and Wright, 2017) differ from these earlier works in allowing firms to compete for buyers both directly and through an intermediary, giving rise to the possibility of discounts and the use of price parity clauses by intermediaries. In Edelman and Wright (2015b) consumers have full information so the intermediary cannot steer consumers in their setting. In Wang and Wright (2017) the intermediary plays the role of an unbiased search platform—it always reveals information truthfully on each firm that is searched, and it just lowers the search cost for consumers compared to if consumers search directly. As a result, in either framework, firms would never have a reason to offer the intermediary positive commissions, and therefore nor do they have an incentive to offer discounts to consumers for direct sales. On the other hand, if the intermediary sets commissions, which is the case considered in these papers, then the intermediary will set positive commissions and firms will discount their direct prices unless they are constrained by a price-parity clause. Thus, both of these frameworks can generate the same overall conclusion we reach, although only in market settings in which steering is not possible, and so for more trivial reasons.

Finally, our paper is related to Johnson (2017) who compares different business models, such as the agency model with the wholesale model, as well as also analyzing price parity restrictions. In each of the four possibilities considered by Johnson, one firm (supplier or retailer) sets the "terms of trade" (given either by a wholesale price or a revenue-sharing term), and the other firm sets the retail price. We also compare business models (who sets commissions), but we differ in that one of our business models involves the same firm setting both the terms of trade and the retail price. This would be of little interest in Johnson's setting given there is no reason in his full-information framework for firms to offer positive commissions. Our purpose in comparing business models is also quite different — to understand why discounts may or may not be offered for direct sales.

3 The model and preliminaries

We adapt the model of Inderst and Ottaviani (2012a), in which firms compete by setting commissions, by allowing firms to sell directly to consumers, potentially at a different price (e.g. at a discount). We will also consider the case in which the intermediary sets commissions. The core of the model is the same as theirs and we will therefore keep our presentation of it brief. We refer the reader to their paper for a more complete discussion.

Suppose there is a continuum of consumers (measure 1), each of whom wants to buy a single unit of one of two products, A or B.[3] Product i is sold by firm i ($i = A, B$). The utility to consumer j of buying good i depends on a state variable θ_j that measures which product (A or B) is more suitable for consumer j. The consumer derives utility v_h from the product if the product matches the state and utility v_l from the product if it doesn't, with $0 < v_l < v_h$. The consumers' outside option gives them zero utility. Firms produce at respective per-unit costs c_i, where $c_B \geq c_A$.

The probability product A is more suitable for consumer j is given by $q = Pr(\theta_j = A)$, which is distributed according to the continuous distribution $G(q)$ with density $g(q) > 0$ over $q \in [0, 1]$, where $G(q)$ is symmetric around $q = 1/2$ with $G(q) = 1 - G(1 - q)$ and has an increasing hazard rate. Consumers and firms do not observe which product is the best match for a specific consumer, but they know G, the distribution of q. On the other hand, there is an intermediary M that observes the realization of q for each consumer. Thus, M knows for any consumer, the expected value of product A is $v_A(q) = qv_h + (1 - q)v_l$ and the expected value of product B is $v_B(q) = (1 - q)v_h + qv_l$. M can make a recommendation to each consumer about which product to buy.[4]

From the ex-ante perspective of a consumer, the expected value of each product is assumed to be less than c_A; i.e.,

$$\int_0^1 v_A(q)\, dG(q) = \int_0^1 v_B(q)\, dG(q) = \frac{v_l + v_h}{2} < c_A. \tag{1}$$

[3]Inderst and Ottaviani (2012a) consider a single representative consumer, in which case product match should be interpreted as vertical quality rather than as a horizontal match value. The analysis that follows can equally apply to this vertical interpretation of quality match.

[4]M's recommendation is assumed to be non-verifiable, so making a statement about the level of q to consumers would be equivalent to recommending which product to buy.

Thus, we maintain Inderst and Ottaviani's critical assumption, that M's advice is essential for selling either product.[5] This rules out firms competing for consumers without making use of M. However, this does not rule out consumers buying directly after obtaining advice from M, which we call "showrooming". Showrooming is potentially a fundamental problem for M—that if firms pay positive commissions to M, they may want to induce consumers to bypass M with lower direct prices after consumers have obtained the relevant advice. To ensure that either product can be sold with advice, we also assume that

$$\int_{\frac{1}{2}}^{1} v_A(q) \frac{g(q)}{1 - G\left(\frac{1}{2}\right)} dq = \int_{0}^{\frac{1}{2}} v_B(q) \frac{g(q)}{G\left(\frac{1}{2}\right)} dq > c_B.$$

In order that M puts some weight on providing good advice, Inderst and Ottaviani (2012a) assume that M obtains additional utility w_h whenever the consumer purchases the more suitable product, additional utility $w_l < w_h$ if the consumer purchases the less suitable product, and no additional utility in case the consumer does not buy either product.[6] Thus, M's concern for suitability depends on the difference $w \equiv w_h - w_l$. Inderst and Ottaviani (2012a) discuss several motivations for their assumption, including that M may face a penalty following a purchase of a product that turns out to be a bad match, that M may be motivated by a concern for the customer's well-being from the match, or may care about the reputational implications of consumers purchasing a bad match.[7] Arguably, each of these motivations also potentially applies just as much to direct sales. E.g. the consumer may still link a bad experience with M's recommendation and M may still care about what happens to the consumer (e.g. in the case of a doctor). Thus, as a benchmark we assume that M's concern for suitability continues to hold, even if the final transaction following M's recommendation is completed directly by the firm. However, we show that our results do not depend critically on this assumption. In Section C of the online appendix, we show that our main results continue to hold even under alternative assumptions, including the case that M puts less, or indeed no weight on suitability for direct sales.

If consumers choose to buy directly after obtaining M's advice, they are assumed to incur a shopping cost s which is drawn from a smooth distribution H over the support $[0, S]$,

[5]We analyze how our main results extend when assumption (1) is relaxed in Section A of an online appendix. The online appendix is also available at http://profile.nus.edu.sg/fass/ecsjkdw/.

[6]We do not require that $w_l > 0$ as assumed by Inderst and Ottaviani (2012a), but instead adopt the weaker assumption that $w_l + w_h \geq 0$. In the appendix, we establish why this is sufficient to ensure M will always want to recommend one of the firms in equilibrium.

[7]Our results do not depend crucially on this formulation for how commissions can steer consumers to buy from one firm or another. As an example, in Section B of the online appendix we show that our main results can also be obtained when M just recommends the firm which is a better match for consumers when M is indifferent, but otherwise recommends the firm which generates higher expected commission revenue.

with $S > 0$ (which need not be finite). Inderst and Ottaviani (2012a) implicitly assumed s was infinite such that consumers would never buy directly after using the intermediary. In practice, consumers can switch and buy directly, although there is likely to be some inconvenience of doing so, and our specification allows this inconvenience s to differ across consumers. This ensures there is a well-defined demand for direct sales.

Denote the price for an intermediated purchase from firm i as p_i^m, and the price for a direct purchase from firm i as p_i^d, where $i = A, B$. We make the same contracting restrictions as in Inderst and Ottaviani (2012a) (so only non-negative commission payments and standard linear pricing can be used), which they provide detailed justifications for. The timing is as follows:

Stage 1: If firms set commissions, then each firm i sets its commission f_i together with its prices to consumers p_i^m and p_i^d (for a sale through M and a direct sale, respectively). If M sets commissions, then M sets f_A and f_B, and firm i sets p_i^m and p_i^d after observing f_i but not f_j.

Stage 2: M sends a message (either A or B).

Stage 3: Consumers observe this message, all prices, their shopping cost s, and possibly commissions. Each consumer makes a final purchase decision, i.e., which firm to purchase from, if any, and whether to complete the purchase directly or through the intermediary.

Regardless of which party sets commissions, our timing assumptions imply that firms do not observe each other's commission when setting their prices. Note that Inderst and Ottaviani assume firms set their commissions prior to setting their prices p_i^m. Our different timing assumption for this case, which is made for convenience, would not affect their results, even in the case commissions are observable. This reflects that a firm will not take into account how its commission affects its rival's price given that firms do not compete in prices.

Our equilibrium concept is perfect Bayesian equilibrium. Like Inderst and Ottaviani, we focus on equilibria in which only pure strategies are played and advice is informative (meaning consumers follow M's recommendations) in the stage-2 subgame. However, unlike Inderst and Ottaviani, in our setting it is possible a firm would want to set its intermediated price above consumers' maximum willingness to pay such that all consumers either buy directly or do not buy at all (because their draw of s is too high). This would mean that M does not complete any transactions for the firm. For example, by setting such a price, a firm could signal to sophisticated consumers that do not observe commissions that M will receive no commission payments from it, thereby increasing consumers' willingness to pay for the firm's product (directly) after being recommended the product. To avoid the complications that would arise if firms set such prices (which we view as unrealistic), we use the equilibrium selection rule that if either firm sets such a price in stage 1, then the babbling equilibrium in which advice is uninformative in the stage-2 subgame is selected (i.e. M always recommends the same firm and consumers ignore its recommendation). Thus, when we claim a unique pure strategy equilibrium exists, we mean subject to this equilibrium selection rule. In Section D of the online appendix we explain the sense in which our results still hold when we continue to assume the informative equilibrium is selected in the stage-2 subgame even for such high intermediated prices.

3.1 Preliminaries

Before presenting our main results, we start with some preliminary analysis that applies to both Section 4 and Section 5, i.e., regardless of which party sets commissions.

In stage 3, we know that a consumer that is willing to buy from firm i through M will want to complete the purchase through M if and only if $s \geq \Delta p_i \equiv p_i^m - p_i^d$, where Δp_i is firm i's discount for a direct purchase. That is, the consumer will purchase through M provided the shopping cost of going directly exceeds the discount from a direct purchase. If this condition does not hold, the consumer will switch to purchase directly from firm i.

In stage 2, provided consumers follow M's recommendation of which firm to buy from in the equilibrium in the stage 3 subgame, M will anticipate that by recommending firm i, a fraction $1 - H(\Delta p_i)$ of consumers will buy from firm i through M, and a fraction $H(\Delta p_i)$ of consumer will switch to buy directly. Therefore, M's expected payoff from recommending firm A is $[1 - H(\Delta p_A)]f_A + qw_h + (1-q)w_l$, and that from recommending firm B is $[1 - H(\Delta p_B)]f_B + (1-q)w_h + qw_l$. When both products are recommended with positive probability, M recommends firm A rather than firm B if $q \geq \bar{q}$, where the cutoff is

$$\bar{q} = \frac{1}{2} + \frac{F_B - F_A}{2w}, \tag{2}$$

and $F_i \equiv (1 - H(\Delta p_i))\, f_i$ is M's "effective commission" from firm i. The effective commission takes into account the probability that the commission is collected by M (i.e. consumers do not switch to buy directly). To simplify the exposition, if $F_A > F_B + w$, we set $\bar{q} = 0$, and if $F_B > F_A + w$ we set $\bar{q} = 1$, although these cases do not arise in equilibrium. The same truncation of $\bar{q}$ and expectations of $\bar{q}$ applies throughout the paper.

The cutoff $\bar{q}$ in (2) depends on the commissions as well as the discounts. For any $0 < \bar{q} < 1$

and $F_i > 0$ ($i = A, B$), we have

$$\frac{\partial \bar{q}}{\partial \Delta p_A} = \frac{h(\Delta p_A) f_A}{2w} > 0, \quad \frac{\partial \bar{q}}{\partial \Delta p_B} = -\frac{h(\Delta p_B) f_B}{2w} < 0,$$
$$\frac{\partial \bar{q}}{\partial f_A} = -\frac{(1 - H(\Delta p_A))}{2w} < 0, \quad \frac{\partial \bar{q}}{\partial f_B} = \frac{(1 - H(\Delta p_B))}{2w} > 0.$$

A higher commission paid to M by firm i (or a lower discount for direct purchase), will make M recommend firm i more often (i.e. use a lower cutoff value of q if $i = A$ or a higher cutoff value of q if $i = B$). Note a higher value of $\bar{q}$ corresponds to a higher (respectively lower) chance that a consumer will obtain a good match when buying from firm A (respectively firm B).

Note the cutoff $\bar{q}$ only depends on effective commissions, which captures the fact that M only cares about commissions that it actually will end up receiving. This is the same expression as in Inderst and Ottaviani (2012a) except we have replaced actual commissions with effective commissions. For any $0 \leq \Delta p_i < S$, since there is a one-to-one relationship between the actual commission and the effective commission, we can consider instead each firm setting its effective commission F_i along with Δp_i and p_i^m. In case $\Delta p_i \geq S$, we have $H(\Delta p_i) = 1$ so that the effective commission is always zero regardless of the actual commission. In this case, we can also take f_i to be equal to zero without loss of generality.

In general, each consumer's conditional valuation depends on their expectation of the cutoff $\bar{q}$, which we denote by q^e. For a given expected cutoff q^e, the consumer's expected valuation for each product conditional on it being recommended is determined by

$$P_A(q^e) = \int_{q^e}^{1} v_A(q) \frac{g(q)}{1 - G(q^e)} dq, \tag{3}$$
$$P_B(q^e) = \int_{0}^{q^e} v_B(q) \frac{g(q)}{G(q^e)} dq. \tag{4}$$

For any $0 < q^e < 1$, differentiation of the respective conditional valuations implies

$$\frac{dP_A(q^e)}{dq^e} = \frac{g(q^e)}{1 - G(q^e)} \left(P_A(q^e) - v_A(q^e)\right) > 0, \tag{5}$$
$$\frac{dP_B(q^e)}{dq^e} = -\frac{g(q^e)}{G(q^e)} \left(P_B(q^e) - v_B(q^e)\right) < 0.$$

In any stage-2 subgame, if firm i has set p_i^m so high that no consumer would ever want to buy from it through M, then our equilibrium selection in the stage-2 subgame implies the firm will obtain no profit. This reflects that to obtain a profit the firm must set its direct price above cost ($p_i^d > c_i$), but then (1) implies consumers will never want to buy from the firm given that M's recommendation is not informative. This also rules out the existence of any alternative equilibrium involving such a price. Thus, throughout the analysis we can restrict attention to the prices $p_i^m \leq P_i\left(q^e\right)$.

It will follow that firms will therefore want to set $p_i^m = P_i(q^e)$ so as to fully extract consumers' willingness to pay for the recommended product. Setting $p_i^m < P_i(q^e)$ is clearly suboptimal for the corresponding firm since it reduces the firm's margin without increasing its market share.[8] Assumption (1) ensures that firms cannot do better by setting such a low price that consumers would always buy irrespective of M's recommendation, since this would require a price below their cost. Note that although consumers have their surplus fully extracted in this model, a consumer that buys the non-recommended product will still be strictly worse off. For example, a consumer's expected payoff from buying firm B's product when firm A's product is recommended is negative since

$$\int_{q^e}^{1} v_B(q)\frac{g(q)}{1-G(q^e)}dq - p_B^m < \int_{0}^{q^e} v_B(q)\frac{g(q)}{G(q^e)}dq - p_B^m = 0.$$

This also implies that in equilibrium both products must be sold with positive probability; i.e. $0 < q^* < 1$, where q^* is the equilibrium cutoff. If instead $q^* = 0$, then only firm A will be recommended, in which case consumers' expected valuation for product A is $\int_0^1 v_A(q)dG(q)$, which is less than c_A by assumption (1), so there would be no trade and no profit for firms or M. A similar argument rules out $q^* = 1$.

Given $p_i^m = P_i(q^e)$, firms' profits are

$$\pi_A = [P_A(q^e) - (1 - H(\Delta p_A))f_A - c_A - \Delta p_A H(\Delta p_A)](1 - G(\bar{q})), \tag{6}$$

$$\pi_B = [P_B(q^e) - (1 - H(\Delta p_B))f_B - c_B - \Delta p_B H(\Delta p_B)]G(\bar{q}). \tag{7}$$

4 Firms set commissions

In this section we show that when firms set commissions, they will not discount for direct sales even though the intermediary does not impose any restriction on their pricing. This implies that the showrooming problem does not arise and price parity holds automatically even without any price-parity clause being imposed.

Each firm sets Δp_i and f_i to maximize its profit in (6)-(7). Alternatively, we can also consider firms' competing in setting the effective commissions. By the definition of F_i and

[8] In the more complicated setting of Section 4.2.2, this result is no longer immediately obvious. We will explain in Section 4.2.2 why even in that case the result still holds.

Δp_i, we can rewrite firms' profits from (6)-(7) as

$$\pi_A = [P_A(q^e) - F_A - c_A - \Delta p_A H(\Delta p_A)]\,(1 - G(\bar{q})),$$
$$\pi_B = [P_B(q^e) - F_B - c_B - \Delta p_B H(\Delta p_B)]\,G(\bar{q}).$$

Each firm sets Δp_i and F_i to maximize its profit.

In the following subsections we show how we obtain our key price-parity result under different informational settings, and the implications for the use of price-parity clauses.

4.1 Observable commissions

Suppose consumers observe commissions f_A and f_B. This could reflect that M is required to disclose these to consumers when providing its recommendation. Since consumers observe all prices and commissions, this corresponds to consumers observing the effective commissions. As a result, intermediated prices are determined by (3)-(4) with q^e replaced by $\bar{q}$ as defined in (2). Firms' expected profits are

$$\begin{aligned}\pi_A &= [P_A(\bar{q}) - F_A - c_A - \Delta p_A H(\Delta p_A)](1 - G(\bar{q})),\\ \pi_B &= [P_B(\bar{q}) - F_B - c_B - \Delta p_B H(\Delta p_B)]G(\bar{q}).\end{aligned}$$

Firm i chooses F_i and Δp_i to maximize π_i.

Note that $\bar{q}$ depends only on F_i and not on Δp_i. Thus, for a given effective commission, the number of consumers a firm attracts to buy its product and how much they are willing to pay for its product does not depend on the firm's discount for direct sales. However, conditional on its choice of effective commission, each firm's profit from these customers depends negatively on Δp_i, the discount for direct sales. Given that there is a one-to-one relationship between the effective commission and the actual commission for any given Δp_i, firm i can always lower Δp_i and in the meantime reduce its commission f_i, to keep F_i unchanged. Therefore, for any F_i which is non-negative, firm i does best by setting $\Delta p_i = 0$ regardless of the discount set by the other firm. This implies $F_i = f_i$.

The key feature we exploited to establish the result is that by changing the discount for direct sales and the commission paid, a firm cannot make consumers think the product it offers is of a higher expected quality after they learn M has recommended it without actually decreasing the effective commission received by M. When commissions are observed, consumers observe the firms' effective commissions, and as shown above, this is all that matters for interpreting M's recommendation. Then, to lower effective commissions, firms should always lower actual commissions (which are targeted) and not increase discounts (which are not targeted).

Given there is no direct purchase, Section IV of Inderst and Ottavani (2012a) characterizes the equilibrium which they show is the unique pure strategy informative equilibrium. Thus, we have established that:

Proposition 1. *When firms set observable commissions, there is a unique pure strategy equilibrium in which firms do not offer any discounts for direct sales.*

The intuition behind Proposition 1 can be illustrated with two simple examples. Suppose initially the discount is so large that all of a firm's customers get advice from the intermediary but then switch to buy directly. In a proposed equilibrium in which each firm is doing this, firms do not pay any commission. But a firm can then completely remove the discount and pay no commission, and the payment to the intermediary and its incentives to recommend the firm would be unchanged. The firm would clearly be better off since it avoids offering the discount on all its consumers but still receives the same number of consumers.

The same logic applies starting with any number of consumers that are buying directly. For example, suppose initially each firm offers a discount such that one half of all consumers switch to buy directly after getting their advice from the intermediary. A firm can do better eliminating its discount and cutting its commission fee in half. Then without any discount, all of the firm's customers will buy through the intermediary (there is no reason for them to switch to buy directly), so the total commission the firm ends up paying the intermediary is unchanged, as is the intermediary's incentive to recommend the firm. However, the firm has saved the discount it was previously offering to the one half of consumers that were previously buying directly.

4.2 Unobservable commissions

Suppose instead consumers do not observe the commissions set by firms. When commissions are not observed, a higher discount for direct sales has the potential to lead consumers to believe M is receiving a lower effective commission, and so to interpret a recommendation to buy from the firm as a more positive signal of the expected quality of its product. This would make consumers willing to pay more for the firm's product, and could possibly make such a discount profitable. The strength of this signaling effect will depend on the nature of consumers' beliefs. In this section, we analyze two extremes in consumer beliefs to show why discounts for direct sales may still not be used even if consumers do not observe commissions.

4.2.1 Naive beliefs

Suppose regardless of the prices observed, consumers hold fixed beliefs about the expected quality of the product being recommended. We call these naive beliefs since consumers do not try to work out the implications of off-equilibrium path prices for M's incentive to make a particular recommendation. Thus, consumers with these beliefs will not view a higher discount as a signal that a recommended firm's product is of higher expected quality. Let q^* be defined as the equilibrium level of $\bar{q}$. Formally, naive beliefs mean that $q^e = q^*$ even when consumers observe a different intermediated price or direct discount than expected. This implies the consumers' conditional expectation of the value from product i also remains fixed at $P_i(q^*)$. Consumers with naive beliefs are rational in the Bayesian sense (i.e. their beliefs are correct in equilibrium).[9]

Facing fixed beliefs about the expected quality of their product conditional on it being recommended, firm i will set $p_i^m = P_i(q^*)$. The firms' profits are

$$\pi_A = [P_A(q^*) - F_A - c_A - \Delta p_A H(\Delta p_A)](1 - G(\bar{q})),$$
$$\pi_B = [P_B(q^*) - F_B - c_B - \Delta p_B H(\Delta p_B)]G(\bar{q}).$$

Note that M's recommendation still depends on the actual cutoff $\bar{q}$, which depends on the effective commissions, $F_i = (1 - H(\Delta p_i))f_i$ for $i = A, B$.

Since $\bar{q}$ depends on F_i but not on Δp_i, and since q^* depends on neither F_i nor Δp_i, conditional on its choice of effective commission, each firm's profit from these customers depends negatively on Δp_i. Therefore, each firm i does best by setting $\Delta p_i = 0$, which implies $F_i = f_i$. The logic is the same as the case with observable commissions.

Given there is no direct purchase, Section III of Inderst and Ottaviani (2012a) characterizes the equilibrium which they show is the unique pure strategy equilibrium. Thus, we have established that[10]

Proposition 2. *When firms set unobservable commissions, if consumers hold naive beliefs, there is a unique pure strategy equilibrium in which firms do not offer any discounts for direct*

[9] The assumption of naive beliefs is equivalent to consumers believing that when they observe a deviation in a firm's discount Δp_i from the equilibrium level, the firm has changed its unobserved commission f_i to keep the effective commission F_i paid to M unchanged.

[10] This same result also holds if consumers are "naive" in the sense of Inderst and Ottavanni (2012b). Specifically, suppose consumers naively assume M sets the cutoff equal to $1/2$. When commissions are observable, naive consumers are irrational whenever effective commissions are unequal since M's cutoff will no longer be $1/2$ in this case. When commissions are unobservable, such consumers are irrational only if the firms' effective commissions are unequal in equilibrium, which happens if the firms are asymmetric (i.e. $c_A < c_B$). In either case, the proof in this section remains valid once we replace $P_i(q^*)$ with $P_i(1/2)$ for $i = A, B$ everywhere.

sales.

4.2.2 Sophisticated beliefs

When a firm sets its commission, it should have in mind the intermediated price and direct price it is also setting at the same time. For this reason, a rational firm would make the same choices for its observable prices and unobservable commissions irrespective of what order it actually sets them in (including the case it sets them at the same time). A sophisticated consumer, realizing this, would form its beliefs about the firms' unobservable commissions (and so the implied cutoff for q) based on this invariance principle. In and Wright (2018) call this approach "Reordering Invariance" and argue it is a way to characterize the reasonable equilibria of games of this type. They show that the equilibrium outcomes associated with such reasonable equilibria can be obtained by finding the equilibrium outcomes of a hypothetical reordered game in which observable actions are chosen before unobservable actions. We adopt this approach by solving a reordered game in which in stage 1a, each firm sets its prices, and in stage 1b, each firm sets its commission without observing the rival's prices from stage 1a.

Consider solving for the equilibrium in the reordered game, in which a candidate equilibrium has commissions equal to f_A^*, f_B^* and prices equal to $p_A^m = P_A(q^*)$, $p_B^m = P_B(q^*)$, Δp_A^*, and Δp_B^*. In stage 1b, given its choice of p_A^m and Δp_A, firm A chooses f_A to maximize its profit

$$\pi_A = [p_A^m - (1 - H(\Delta p_A))f_A - c_A - \Delta p_A H(\Delta p_A)](1 - G(\bar{q}_A)), \tag{8}$$

where M's cutoff is

$$\bar{q}_A = \frac{1}{2} + \frac{(1 - H(\Delta p_B^*))f_B^* - (1 - H(\Delta p_A))f_A}{2w}.$$

A similar argument applies for firm B's choice of f_B. For each Δp_A, where $0 \leq \Delta p_A < S$, taking the derivative of π_A with respective to f_A, we have

$$\frac{d\pi_A}{df_A} = \frac{(1 - H(\Delta p_A))g(\bar{q}_A)}{2w} m_A,$$

where

$$m_A = p_A^m - (1 - H(\Delta p_A)) f_A - c_A - \Delta p_A H(\Delta p_A) - \frac{2w(1 - G(\bar{q}_A))}{g(\bar{q}_A)} \tag{9}$$

is decreasing in f_A given the assumption that $G(q)$ has an increasing hazard rate. Denote the unique commission that solves $m_A = 0$ to be $f_A(p_A^m, \Delta p_A)$, where we set $f_A(p_A^m, \Delta p_A) = 0$ if it is negative. Therefore, we must have $\frac{d\pi_A}{df_A} > 0$ if and only if $0 \leq f_A < f_A(p_A^m, \Delta p_A)$

when $f_A(p_A^m, \Delta p_A) > 0$, and $\frac{d\pi_A}{df_A} < 0$ for $f_A > f_A(p_A^m, \Delta p_A)$. Then $f_A(p_A^m, \Delta p_A)$ must be the optimal commission that maximizes firm A's profit π_A. Note that for any given $\Delta p_A < S$, $f_A(p_A^m, \Delta p_A)$ is increasing in p_A^m, which again follows from the increasing hazard rate property of $G(q)$.

In stage 1a, for any given Δp_A, since firm A's profit is strictly increasing in p_A^m subject to $p_A^m \leq P_A\left(q_A^e\right)$, where

$$q_A^e = \frac{1}{2} + \frac{(1 - H(\Delta p_B^*))f_B^* - (1 - H(\Delta p_A))f_A(p_A^m, \Delta p_A)}{2w}, \tag{10}$$

it is optimal for firm A to set $p_A^m = P_A(q_A^e)$. For any given Δp_A, denote the unique p_A^m that solves $p_A^m = P_A(q_A^e)$ to be $p_A^m(\Delta p_A)$. To show that $p_A^m(\Delta p_A)$ is uniquely defined, it is sufficient to show that $P_A(q_A^e)$ is decreasing in p_A^m. To see this, note that $P_A(q_A^e)$ is increasing in q_A^e from (5), q_A^e is decreasing in $f_A(p_A^m, \Delta p_A)$, and finally $f_A(p_A^m, \Delta p_A)$ is increasing in p_A^m as noted above. We further denote q_A^e defined in (10) by $q_A^e(\Delta p_A)$ when we replace p_A^m with $p_A^m(\Delta p_A)$, and denote $f_A(p_A^m, \Delta p_A)$ by $f_A(\Delta p_A)$ when we also replace p_A^m with $p_A^m(\Delta p_A)$.

Replacing p_A^m in (8) and (9) with $P_A(q_A^e(\Delta p_A))$, and differentiating (8) with respect to Δp_A, and using the envelope theorem to ignore the effect through the f_A set in stage 1b, we obtain

$$\frac{\partial \pi_A}{\partial \Delta p_A} = \left(\underbrace{\frac{dP_A(q_A^e(\Delta p_A))}{dq_A^e(\Delta p_A)} \frac{dq_A^e(\Delta p_A)}{d\Delta p_A}}_{\text{signaling effect}} - \underbrace{\Delta p_A h(\Delta p_A) - H(\Delta p_A)}_{\text{commission-saving effect}} \right)(1 - G(\bar{q}_A)).$$

If $\frac{\partial \pi_A}{\partial \Delta p_A} \leq 0$ for any Δp_A, then firm A would not want to offer any discount in equilibrium. Intuitively, an increase in the discount for direct sales has two effects on firm A's profit. On the one hand, it increases the expected quality of firm A's product, and so enables the firm to charge more for its intermediated transactions (the signaling effect). On the other hand, an increase in the discount implies the firm obtains less from inframarginal consumers who would anyway buy directly (commission-saving effect). Hence, the net effect of offering a direct discount on firm A's profit depends on which of these two effects dominates. If the signaling effect is always dominated, then firm A would never want to offer any discount.

To proceed, we focus on the case that $G(\cdot)$ is the uniform distribution over $[0, 1]$. It can then be confirmed that when the optimal commission $f_A(\Delta p_A)$ is strictly positive, $\frac{dq_A^e(\Delta p_A)}{d\Delta p_A} = \frac{2(\Delta p_A h(\Delta p_A) + H(\Delta p_A))}{(8w + v_h - v_l)}$ and

$$\frac{\partial \pi_A}{\partial \Delta p_A} = -\frac{8w(\Delta p_A h(\Delta p_A) + H(\Delta p_A))}{(8w + v_h - v_l)}(1 - \bar{q}_A).$$

When the optimal commission $f_A(\Delta p_A)$ is zero, $\frac{dq_A^e(\Delta p_A)}{d\Delta p_A} = 0$, and

$$\frac{\partial \pi_A}{\partial \Delta p_A} = -(\Delta p_A h(\Delta p_A) + H(\Delta p_A))(1 - \bar{q}_A).$$

In both cases, we can verify that $\frac{\partial \pi_A}{\partial \Delta p_A} \leq 0$ for any Δp_A, so that firm A will not want to offer any positive discount in equilibrium. A similar analysis applies for firm B.

Given there is no direct purchase, Section III of Inderst and Ottaviani (2012a) characterizes the equilibrium which they show is the unique pure strategy informative equilibrium. Thus, we have established[11]:

Proposition 3. *When firms set unobservable commissions, if consumers hold sophisticated beliefs (as defined above) and G is the uniform distribution on $[0, 1]$, there is a unique pure strategy equilibrium in which firms do not offer any discounts for direct sales.*

This result reflects that although a higher discount does lead consumers to believe that the effective commission received by the recommended firm is lower, this positive effect on a firm's profit is still not strong enough to offset the negative effect of offering a discount (that a discount for direct sales is an inefficient way of reducing effective commissions given a discount is not targeted at intermediated transactions).[12]

4.3 Price-parity clauses

Suppose the intermediary actually impose a price-party clause. Given the positive shopping cost in our model, all buyers would always want to purchase through the intermediary so that no buyer would ever want to purchase directly. The result is exactly equivalent to the setting in which direct sales are not allowed in Inderst and Ottaviani (2012a). Since we showed in each of the cases in Sections 4.1-4.2.2, the equilibrium that arises when firms set commissions is identical to the corresponding case in Inderst and Ottaviani (2012a) without direct sales, introducing a price-party clause has no effect on the resulting equilibrium. Thus, we immediately obtain the following result.

Proposition 4. *When firms sets commissions, the intermediary has no reason to impose a price-parity clause that rules out firms setting discounts for direct sales.*

[11] The full workings to establish these results when G is the uniform distribution are given in the appendix.

[12] In Section E of the online appendix, we show that for beliefs that lie between naive beliefs and the sophisticated beliefs above, the positive effect of a higher discount on a firm's profit is even weaker, and so the result in Proposition 3 continues to hold.

5 Intermediary sets commissions

In this section we assume that the intermediary rather than firms set commissions. With M setting commissions, firms can no longer offset a lower discount with a lower commission. In contrast to the previous section, we show firms will now always want to set positive discounts for direct sales in equilibrium, and showrooming will become a problem for M. We then show M can do better imposing a price-parity clause as part of its contract with the firms to rule out such discounting.

5.1 Observable commissions

Suppose commissions become public at the point that consumers make their decisions. Recall, M's expected payoff from recommending firm A is $[1-H(\Delta p_A)]f_A+qw_h+(1-q)w_l$, and from recommending firm B is $[1-H(\Delta p_B)]f_B+(1-q)w_h+qw_l$. The expected payoff for M from an individual consumer based on a cutoff of $\bar{q}$, assuming consumers will follow M's recommendation (which they will do in equilibrium), is

$$\begin{aligned}\Pi_M &= \int_{\bar{q}}^{1}\left[\left(\left(1-H(\Delta p_A)\right)f_A+qw_h+(1-q)w_l\right)g\left(q\right)\right]dq \\ &\quad+\int_{0}^{\bar{q}}\left[\left(\left(1-H(\Delta p_B)\right)f_B+(1-q)w_h+qw_l\right)g\left(q\right)\right]dq.\end{aligned}$$

Differentiating with respect to the choice of cutoff $\bar{q}$ gives the usual formula

$$\bar{q}=\frac{1}{2}+\frac{F_B-F_A}{2w},$$

where $F_i=(1-H(\Delta p_i))f_i$ for $i=A,B$.

When differentiating Π_M with respect to f_i, we can ignore the effect through $\bar{q}$ due to the envelope theorem. Therefore, M's choice of f_i is to maximize

$$\int_{\bar{q}}^{1}\left[F_A g\left(q\right)\right]dq+\int_{0}^{\bar{q}}\left[F_B g\left(q\right)\right]dq,$$

holding $\bar{q}$ as given, subject to firms' participation constraints. Provided both firms are making sales, the only thing constraining commissions is therefore that firms could discount more if commissions are higher.

Note since each firm does not observe the commission charged to the other firm at the time it sets its prices, firm B's choice of Δp_B cannot depend on f_A and firm A's choice of Δp_A cannot depend on f_B. Therefore f_i just maximizes $F_i=\left(1-H(\Delta p_i)\right)f_i$. When firms' participation constraints are not binding, f_i is determined by the first-order condition

$$f_i = \frac{(1 - H(\Delta p_i))}{h\left(\Delta p_i\right)} \frac{1}{\frac{d\Delta p_i}{df_i}}. \tag{11}$$

Consider firm A's problem of setting p_A^m and Δp_A. Since firm A does not observe f_B, the perceived demand faced by firm A depends on the firm's expectation of the cutoff $\bar{q}$, denoted by $\hat{q}_A$, where

$$\hat{q}_A = \frac{1}{2} + \frac{(1 - H(\Delta\widehat{p}_B))\hat{f}_B - (1 - H(\Delta p_A))f_A}{2w},$$

and $\hat{f}_B$ is firm A's expectation of f_B. We suppose firm A's expectation of f_B is fixed at the equilibrium level f_B^* so it does not depend on the f_A the firm observes. This is natural given (11) implies M's optimal commission for each firm is independent of the commission charged to the other firm (note $\frac{d\Delta p_i}{df_i}$ cannot depend on f_j since f_j is not observed by firm i when setting Δp_i). For the same reason, firm A's expectation of Δp_B should not depend on the observed f_A. Therefore we have $\Delta\widehat{p}_B = \Delta p_B^*$, and

$$\hat{q}_A = \frac{1}{2} + \frac{(1 - H(\Delta p_B^*))f_B^* - (1 - H(\Delta p_A))f_A}{2w}.$$

Then firm A's expected profit becomes

$$\pi_A = [P_A(\hat{q}_A) - c_A - (1 - H\left(\Delta p_A\right)) f_A - \Delta p_A H(\Delta p_A)](1 - G(\hat{q}_A)),$$

where

$$P_A(\hat{q}_A) = \int_{\hat{q}_A}^{1} v_A(q) \frac{g(q)}{1 - G(\hat{q}_A)} dq. \tag{12}$$

Here when setting its price p_A^m, firm A has to form an expectation of how much consumers will be willing to pay. It does so based on its expectation of f_B and Δp_B, which is why we use $\hat{q}_A$ still even though consumers actually observe f_B and Δp_B. Therefore, the first-order condition for firm A's choice of discount is given by

$$\begin{aligned} \frac{d\pi_A}{d\Delta p_A} &= \left[\frac{dP_A(\hat{q}_A)}{d\hat{q}_A}\frac{h(\Delta p_A)f_A}{2w} + (f_A - \Delta p_A)h(\Delta p_A) - H(\Delta p_A)\right](1 - G(\hat{q}_A)) \\ &\quad -[P_A(\hat{q}_A) - c_A - (1 - H\left(\Delta p_A\right)) f_A - \Delta p_A H(\Delta p_A)]g(\hat{q}_A)\frac{h(\Delta p_A)f_A}{2w} \leq 0, \end{aligned} \tag{13}$$

where the equality holds if firm A's choice of discount is strictly positive, i.e. $\Delta p_A > 0$. For some levels of the commission f_A, it is possible that firm A will optimally choose $\Delta p_A = 0$ so that the inequality in (13) holds. However, it can never happen in any equilibrium, since in that case M can always increase f_A without affecting firm A's choice of Δp_A (which remains zero). Therefore, for the analysis below, we just need to focus on the case that (13) holds with equality.

Provided $(f_A - \Delta p_A)h(\Delta p_A) > H(\Delta p_A)$ (i.e. starting from no discount or a low discount), a higher discount for direct sales reduces the amount firm A can expect to pay in total commissions and discounts. It also raises $\hat{q}_A$ and so the amount firm A expects consumers are willing to pay for its product. Both factors allow firm A to increase its profit margin on each sale—the first line in (13). On the other hand, a higher discount will reduce the probability the firm is recommended to a consumer, which will reduce the firm's profit based on its equilibrium margin—the second line in (13). Totally differentiating (13) with respect to f_A and Δp_A to obtain $d\Delta p_A/df_A$ (and similarly for firm B), we can show the overall effect of offering a discount on firm A's profit is positive. Thus, we obtain the following proposition (the proof is in the appendix).

Proposition 5. *When M sets observable commissions, firms set positive discounts for direct sales.*

5.2 Unobservable commissions

Suppose M's commissions are not observed by consumers. The conditions that characterize profits, commissions and optimal discounts are the same as those in Section 5.1 given that in both cases we assume each firm cannot observe what commission their rival is charged. The only difference is that consumers' expectation of the cutoff is now based on the expected commissions for both firms. This affects how much consumers are willing to pay for a good. Consumers' conditional valuation of firm A's product if it is recommended is no longer (12) but instead is based on their expectation of the cutoff. With naive beliefs, this remains fixed at the equilibrium level.[13] This means firm A's incentive to increase its discount Δp_A given by the derivative (13) is now equal to

$$\begin{aligned} \frac{d\pi_A}{d\Delta p_A} &= \left[(f_A - \Delta p_A)h(\Delta p_A) - H(\Delta p_A)\right](1 - G(\hat{q}_A)) \\ &\quad - [P_A(q^*) - c_A - (1 - H(\Delta p_A)) f_A - \Delta p_A H(\Delta p_A)] g(\hat{q}_A) \frac{h(\Delta p_A) f_A}{2w}. \end{aligned} \tag{14}$$

[13]Note we can no longer apply the invariance principle used in Section 4.2.2 in this case since it is a different party that sets prices from the one setting commissions. However, note in Section E of the online appendix we show that for other types of beliefs in which consumers interpret a higher discount to mean any firm that is still recommended is more likely to have a suitable product, firms will have even more reason to set positive discounts than with naive beliefs (provided a mild additional condition is satisfied).

Following a similar logic to that used in Section 5.1, the derivative $d\pi_A/d\Delta p_A$ must be equal to zero in any equilibrium.

Compared to the case with observable commissions, setting a higher discount no longer increases consumers' expected valuation of a firm's product, when consumers hold naive beliefs. This implies that firms have less incentive to set positive discounts compared to the case with observable commissions. Despite this, following a similar proof (see the appendix) to that for Proposition 5, we obtain[14]:

Proposition 6. *When M sets unobservable commissions, if consumers hold naive beliefs, firms set positive discounts for direct sales.*

5.3 Price-parity clauses

Given that firms set positive discounts for direct sales when M sets commissions, in this section we consider whether M could do better imposing a price-parity clause that rules out such discounts. We will show M is always better off doing so.

With a price-parity clause imposed, consider M's optimal choice of commissions. First we know that M always wants to set f_A and f_B such that $0 < \bar{q} < 1$, i.e. both firms are recommended with positive probability and thus capture a positive market share. Otherwise neither product can be sold given (1). Then M's maximization problem becomes

$$\max_{\bar{q}, f_A, f_B} \int_{\bar{q}}^{1} \left[\left(f_A + q w_h + (1-q) w_l \right) g\left(q\right) \right] dq + \int_{0}^{\bar{q}} \left[\left(f_B + (1-q) w_h + q w_l \right) g\left(q\right) \right] dq,$$

subject to the firms' break-even constraints

$$\begin{aligned} (P_A(q^e) - f_A - c_A)(1 - G(\bar{q})) &\geq 0, \\ (P_B(q^e) - f_B - c_B)G(\bar{q}) &\geq 0. \end{aligned}$$

Note $q^e = \bar{q}$ when commissions are observable, $q^e = q^*$ when commissions are unobservable and consumers hold naive beliefs, and $q^e = 1/2$ when consumers are naive.

The two constraints must both be binding at the optimum, since otherwise M can always increase one of the commissions and adjust the cutoff $\bar{q}$ accordingly to obtain a higher surplus. By eliminating the ability of firms to respond to high commissions with positive discounts, M is able to raise commissions to the point that each firm's profit margin becomes zero.

[14] If instead consumers are naive, and simply assume M always sets the cutoff equal to $1/2$, then regardless of whether commissions are observable or not, the proof of Proposition 6 remains valid once we replace $P_i\left(q^*\right)$ with $P_i\left(1/2\right)$ for $i = A, B$ everywhere.

That is, M is able to fully extract each firm's surplus. In equilibrium, the commissions are determined by

$$f_i^P = P_i(q^P) - c_i,$$

for $i = A, B$ and $q^P = \frac{1}{2} + \frac{f_B^P - f_A^P}{2w}$. We must have $f_A^P \geq f_B^P$, where the equality holds if and only if $c_A = c_B$, and $f_A^P > 0$.

We denote M's equilibrium profit by Π_M^P, where

$$\Pi_M^P = \int_{q^P}^{1} \left[\left(f_A^P + q w_h + (1-q) w_l \right) g\left(q\right) \right] dq + \int_0^{q^P} \left[\left(f_B^P + (1-q) w_h + q w_l \right) g\left(q\right) \right] dq.$$

We claim that M is no worse-off imposing a price-parity clause. Without the price-parity clause, suppose the equilibrium is $(P_A(q^*), P_B(q^*))$, $(\Delta p_A^*, \Delta p_B^*)$ and (f_A^*, f_B^*). In equilibrium, firm i's profit must be non-negative, i.e.

$$P_i(q^*) - (1 - H(\Delta p_i^*)) f_i^* - c_i - \Delta p_i^* H(\Delta p_i^*) \geq 0.$$

By imposing the price-parity clause, M can always set $f_i' = (1 - H(\Delta p_i^*)) f_i^*$ so as to keep its recommendation unchanged. With the commissions (f_A', f_B') set, firm i's profit cannot be lower and will be strictly higher if $\Delta p_i^* > 0$. Thus, M's profit cannot be lower given that

$$\begin{aligned} \Pi_M^P &\geq \int_{q^*}^{1} \left[\left(f_A' + q w_h + (1-q) w_l \right) g\left(q\right) \right] dq + \int_0^{q^*} \left[\left(f_B' + (1-q) w_h + q w_l \right) g\left(q\right) \right] dq, \\ &= \int_{q^*}^{1} \left[\left((1 - H(\Delta p_A^*)) f_A^* + q w_h + (1-q) w_l \right) g\left(q\right) \right] dq \\ &\quad + \int_0^{q^*} \left[\left((1 - H(\Delta p_B^*)) f_B^* + (1-q) w_h + q w_l \right) g\left(q\right) \right] dq. \end{aligned}$$

And indeed M is strictly better-off whenever the equilibrium without the price-parity clause is not exactly identical to the one with price-parity clause. We show this next by showing that if M sets the above commissions f_i^P, at least one firm will want to offer a positive discount if it is not restricted from doing so.

Consider M removing its price-parity clause, while still setting the commissions f_i^P. Consider firm A, for example. Taking the derivative of π_A with respect to Δp_A and evaluating at $\Delta p_A = 0$, we have

$$\left. \frac{d\pi_A}{d\Delta p_A} \right|_{\Delta p_A = 0} = \left(\left. \frac{dP_A(q_A^e)}{dq_A^e} \right|_{q_A^e = q^P} \frac{1}{2w} + 1 \right) h(0) f_A^P \left(1 - G(q^P) \right).$$

Since $\frac{dP_A\left(q_A^e\right)}{dq_A^e}\Big|_{q_A^e=q^P}$ is positive when commissions are observable, and zero when commissions are unobservable and consumers hold naive beliefs, or when consumers are naive, we always have $\frac{d\pi_A}{d\Delta p_A}\Big|_{\Delta p_A=0} > 0$ since $f_A^P > 0$, and firm A wants to offer a positive discount. Thus, we have shown:

Proposition 7. *When M sets commissions, it will always do better imposing a price-parity clause that rules out firms setting discounts for direct sales.*

In Edelman and Wright (2015b), price-parity clauses work by making all buyers share the intermediary's commissions fees to sellers, thereby raising the price for direct sales, and so shifting more consumers to buy through the intermediary. In Wang and Wright (2017), price-parity clauses work by (i) closing down substitution to the direct channel by ensuring consumers have no incentive to search directly and (ii) ensuring there is no incentive for consumers to search on the intermediary and then switch to buy directly. In either case, price-parity clauses raise prices and shift surplus from consumers to the intermediary, but firms' profits are unaffected. In our setting, firms price as monopolists having attracted consumers through recommendations, and so the main purpose of price-parity clauses is just to prevent firms' free riding on advice by discounting direct sales (this is similar to the second mechanism in Wang and Wright). In contrast to these previous papers and consistent with firms' complaints about these clauses, we find the main effect of price-parity clauses is to shift surplus from firms to the intermediary. Consumers who previously purchased directly are also worse off due to price-parity clauses (without price-parity clauses they received a discount from the monopoly price). On the other hand, the imposition of price-parity clauses actually increases *total* welfare in our setting given it prevents consumers from using the socially costly direct sales channel. This reflects that consumers incur costs to switch to the direct channel which will be avoided if firms cannot set discounts for direct purchases.

6 Concluding remarks

This paper has three main contributions. It provides an explanation for the puzzle of why firms sometimes do not free ride on the advice of an intermediary by discounting for direct sales. It also provides a theory to predict in which market settings an intermediary will want to impose price-parity clauses to prevent such free riding and in which market settings it will not. Finally, it provides a new theory of price-parity clauses which has as its main effect that surplus is shifted from firms to the intermediary.

The theory predicts that when firms compete by setting the commissions they pay an intermediary that provides advice, they have no incentive to offer a discount to attract consumers to switch to buy directly. This is because providing discounts for direct sales is dominated by lowering the commission firms pay the intermediary for intermediated sales. Discounts involve a reduction in the margin on inframarginal consumers who would buy directly anyway. Unlike commissions, this loss in revenue does not help incentivize the intermediary to recommend the firm more often. With firms choosing not to offer discounts for direct sales, there is no need for the intermediary to impose price-parity clauses. On the other hand, if the intermediary is the party determining commissions, the firm can no longer offset a lower discount with a lower commission. In this case, our theory predicts firms will discount direct sales in response to high commissions, thereby free riding on the intermediary's advice. The intermediary can do better if it can rule this out by imposing price-parity clauses, thereby shifting surplus from firms to the intermediary.

Since our theory predicts that the use of discounting for direct sales (or price-parity clauses to prevent such discounting) is more likely when intermediaries set commissions, it is natural to ask in what situations one party or the other would set commissions. While we leave an analysis of this issue to future research, we think one plausible explanation runs as follows.

For some types of advice, the intermediary needs to meet with the consumer to determine the consumer's ideal match. For example, doctors meet patients for discussions, physical examinations, and to conduct tests. As a result of time constraints, each doctor can only handle a relatively small number of patients. This means there will be many doctors relative to the number of competing firms providing a particular pharmaceutical product. A similar situation exists for insurance brokers, financial advisors or other offline experts. With few firms relative to intermediaries, it is natural that firms set commissions, which based on our theory would explain why for these types of intermediaries we do not observe discounting of direct sales or the use of price-parity clauses. Note while our model assumes there is a single intermediary, one could think of this as representing one of many intermediaries that serves its own small set of "local" consumers.

On the other hand, for recommendations that can be provided through an online platform (e.g. based on online product search queries), one intermediary may be able to serve all consumers. In such a setting, it is natural that intermediaries set commissions, which based on our theory would explain why for online platforms we do tend to see the use of price-parity clauses to prevent firms offering discounts for direct sales (see Edelman and Wright, 2015a). Thus, the technology used to provide advice may ultimately explain why sometimes an intermediary needs to impose price-parity clauses to rule out free riding on its advice and other times it does not need to. It may also explain why, as technology has evolved towards advice increasingly being offered through online platforms, free riding and price-parity clauses have become more prominent concerns.

We have demonstrated the robustness of our results to whether consumers observe commissions or not, although in the latter case, only under some restrictions on beliefs which ensure that the signaling effect of discounts is not too strong. Moreover, as discussed in the literature review, our results also hold in market settings in which steering is not possible such as when consumers have full information about the quality match, although for more trivial reasons. There are, however, some settings in which our results could change. If firms can offer private discounts to individual consumers that are not observed by the intermediary, then firms will want to offer discounts for direct sales even when they set commissions. However, the logic of our paper based on effective commissions should be restored provided the intermediary can observe these discounts with some probability. On the other hand, if consumers cannot observe discounts before switching, firms would not want to offer discounts which would represent a pure give-away to consumers. This is true regardless of which party sets commissions. Thus, settings in which consumers have to incur significant search costs to find out direct prices may provide an alternative explanation for the lack of direct discounts. If in online settings these search costs are much less important than in offline settings, this provides an alternative story for why free riding and price-parity clauses arise online but not so much offline. Future work could try to provide evidence to test between our explanation and this alternative.

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Appendix: Proofs

Proof that M always makes a recommendation. We want to show that the assumption $w_l + w_h \geq 0$ ensures M always makes a recommendation in equilibrium.

Consider any equilibrium profile, (f_A^*, f_B^*), (p_A^m, p_B^m), $(\Delta p_A^*, \Delta p_B^*)$, and q^*, where

$$q^* = \frac{1}{2} + \frac{(1 - H(\Delta p_B^*))f_B^* - (1 - H(\Delta p_A^*))f_A^*}{2w}.$$

Since M will recommend A whenever $q \geq q^*$, the lowest payoff M gets when recommending A is

$$[1 - H(\Delta p_A^*)]f_A^* + q^* w_h + (1 - q^*)w_l.$$

Similarly, the lowest payoff M gets when recommending B is

$$[1 - H(\Delta p_B^*)]f_B^* + (1 - q^*)w_h + q^* w_l.$$

Therefore, to ensure M always gets a non-negative payoff from making a recommendation, we need

$$[1 - H(\Delta p_A^*)]f_A^* + [1 - H(\Delta p_B^*)]f_B^* + w_l + w_h \geq 0,$$

which is clearly implied by the assumption $w_l + w_h \geq 0$. □

Proof of Proposition 3. We directly derive the equilibrium results assuming $G(q) = q$ on $[0, 1]$.

Consider firm A's problem. In stage 1b, given its choice of p_A^m and Δp_A, firm A chooses f_A to maximize its profit

$$\begin{aligned} \pi_A &= \left(p_A^m - (1 - H(\Delta p_A))\, f_A - c_A - \Delta p_A H(\Delta p_A)\right) \\ &\quad \times \left(1 - \left(\frac{1}{2} + \frac{(1 - H(\Delta p_B^*))\, f_B^* - (1 - H(\Delta p_A))\, f_A}{2w}\right)\right). \end{aligned}$$

The optimal commission $f_A(p_A^m, \Delta p_A)$ is

$$\begin{aligned} f_A(p_A^m, \Delta p_A) &= \arg\max_{f_A} \pi_A \\ &= \frac{p_A^m - c_A - w - \Delta p_A H(\Delta p_A) + (1 - H(\Delta p_B^*))\, f_B^*}{2\,(1 - H(\Delta p_A))}, \end{aligned} \tag{15}$$

when this is strictly positive, and otherwise is $f_A(p_A^m, \Delta p_A) = 0$.

In stage 1a, for any given Δp_A, firm A sets p_A^m to maximize π_A subject to

$$p_A^m \leq \frac{(1 + q_A^e)v_h + (1 - q_A^e)v_l}{2}, \tag{16}$$

where

$$q_A^e \;=\; \frac{1}{2} + \frac{(1-H(\Delta p_B^*))\, f_B^* - (1-H(\Delta p_A))\, f_A(p_A^m, \Delta p_A)}{2w}, \tag{17}$$

Substituting the positive $f_A(p_A^m, \Delta p_A)$ obtained from (15) into (17) yields

$$q_A^e = \frac{1}{2} + \frac{(1-H(\Delta p_B^*))\, f_B^* - p_A^m + c_A + w + \Delta p_A H(\Delta p_A)}{4w}, \tag{18}$$

which from (16) implies that

$$p_A^m \leq \frac{3v_h + v_l}{4} + \frac{\Delta v\left[(1-H(\Delta p_B^*))\, f_B^* - p_A^m + c_A + w + \Delta p_A H(\Delta p_A)\right]}{8w},$$

where $\Delta v = v_h - v_l$. Thus, it is optimal for firm A to set p_A^m such that the above constraint is binding, which leads to

$$p_A^m(\Delta p_A) = \frac{v_h + v_l}{2} + \frac{\Delta v}{8w + \Delta v}\left((1-H(\Delta p_B^*))\, f_B^* + c_A + \Delta p_A H(\Delta p_A) + 3w - \frac{(v_h + v_l)}{2}\right). \tag{19}$$

Substituting (19) into (15) and (18) yields

$$q_A^e(\Delta p_A) \;=\; \frac{2\left((1-H(\Delta p_B^*))\, f_B^* + c_A + \Delta p_A H(\Delta p_A) + 3w\right) - (v_h + v_l)}{8w + \Delta v}, \tag{20}$$

$$f_A(\Delta p_A) \;=\; \frac{w}{(1-H(\Delta p_A))(8w + \Delta v)}\left(\begin{array}{c} 3v_h + v_l - 4\left(w + c_A + \Delta p_A H(\Delta p_A)\right) \\ +\left(4 + \frac{\Delta v}{w}\right)(1-H(\Delta p_B^*))\, f_B^* \end{array}\right).$$

This implies $\frac{\partial q_A^e(\Delta p_A)}{\partial \Delta p_A} = \frac{2(\Delta p_A h(\Delta p_A) + H(\Delta p_A))}{(8w + \Delta v)}$ as noted in Section 4.2.2. Note that $f_A(\Delta p_A)$ is strictly positive if and only if

$$\frac{3v_h + v_l}{4} + \left(1 + \frac{\Delta v}{4w}\right)(1-H(\Delta p_B^*)) f_B^* - \Delta p_A H(\Delta p_A) > w + c_A. \tag{21}$$

Otherwise, since $f_A(\Delta p_A) = 0$, we get

$$q_A^e(\Delta p_A) = \frac{1}{2} + \frac{(1-H(\Delta p_B^*))\, f_B^*}{2w},$$

and

$$p_A^m(\Delta p_A) = \frac{3v_h + v_l}{4} + \frac{\Delta v(1-H(\Delta p_B^*)) f_B^*}{4w}.$$

If (21) holds so that $f_A(\Delta p_A) > 0$, firm A's profit is

$$\pi_A = \frac{8w}{(8w+\Delta v)^2}\left(v_h + w - c_A - (1 - H(\Delta p_B^*))\, f_B^* - \Delta p_A H(\Delta p_A)\right)^2.$$

From (20), we have

$$1 - q_A^e(\Delta p_A) = \frac{2}{8w+\Delta v}\left(v_h + w - c_A - (1 - H(\Delta p_B^*))\, f_B^* - \Delta p_A H(\Delta p_A)\right),$$

and firm A's profit function can be rewritten as

$$\pi_A = 2w(1 - q_A^e(\Delta p_A))^2.$$

Since we must have $1 - q_A^e(\Delta p_A) > 0$, i.e. firm A's demand should be positive, firm A will maximize π_A by setting $\Delta p_A = 0$.

If (21) does not hold and $f_A(\Delta p_A) = 0$, then firm A's profit is

$$\pi_A = \left(\frac{3v_h + v_l}{4} + \frac{\Delta v(1 - H(\Delta p_B^*))f_B^*}{4w} - c_A - \Delta p_A H(\Delta p_A)\right)\left(\frac{1}{2} - \frac{(1 - H(\Delta p_B^*))f_B^*}{2w}\right),$$

which is also maximized at $\Delta p_A = 0$.

A similar argument applies to firm B. Therefore, price parity holds for both firms in equilibrium. □

Proof of Proposition 5. We want to show that both firms must offer positive discounts in equilibrium. Suppose to the contrary there exists an equilibrium in which price parity holds for firm A, i.e. $f_A^*, f_B^* \geq 0$, $\Delta p_A^* = 0$ and $\Delta p_B^* \geq 0$. We need to consider two cases.

First, suppose the equilibrium involves an interior solution, so that it satisfies (11) for firm A. From (11), we know that setting $f_i = 0$ is never optimal for M. Thus, we just need to focus on $f_i > 0$. Following the argument in Section 5.1, the optimal choice of Δp_A by firm A in equilibrium is given by (13), which holds with equality. It can further be written as

$$\begin{aligned}\Phi(f_A, \Delta p_A) \equiv\; & v_A(\hat{q}_A) - c_A - (1 - H(\Delta p_A))f_A - \Delta p_A H(\Delta p_A) \\ & - \left(1 - \frac{1}{f_A}\left(\Delta p_A + \frac{H(\Delta p_A)}{h(\Delta p_A)}\right)\right)\frac{2w(1 - G(\hat{q}_A))}{g(\hat{q}_A)} = 0,\end{aligned} \tag{22}$$

where we have used (5).

Totally differentiating (22) with respect to f_A and Δp_A yields

$$\frac{\partial \Phi}{\partial f_A} df_A + \frac{\partial \Phi}{\partial \Delta p_A} d\Delta p_A = 0,$$

where after evaluating $\frac{\partial \Phi}{\partial f_A}$ and $\frac{\partial \Phi}{\partial \Delta p_A}$ at the proposed equilibrium in which $\Delta p_A = \Delta p_A^* = 0$ we have

$$\begin{aligned}
\frac{\partial \Phi}{\partial f_A} &= -\frac{3}{2} + \left(\frac{d}{d\hat{q}_A} \frac{(1-G(\hat{q}_A))}{g(\hat{q}_A)} \right) \bigg|_{\hat{q}_A = q^*}, \\
\frac{\partial \Phi}{\partial \Delta p_A} &= h(0) f_A^* \left(\frac{3}{2} + \frac{4w(1-G(q^*))}{(f_A^*)^2 h(0) g(q^*)} - \left(\frac{d}{d\hat{q}_A} \frac{(1-G(\hat{q}_A))}{g(\hat{q}_A)} \right) \bigg|_{\hat{q}_A = q^*} \right).
\end{aligned}$$

Thus,

$$\frac{d\Delta p_A}{df_A} = -\frac{\frac{\partial \Phi}{\partial f_A}}{\frac{\partial \Phi}{\partial \Delta p_A}} = \frac{\frac{3}{2} - \left(\frac{d}{d\hat{q}_A} \frac{(1-G(\hat{q}_A))}{g(\hat{q}_A)} \right) \big|_{\hat{q}_A = q^*}}{\frac{3}{2} - \left(\frac{d}{d\hat{q}_A} \frac{(1-G(\hat{q}_A))}{g(\hat{q}_A)} \right) \big|_{\hat{q}_A = q^*} + \frac{4w(1-G(q^*))}{(f_A^*)^2 h(0) g(q^*)}} \frac{1}{h(0) f_A^*},$$

which implies that

$$0 < \frac{d\Delta p_A}{df_A} < \frac{1}{h(0) f_A^*}, \tag{23}$$

given $\frac{(1-G(\hat{q}_A))}{g(\hat{q}_A)}$ is decreasing in $\hat{q}_A$ following the increasing hazard rate of $G(q)$.

It follows that f_A^* cannot maximize M's profit, i.e. f_A^* does not solve (11). This is because M is always better off setting f_A above f_A^* reflecting that

$$\begin{aligned}
\frac{d[(1-H(\Delta p_A)) f_A]}{df_A} \bigg|_{f_A = f_A^*} &= (1 - H(\Delta p_A^*)) - h(\Delta p_A^*) f_A^* \frac{d\Delta p_A}{df_A} \\
&= 1 - h(0) f_A^* \frac{d\Delta p_A}{df_A} \\
&> 0,
\end{aligned}$$

where the inequality follows from (23). Therefore, from the proposed equilibrium, M would always want to set $f_A > f_A^*$ which would induce a positive discount by firm A. A similar argument implies that the discount set by firm B also cannot be zero in equilibrium.

Second, suppose the equilibrium involves a corner solution, so that it does not satisfy (11) for firm A. This implies that firm A just breaks even in equilibrium and its profit margin is zero; i.e.

$$P_A(q^*) - c_A - (1 - H(\Delta p_A^*)) f_A^* - \Delta p_A^* H(\Delta p_A^*) = 0, \tag{24}$$

where $q^* = \frac{1}{2} + \frac{(1-H(\Delta p_B^*)) f_B^* - (1-H(\Delta p_A^*)) f_A^*}{2w}$. By definition, f_A^* and Δp_i^* solve (13). Together with (24), this implies

$$\frac{dP_A(\hat{q}_A)}{d\hat{q}_A}\Big|_{\hat{q}_A=q^*}\frac{h(\Delta p_A^*)f_A^*}{2w}+(f_A^*-\Delta p_A^*)h(\Delta p_A^*)-H(\Delta p_A^*)=0. \tag{25}$$

If $\Delta p_A^* = 0$, then (25) implies that $f_A^* = 0$. However, this contradicts (24), since $q^* = \frac{1}{2}+\frac{(1-H(\Delta p_B^*))f_B^*}{2w} \geq \frac{1}{2}$, $P_A(1/2)-c_A>0$ and $P_A(q)$ is increasing. Therefore, it must be that $\Delta p_A^* > 0$. A similar argument applies to firm B. □

Proof of Proposition 6. Suppose there exists an equilibrium in which firm A offers no discount. I.e. $f_A^*, f_B^* \geq 0$, $\Delta p_A^* = 0$ and $\Delta p_B^* \geq 0$. Following the proof of Proposition 5, we also consider two cases.

First, suppose the equilibrium involves an interior solution, so that it satisfies (11) for firm A. For any commission f_A, the optimal choice of Δp_A by firm A is given by equating (14) to zero. Totally differentiating the resulting expression with respect to f_A and Δp_A, and following the same steps as in the proof of Proposition 5 we get that

$$\frac{d\Delta p_A}{df_A}=\frac{1-\left(\frac{d}{d\hat{q}_A}\frac{(1-G(\hat{q}_A))}{g(\hat{q}_A)}\right)\big|_{\hat{q}_A=q^*}}{1-\left(\frac{d}{d\hat{q}_A}\frac{(1-G(\hat{q}_A))}{g(\hat{q}_A)}\right)\big|_{\hat{q}_A=q^*}+\frac{4w(1-G(q^*))}{(f_A^*)^2h(0)g(q^*)}}\frac{1}{h(0)f_A^*}.$$

This implies (23) given $\frac{(1-G(\hat{q}_A))}{g(\hat{q}_A)}$ is decreasing in $\hat{q}_A$ following the increasing hazard rate of $G(q)$. The remainder of the proof of this first case is identical to that of the proof of Proposition 5.

Second, suppose the equilibrium involves a corner solution, so that it does not satisfy (11) for firm A. This implies that firm A just breaks even in equilibrium and its profit margin is zero; i.e. (24) holds. By definition, f_A^* and Δp_i^* solve (14). Together with (24), this implies

$$(f_A^*-\Delta p_A^*)h(\Delta p_A^*)-H(\Delta p_A^*)=0. \tag{26}$$

If $\Delta p_A^* = 0$, then (26) implies that $f_A^* = 0$. However, this leads to a contradiction following the same argument as in the proof of Proposition 5. Therefore, it must be that $\Delta p_A^* > 0$. A similar argument applies to firm B. □