Search platforms: Showrooming and price parity clauses*

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Abstract

We provide a model in which consumers search for firms directly or through platforms. Platforms lower search costs but charge firms for the transactions they facilitate. Platform fees raise the possibility of showrooming, in which consumers search on a platform but then switch and buy directly to take advantage of lower direct prices. In settings like this, search platforms like Booking.com have adopted price parity clauses, requiring firms offer their best prices on the platform, arguing this is needed to prevent showrooming. However, despite allowing for showrooming in our model, we find that price parity clauses often harm consumers.

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1 Introduction

A growing number of intermediaries act as platforms over which firms sell to consumers. Well known examples include third-party marketplaces such as Amazon.com, online travel agencies such as Expedia, and hotel booking services such as Booking.com. Key features of these platforms are that (i) firms set prices on the platforms; (ii) consumers search for firms and complete their purchases through the platforms; and (iii) when consumers complete a purchase through a platform, firms pay a commission fee to the platform. Many booking and reservation systems including the global distribution systems Amadeus, Sabre and Travelport[1] as well as restaurant booking services also share these features, as do some price comparison websites (e.g. for automotive insurance in the U.K.). An additional feature of most of the markets in which these platforms operate is that firms can also sell to consumers directly, potentially setting different prices. Consumers can therefore search directly for firms instead of on a platform, or they can search on the platform and then switch to purchase directly. This paper provides a model that matches these features.

Our interest in modeling these markets stems from recent policy investigations into the use of price parity clauses (PPCs) by platforms. Two types of clauses are relevant. A wide-PPC requires that the price a firm sets on the platform be no higher than the price the same firm charges for the same good through any other channel, including when it sells directly and when it sells through a rival platform. A narrow-PPC requires only that the price a firm sets on the platform be no higher than the price the firm sets when it sells directly.[2]

PPCs have been used by platforms in most of the markets we are interested in. For example, Amazon's General Pricing Rule requires that the item price and total price of an item a seller lists on Amazon.com be at or below the item price and total price at which the seller offers the item via any other online sales channel. In 2012, German and U.K. authorities investigated Amazon's rule, and Amazon responded by removing the rule from its marketplace contracts in Europe from 2013, although it kept the rule in place in the U.S. until March 2019 when it was removed amid growing political pressure. Similarly, in 2015, after investigations by several European authorities into their use of PPCs, Booking.com and Expedia, the

[1] These are worldwide computerized reservation networks used as a single point of access for reserving airline seats, hotel rooms, and rental cars.

[2] These types of restrictions are also sometimes called "platform MFNs", "vertical MFNs" and "best-price clauses".

two largest booking platforms for hotels, made commitments to remove wide-PPCs in Europe but retained their narrow-PPCs. However, the Austrian, Belgian, French, and Italian parliaments have each passed laws making both types of PPC illegal, while a German court has upheld a similar ruling with respect to Booking.com and local competitor HRS, although not yet Expedia.[3]

The main defense put forward for PPCs is that they are needed to prevent "showrooming". Consumers might use the platform to search for a suitable firm but then complete their purchase on the firm's own website if the firm offers a lower price when it sells directly (which it may do to avoid the platform's fee). Showrooming, which is a form of free-riding, may therefore undermine a platform's ability to operate. A PPC (either narrow or wide) eliminates the restriction on the platform's fee implied by such showrooming.

In this paper we develop a model of search platforms that is used to explore the implications of showrooming and PPCs. Consumers search sequentially for firms either directly or through a platform. Search reveals information on a firm's match value and price. We allow for two types of consumers—regular consumers, who consider whether to use the platform or the direct channel, and direct consumers, who only search and buy on the direct channel. Regular consumers can complete purchases on the channel they search on, or can switch channels to complete a purchase. The platform lowers search costs to consumers but charges firms for the transactions it facilitates. Among the questions we address are whether showrooming provides a legitimate defense for the use of PPCs, and what is the effect of PPCs on consumers. We address these questions first for a monopoly platform and then in the context of horizontally differentiated competing platforms.

We first consider a monopoly platform without a PPC. Lower search costs on the platform lead to higher expected match values for consumers and more intense price competition among firms. The platform would like to set its fee to extract this added surplus. However, if the platform's fee is too high (e.g. equal to or exceeding the difference in the firms' markups across the two channels), firms will want to set their direct prices below on-platform prices to induce regular consumers to showroom. Can then the platform still attract business with a positive fee? We show it can because firms will not want to lower their direct price so as to induce showrooming and thereby sacrifice their margins on direct consumers if the platform fee is low enough. The possibility of showrooming can therefore be good for consumers by restricting the fee set by a monopoly platform, provided the platform

[3]See Hunold et al. (2018) for more details and for an empirical evaluation of these clauses.

remains viable (i.e. the positive fee it can sustain is sufficiently high to cover the platform's cost).

A PPC, which requires firms to offer their best prices when selling through platforms, might appear pro-competitive at first glance. Indeed it ensures consumers have no incentive to switch to purchasing directly after searching on the platform, thereby ruling out showrooming. However, a PPC also removes the restriction on the platform's fee implied by consumers choice of which channel to search on. Regular consumers will always prefer to search on the platform rather than directly given prices are never higher, regardless of the fees charged to firms (provided firms still list on the platform). This allows the platform to increase its fee, which is why the platform always profits by imposing a PPC. The platform's fee is only constrained by prices becoming so high that either consumers do not want to search at all or that firms prefer to delist and just sell to direct consumers. We show the resulting equilibrium fee and prices (on both channels) are always higher than the case without a PPC, so direct consumers are always worse off, and total consumer surplus is always lower. If a PPC is not needed for the viability of the platform, then the same is true for regular consumers. On the other hand, if a PPC is needed for the viability of the platform, then regular consumers benefit from lower search costs which can more than offset the higher prices they face due to the PPC.

Platform competition acts as an alternative constraint on platform fees. Allowing competing platforms to retain narrow-PPCs ensures the constraint implied by platform competition still applies, even though the constraint implied by showrooming is eliminated. This is good for consumers provided platform competition is sufficiently effective and the viability of the platforms depends on eliminating showrooming. Otherwise, it is bad for consumers since it removes the showrooming constraint on fees and prices. Consumers are always worse off with wide-PPCs compared to narrow-PPCs since wide-PPCs remove the constraint on fees implied by platform competition as well as the constraint implied by showrooming. Indeed, in our model, by adopting wide-PPCs, competing platforms restore the same high fees and prices that arise when a monopoly platform imposes a PPC.

1.1 Related literature

Our paper relates to other recent theories of PPCs. Boik and Corts (2016) and Johnson (2017) assume consumers must use one of two differentiated platforms, and focus on how wide-PPCs result in each platform's demand becoming less responsive

to its fees, resulting in higher equilibrium fees and prices. Carlton and Winter (2018) extend these works by allowing for a direct channel. They focus on the case with perfectly competitive firms that must list on the platform, applying their theory of a PPC to show the harm caused by the no-surcharge rule of credit card platforms. Johansen and Vergè (2017) also allow for a direct channel, but assume consumers view each of the channels (the two different platforms and the direct channel) as well as each of the firms as being exogenously differentiated. They focus on the effects of allowing firms to delist from the platforms, finding PPCs can decrease prices if suppliers are sufficiently close substitutes and the direct channel is a sufficiently close substitute for the platform channels. Edelman and Wright (2015) also allow for a direct channel and allow firms to delist, but show PPCs are harmful by taking into account that high platform fees can be used to fund platform benefits (including rewards) to consumers, resulting in high direct prices, an excessive number of consumers joining and using the platform and an over-investment in the provision of platform benefits.

A key difference in our theory from these other works is that consumers have to search for price and match information, and that platforms facilitate this search. Facilitating search is a key feature of many of the platforms (booking websites, marketplaces, and price comparison websites) that have applied PPCs. The need for consumers to search gives rise to the possibility of showrooming, and our micro-founded search framework allows us to capture this effect. It also predicts that firms compete more intensely on the platform. This helps explain why the platform may be viable even in the face of showrooming—firms do not want to have to lower their direct prices to attract consumers to switch, thereby sacrificing their higher margins on direct consumers. At the same time, the fact consumers must search for firms reduces the incentives of a firm to delist from a platform that charges a high fee, since the firm will not be found by the consumers who continue to search on the high-fee platform. Unlike most of these other works, we also distinguish between narrow and wide PPCs.

In modeling the platform's role in facilitating search, our paper is also related to the literature modelling price comparison websites (PCWs). The seminal model of a PCW is Baye and Morgan (2001), in which consumers can use a PCW to find the lowest priced seller or go to their local monopolist. Extending Baye and Morgan's framework to a setting where firms offer horizontally differentiated products, Galeotti and Moraga-González (2009) and Moraga-González and Wildenbeest (2012) find that requiring prices be the same on the PCW as in the direct market does not harm

consumers, which reflects that the PCW is assumed to only use fixed fees. Ronayne (2015) and Ronayne and Taylor (2018) find a PCW's adoption of a PPC can harm consumers by considering the more realistic case in which platforms instead charge firms a per-transaction fee, and that there are shoppers who can search without the PCW. In an extension of their model, Ronayne and Taylor (2018) analyze the impact of a PPC allowing for a form of showrooming (when consumers search on the PCW, they also get to see all the listed firms' direct prices), showing how the PPC increases the PCW's fee and prices, making consumers unambiguously worse off. However, like Baye and Morgan, they assume firms are homogeneous.

The showrooming we consider is related to the literature on free riding in retail services, and in particular Mathewson and Winter (1983), in which discount stores free ride on the costly efforts of full-service stores which provide consumers with product and price information. A manufacturer addresses showrooming by imposing that all retailers set a minimum price (RPM). A fundamental difference is that in their setting the principal (the manufacturer) addresses showrooming across agents, whereas in our setting, the principal (the platform) addresses showrooming between itself and agents. A PPC in their setting would be akin to a full-service retailer requiring that discount retailers cannot undercut it, which would obviously be anticompetitive. This difference explains why unlike the manufacturer's use of RPM, a platform would still want to use a PPC even absent any showrooming problem.

Finally, our model of search builds on the classic works of Anderson and Renault (1999) and Wolinsky (1986) by introducing search with channel switching. However, it abstracts from the many interesting design choices faced by platforms in search contexts (e.g. see Athey and Ellison, 2011, de Corniére, 2016, Eliaz and Spiegler, 2011, Hagiu and Jullien, 2011, and Renault, 2014).

2 The Model

We assume a continuum of consumers (or buyers) and firms (or sellers), of measure 1 in each case. Each firm produces a horizontally differentiated product. We normalize the firms' production cost to zero. In the baseline setting, there is a single platform (M) which facilitates trades between the firms and consumers. Among the consumers, a fraction $n_r \in (0, 1]$ which we call "regular" consumers can search on M or directly, and can likewise buy on M or directly. The remaining fraction $n_d = 1 - n_r$ of consumers are what we call "direct" consumers, in that they can only search and buy directly. They may be consumers that dislike using M to search, or

are not aware of it. For tractability we do not model their preferences or possible lack of information but rather treat n_r and n_d as exogenously given parameters.

□ **Preferences**. Each consumer has a taste for firm i (i.e. to buy one unit of its product) described by the gross utility (ignoring any search cost) of the form $v^i - p^i$ if she buys from i at price p^i and draws the match value v^i. The match value v^i is drawn i.i.d. from the common distribution function G over $[0, \overline{v}]$ for each consumer and each firm. We assume G is twice continuously differentiable with a strictly increasing hazard rate and a strictly positive density function g over $[0, \overline{v}]$. A strictly increasing hazard rate, together with other assumptions we will make, will ensure a firm's optimal pricing problem is characterized by the usual first-order condition. The assumption will also ensure M can charge a positive fee and still attract transactions under showrooming, as we show in Section 3.2.

Defining the inverse hazard rate $\lambda(z) = \frac{1-G(z)}{g(z)}$, our assumption of strictly increasing hazard rate implies $\lambda'(z) < 0$.

□ **Consumer search**. All firms are available for consumers to search directly even if M is absent. For consumers who search directly, they incur a search cost $s_d > 0$ every time they sample a firm. By sampling firm i, a consumer learns its price p_d^i and its match value v^i. We interpret the search cost as the cost of investigating each firm's offerings, so as to learn p_d^i and v^i (e.g. so the consumer can work out the value she obtains from the hotel's location, facilities, feedback, room type and prices for particular dates; or an airline's flight times, fares, connections, aircraft type, cancellation policy and baggage policy). Note this is not the cost of going from one link to another on a website, which is likely to be trivial. Consumers search sequentially with perfect recall.

The utility of a consumer is given by $v^i - p_d^i - ks_d$ if she buys from firm i at price p_d^i at the kth firm she visits and obtains the match value v^i. We assume the search cost s_d is sufficiently low so that consumers would want to search directly if this were their only option (i.e. in the absence of M). This assumption will be formalized in Section 3.1.

□ **Platform**. A platform M provides search services to consumers. If a firm i also sells over M, its price on M is denoted p_m^i. When (regular) consumers search via M instead of directly, we assume search works in the same way[4] but their search cost reduces to $s_m \in (0, s_d)$. Thus, we assume M provides a less costly search environment for consumers (e.g. because it standardizes the relevant information

[4]By sampling firm i on M, a consumer learns its price p_m^i and the match value v^i.

about each firm).

We assume M incurs a fixed cost $c > 0$ in order to operate. We assume c is low enough so that M is viable when it charges its monopoly fees and regular consumers cannot switch to buy directly (i.e. without showrooming). This assumption will be formalized in Section 3.1.

□ **Showrooming**. Showrooming is possible only if consumers can observe a firm's identity when they search on M since otherwise switching to buy directly would involve starting the search over again.[5] To be as general as possible, we also allow regular consumers to search directly but having identified a good match, switch to buy on M. When consumers switch (in either direction), they can choose to stop and purchase from the firm (or any previous firm they have already searched), continue to search on the channel they have switched to, or switch back again. We assume that having identified a firm and its match value, there is no cost to the consumer of such switching. In practice, any such cost is likely to be trivial in the case where the purchases are all online. Costless switching implies that having incurred the search cost to identify and evaluate a particular firm on one channel, consumers can costlessly observe the firm's prices on all channels, including its direct channel. This is consistent with consumers being able to use a metasearch site that shows prices corresponding to all available channels for any particular firm.

□ **Instruments**. We allow M to charge a per-transaction fee f to firms when consumers make a transaction through M. All the platforms discussed in the Introduction charge firms a transaction fee when they sell through the platform. We assume M cannot charge a transaction fee to buyers, which is done for notational brevity and is without loss of generality. This reflects that either the firms' prices are not constrained by a PPC, in which case a buyer fee is a redundant instrument given that M can always achieve the same outcome by altering the fee charged to firms (see Gans and King, 2003, for a general statement of this type of neutrality result), or firms' prices are constrained by a PPC, in which case setting the buyer fee to zero will be strictly optimal if it is required to prevent consumers wanting to switch to buy directly.[6] Moreover, note that consistent with these results, among the search

[5]For this reason, in the absence of a PPC, M would want to hide the identity of users if it could. In Section 3.1 we analyze this benchmark case (i.e. where showrooming is not a problem). However, even in this case, we will show M will want to impose a PPC. Furthermore, M may not be able to hide the identity of users in practice which is why our main focus is on the case with showrooming.

[6]Indeed, under PPCs, platforms will want to use negative fees (rewards) if possible, to shift consumers to use their platform, as shown by Edelman and Wright (2015).

and booking platforms that have used PPCs, we do not see buyer-side transaction fees being used. In practice, most platforms also do not charge users registration fees for joining. We discuss the possible role of registration fees, per-click fees and referral fees in Section 5.

□ **Timing and equilibrium selection**. The timing of the game is as follows:

1. The platform M decides whether to operate, and if it does, sets the fee f to maximize its profits. Firms and consumers observe f.

2. Firms decide whether to join M and set their price(s).

3. Without observing firms' decisions, consumers decide whether to search. If they want to search, regular consumers decide whether to search on M or search directly (possibly switching search channels along the way), and carry out sequential search until they stop search or complete a purchase, while direct consumers carry out sequential search among the firms directly until they stop search or complete a purchase.

Note although consumers don't observe whether firms join M or not, they form their beliefs about whether they do (along with the prices they expect to face when searching) rationally based on the observed level of f. If instead f cannot be observed by consumers, then based on the logic of Janssen and Shelegia (2015), we may expect that the platform will be able to profitably increases its fee. On the other hand, if consumers anticipate this, and expect a negative surplus from search, they will not search using the platform. In Online Appendix A we analyze this possibility, showing that our results can still hold but this requires an additional parameter restriction for the case that M imposes a PPC.

In case firms' prices are not pinned down on a channel (say channel j) because no consumers are considering firms' offers on channel j (either by searching on channel j, or searching on another channel and switching), then we pin down the equilibrium price p_j for channel j using the following refinement.

- We determine the hypothetical equilibrium prices $p_j(n)$ in the user subgame in which there is an exogenous positive mass n of consumers that *only* search and buy through channel j ($j = d$ if this is the direct channel and $j = m$ if this is M). We determine p_j to be the limit of $p_j(n)$ as $n \to 0$.

This approach, which we apply throughout the paper, simplifies the analysis that follows, while still making sure our equilibrium satisfies the requirements for a perfect Bayesian equilibrium.

Finally, we focus on symmetric perfect Bayesian equilibria where all firms make the same joining decisions and set the same prices. We adopt the usual assumption that consumers hold passive beliefs about the distribution of future prices upon observing any sequence of prices. This is natural since all firms set their prices before consumer search starts. Note there will always be a trivial equilibrium in which consumers do not search through M because they expect no firms to join, and firms do not join because they do not expect any consumers to search through M. Such an equilibrium is the worst outcome for M. Thus, for any user subgame (i.e. the subgame starting from stage 2), if there is a symmetric equilibrium in which all firms join M, then this equilibrium will be selected instead of the trivial equilibrium. We can think of M as being able to coordinate users on the equilibrium that is better for itself.

3 Analysis with a monopoly platform

In this section, we analyze the model in which there is a single platform M. In Section 3.1 we consider a benchmark setting in which showrooming is not possible. Section 3.2 relaxes this assumption by exploring the possibility of showrooming. Section 3.3 considers what happens when M uses a PPC.

3.1 No free-riding benchmark

Initially, we consider a benchmark setting in which regular consumers cannot observe a firm's identity when they search for the firm via M, thereby ruling out the possibility of showrooming. Indeed, sometimes platforms deliberately conceal or obscure such information in an attempt to prevent showrooming.

□ **Consumer search**. Regular consumers are free to search directly, search through M, or switch the channel on which they are searching at any point. For expositional purposes, we first separately study the cases where such consumers only search and make purchases directly, and where such consumers only search and make purchases through M, and later combine these to study regular consumers' searching and purchasing choices when both channels are available.

Define the reservation value x_d for a consumer that only searches and buys directly such that

$$\int_{x_d}^{\overline{v}} (v - x_d) dG(v) = s_d, \tag{1}$$

so that the incremental expected benefit from one more direct search is equal to the search cost. We assume s_d is sufficiently small such that a unique value of x_d exists satisfying $0 < \lambda(x_d) < x_d$. Specifically, we assume $s_d < \underline{s}$, where $\underline{s} = \int_{\underline{x}}^{\overline{v}} (v - \underline{x}) dG(v)$ and $\underline{x}$ is uniquely defined by $\underline{x} = \lambda(\underline{x})$.[7]

Denote the equilibrium direct price as p_d. It is well understood from Kohn and Shavell (1974) and Weitzman (1979) that the optimal search rule in this environment is stationary and consumers use a cutoff strategy. When searching and buying only directly, each consumer employs the following cutoff strategy: (i) she starts searching if and only if $x_d \geq p_d$; (ii) she stops and buys from firm i if she finds a price p_d^i and match value v^i such that $v^i - p_d^i \geq x_d - p_d$; and (iii) she continues to search the next firm otherwise. The rule for stopping and buying from firm i says that a consumer's actual net utility from consuming from firm i (i.e. $v^i - p_d^i$) must be at least equal to this cutoff (i.e. $x_d - p_d$). After each search, expecting that firms charge symmetric prices p_d, a consumer's search ends with probability $1 - G(x_d)$ and continues with probability $G(x_d)$. A consumer's expected search cost is therefore $\frac{s_d}{1-G(x_d)}$. Given that there is a continuum of firms, each consumer will eventually buy a product with value $v \geq x_d$ at price p_d. The expected match value is $\mathbb{E}[v|v \geq x_d]$. The consumer's expected value of initiating such a search is therefore

$$\frac{\int_{x_d}^{\overline{v}} v dG(v)}{1 - G(x_d)} - p_d - \frac{s_d}{1 - G(x_d)} = x_d - p_d.$$

The equality is obtained by using (1). Note that x_d is a consumer's gross surplus (including search cost) from searching and buying on the direct channel only.

With all firms available on M, the optimal stopping rule for a regular consumer searching and buying only on M is the same but with the reservation value x_m defined by

$$\int_{x_m}^{\overline{v}} (v - x_m) dG(v) = s_m$$

to reflect the lower search cost s_m, and with the prices p_d^i and p_d replaced by p_m^i and p_m respectively, where p_m is the symmetric equilibrium price on M. Consumers would start such a search if and only if $x_m \geq p_m$.

Since $s_m < s_d$ and the left-hand side of (1) is decreasing in x_d, we have $x_m > x_d$.

[7]Note that, since $\underline{x} > 0$, we have $s_d < \int_{\underline{x}}^{\overline{v}} (v - \underline{x}) dG(v) < \int_0^{\overline{v}} v dG(v)$. This, together with the fact the left-hand side of (1) is strictly decreasing in x_d and equals zero when $x_d = \overline{v}$ ensures a unique value of (1) exists satisfying $\underline{x} < x_d < \overline{v}$. It will become clear later that the assumption $s_d < \underline{s}$ ensures that the net expected value is always positive and consumers will search in equilibrium.

Consumers tend to search more when using M due to the low search cost; i.e. they hold out for a higher match value. We denote this difference in the gross surplus from searching through M and directly as

$$\triangle_s = x_m - x_d,$$

and call it the *surplus differential* of the platform. It reflects the additional surplus each regular consumer enjoys from being able to search at a lower cost on M, ignoring any difference in prices. Note that if

$$c < n_r \triangle_s, \tag{2}$$

M's fixed cost of operating is less than the surplus differential it creates for regular consumers, who can use it. Thus, (2) implies that the existence of the platform M will increase welfare compared to the case where there is no platform.

Finally, consider the case both M and the direct channel are available. During their search, regular consumers are free to switch channels so as to continue searching on the other channel. However, given that we have not allowed for showrooming yet, regular consumers can still only complete a purchase from a particular firm on the channel that they found the firm on. With all firms available for searching on M, the expected utility (including search cost) that a consumer can get from searching on M only is $x_m - p_m$. Similarly, if the consumer only searches directly, her expected utility is $x_d - p_d$. Then, regardless of which channel regular consumers are currently using to search, their reservation value for stopping is $\max\{x_m - p_m, x_d - p_d\}$. If consumers continue to search, they will always use the channel which yields this reservation value. Considering which channel to search on initially, consumers will prefer to start their search through M provided the resulting expected payoff is weakly higher than that from searching directly. That is,

$$x_m - p_m \geq x_d - p_d. \tag{3}$$

□ **Firms' pricing**. We consider an equilibrium in which all firms join M and set the price p_m for consumers who purchase through M and the price p_d for consumers who purchase directly.

Suppose to start with that $\max\{x_m - p_m, x_d - p_d\} = x_d - p_d$ and all consumers search and complete transactions directly in equilibrium. Suppose a firm i deviates and sets its direct price to $p_d^i \neq p_d$. The limit version of Wolinsky (1986), and

more recently, Bar-Isaac *et al.* (2012) consider exactly this case and our argument follows theirs. The probability that a consumer who visits a random non-deviating firm buys from that firm is $\rho = 1 - G(x_d)$. This probability is exogenous from the deviator's perspective. The expected number of consumers who visit the deviating firm in the first round is 1. A further $(1-\rho)$ consumers visit the firm in the second round after an unsuccessful visit to some other firm, a further $(1-\rho)^2$ visit in the third round, and so on. From (ii) in the optimal stopping rule above, consumers buy from the deviating firm i only if $v^i - p_d^i \geq x_d - p_d$. Therefore, firm i's expected demand from consumers who search directly is given by

$$\sum_{z=0}^{\infty}(1-\rho)^z(1-G(x_d-p_d+p_d^i)) = \frac{(1-G(x_d-p_d+p_d^i))}{1-G(x_d)},$$

and its expected profit from these consumers is given by

$$\pi_d^i = p_d^i \frac{(1-G(x_d-p_d+p_d^i))}{1-G(x_d)}. \tag{4}$$

The increasing hazard rate property of G ensures the usual first-order condition from differentiating (4) with respect to p_d^i and setting the derivative equal to zero determines the optimal solution. Imposing symmetry on the first-order condition, the symmetric equilibrium price for direct sales is

$$p_d = \lambda(x_d), \tag{5}$$

and each firm's expected profit is $\pi_d = p_d = \lambda(x_d)$. Since $x_d > \lambda(x_d)$, we have that $x_d > p_d$, so consumers expect a positive surplus from searching in the first place.

Suppose instead $\max\{x_m - p_m, x_d - p_d\} = x_m - p_m$ and regular consumers search and complete transactions through M in equilibrium. Using the same argument as above but taking into account firms pay f to M for each transaction, a deviating firm's expected profit from regular consumers is given by

$$\pi_m^i = n_r(p_m^i - f)\frac{(1-G(x_m-p_m+p_m^i))}{1-G(x_m)}. \tag{6}$$

Solving the first-order condition by differentiating (6) with respect to p_m^i and setting equal to zero, and applying symmetry, the equilibrium price for intermediated search is

$$p_m(f) = f + \lambda(x_m), \tag{7}$$

and each firm's expected profit is given by $\pi_m = n_r \lambda(x_m)$.

We define the difference in the inverse hazard rates evaluated at the respective reservation values as

$$\triangle_m = \lambda(x_d) - \lambda(x_m).$$

We call this the *markup differential* of the direct channel since it captures the difference in the firms' equilibrium markups across the two channels.

□ **Platform pricing**. M can only make a positive profit if regular consumers choose to use it. Regular consumers compare the expected surplus from using each channel. M can influence their expected surplus through its fee f. Provided regular consumers expect all firms to join M, they are better off using M to search if (3) holds. Whether (3) holds depends on the prices firms charge on each channel. Substituting (5) and (7) into (3), regular consumers will use M to search if and only if

$$f \leq \triangle_s + \triangle_m. \tag{8}$$

This ensures consumers come on board. Note firms are always willing to join M in the absence of any showrooming or PPC concern. Consumers benefit from M due to lower search costs (the surplus differential) and intensified competition (the markup differential). Equation (8) says that in order to attract consumers, M's per-transaction fee cannot exceed the sum of these two benefits. M's profit in this case is $\Pi = n_r f - c$.

Maximizing f subject to (8), M's optimal fee makes the constraint bind. We assume

$$c \leq n_r(\triangle_s + \triangle_m), \tag{9}$$

so that M is viable in this benchmark setting. The following proposition summarizes the equilibrium outcome that follows.

Proposition 1. (No free-riding benchmark)
M operates and sets the fee $f^* = \triangle_s + \triangle_m$, with firms pricing at $p_d^* = \lambda(x_d)$ and $p_m^* = \triangle_s + \lambda(x_d)$ on the direct channel and on M respectively. Regular consumers search and buy on M.

The firms' equilibrium prices in Proposition 1 follow from substituting f^* from the proposition into (7). We know without any platform fee, on-platform prices would be lower than direct prices due to lower search costs making firms price more competitively on M. Collectively, firms would prefer an equilibrium where all trade

happens directly. However, each individual firm strictly prefers to join M given regular consumers are expected to search on M. The platform can take advantage of this by increasing its fee so that prices on M are equal to direct prices. This is the markup differential term in f^*. But this is not the end of the story. With equal prices, regular consumers would still strictly prefer to search on M due to the lower search costs. Without any possibility of showrooming, M will increase its fee until the higher prices on M just offset the surplus differential $\triangle_s$, and regular consumers are indifferent between searching on M and searching directly.

3.2 Showrooming

Suppose now regular consumers obtain a firm's identity when they search the firm on M. This will enable them to switch to buying directly having found a good match through M, potentially at a lower price. For instance, the equilibrium in the previous section in which prices are higher on M by the amount $\triangle_s$ would not be sustainable. Facing the equilibrium prices in Proposition 1, regular consumers would search on M and then switch to purchase directly. As a result, M would obtain no revenue, and would want to lower the fee f it charges firms provided it can still recover its cost c.

The implication of switching is more complicated than this, however, since a firm may want to raise its price on M and/or lower its direct price to induce consumers to switch, given the firm can avoid paying the fee f on any consumer who purchases directly. In this section, we take into account this possibility.

We first note that given firms obtain a higher markup selling to direct consumers, they will not want to sacrifice this higher markup by lowering their direct price to induce showrooming by regular consumers if M's fee is low enough. Since M will want to set a fee low enough to avoid showrooming, it is therefore natural to focus on a symmetric equilibrium in which regular consumers search and buy on M, while direct consumers search and buy directly. In the proof of the following proposition, which is given in the appendix, we show there exists a unique fee within $(0, \triangle_m)$ which just makes firms not willing to engage in showrooming, and so is the highest fee that M can charge without losing all transactions. We denote this cutoff level of f as τ. Then whether M is viable just depends on whether earning this fee on regular consumer is enough to cover its fixed cost.

Proposition 2. (Showrooming equilibrium)

(i) *Suppose* $c \leq n_r\tau$. *M operates, and sets the fee* $f^* = \tau$, *where* $0 < \tau < \triangle_m$.

Regular consumers search and buy on M. Firms price at $p_d^* = \lambda(x_d)$ and $p_m^* = \tau + \lambda(x_m)$, where $p_m^* < p_d^*$.

(ii) *Suppose $c > n_r\tau$. M will not operate. All consumers search and buy directly. Firms price at $p_d^* = \lambda(x_d)$.*

Proposition 2 characterizes the equilibrium when showrooming is possible. When M's fee is above τ, a firm can do better inducing consumers to switch to buy directly. In the off-equilibrium subgame that results, regular consumers search on M but switch to purchase directly with direct prices determined as if firms competed on M but without facing any fee (i.e. direct prices are less than $\lambda(x_d)$). To rule out this switching equilibrium, M has to lower its fee to τ. Without any switching in equilibrium, the firms' prices are determined as before by (5) and (7).

To understand this result, note that given competition is more intense for regular consumers (i.e. due to lower search costs), firms would like to offer regular consumers a lower price if they buy directly than the price they set to direct consumers. However, since firms cannot distinguish consumers, they cannot price discriminate in this way when selling directly. This implies that any price reduction in order to induce switching also reduces the markup firms get from selling to direct consumers.[8] To mitigate such a loss, firms do not want to reduce their direct price too much, which in turn allows M to still set a positive fee. However, the fee cannot be as much as $\triangle_m$. If f were equal to $\triangle_m$, then firms will always prefer to lower their direct price to induce regular consumers to switch to buy directly (and thereby save the fee f) since they would only have to do so by an infinitesimal amount relative to the equilibrium prices that would arise without any showrooming. Thus, showrooming helps constrain M's fee, benefiting consumers provided M remains viable.

The possibility of showrooming prevents M from extracting the surplus differential Δ_s. Regular consumers are free to switch to buy directly if the on-platform price is too high, even though they benefit from searching at a lower cost on M. However, M can still extract some of the markup differential $\tau \in (0, \Delta_m)$ under showrooming. That τ is strictly positive relies on the various key ingredients of our model. First, it relies on our assumption of an increasing hazard rate. If instead the hazard rate was either constant (corresponding to the exponential distribution for G) or decreasing, then $\Delta_m \leq 0$, implying M would not be able to charge a positive fee without causing consumers switching to buy directly. Second, it relies on the

[8]Note, if the firm simply increased its price on M instead, this will be suboptimal since that would cause too many consumers to continue searching on M rather than buying from the firm on M or switching to buy directly from the firm.

existence of an active direct market. In the limit cases in which $s_d \to \infty$ or $n_d \to 0$, there will be no direct consumers searching. Without direct consumers searching, firms do not lose any profit from direct consumers when they lower their direct price to induce regular consumers to showroom. Thus, when $s_d \to \infty$ or $n_d \to 0$, showrooming will arise for any positive fee charged by M. As a result, M would no longer be viable in this case as well. Finally, a positive τ relies on search costs being lower on the platform. When $s_m \to s_d$, then $\Delta_m \to 0$, implying M would not be able to charge a positive fee without consumers switching to buy directly. In any of these alternative scenarios, the outcome is captured by case (ii) in Proposition 2 in which M is not viable.

3.3 Price parity

One way M can eliminate showrooming and the constraint it implies for its fee is to impose a PPC. If a firm joins M and thereby accepts the PPC, its direct price must be at least as high as its price on M. For a given f, as long as all firms (including firm i) join M, an individual firm i chooses p_m^i and p_d^i to solve

$$\max_{p_m^i, p_d^i} \left\{ (p_m^i - f) n_r \left[\frac{1 - G(x_m - p_m(f) + p_m^i)}{1 - G(x_m)} \right] + p_d^i n_d \left[\frac{1 - G(x_d - p_d(f) + p_d^i)}{1 - G(x_d)} \right] \right\},$$

subject to $p_m^i \leq p_d^i$, with all other firms choosing the symmetric equilibrium prices $p_m(f)$ and $p_d(f)$ where $p_m(f) \leq p_d(f)$.[9]

We first note that if $f < \triangle_m$, then the pricing equilibrium from the user subgame in Section 3.1 applies, in which $p_d(f) = \lambda(x_d)$ and $p_m(f) = f + \lambda(x_m)$, given firms cannot induce switching as they could in Section 3.2 and given $p_m(f) < p_d(f)$ does not violate the PPC. For $f \geq \triangle_m$, it is straightforward to show that $p_d(f) > p_m(f)$ leads to a contradiction, given firms' unconstrained direct and on-platform prices are given by (5) and (7). Thus, given a PPC, we can focus on the case that the prices are common, so $p_d(f) = p_m(f)$ when $f \geq \triangle_m$.[10]

We will first characterize equilibrium prices for a given $f \geq \Delta_m$ when M is unconstrained by the possibility of firms delisting from it, and later provide a full characterization when delisting is taken into account. Let $p_c(f)$ be the equilibrium common price for a given fee $f \geq \Delta_m$ conditional on firms joining M, which is

[9] Since p_d cannot be less than $p_m(f)$, it will also generally depend on f, which is why we write it as $p_d(f)$.

[10] This assumes that the direct channel remains active. We will show in the next paragraph that our equilibrium selection rule implies the price on each channel will remain the same even if the direct channel is not active.

defined as[11]

$$p_c(f) = \arg\max_p \left\{ (p-f) n_r \left[\frac{(1 - G(x_m - p_c(f) + p))}{1 - G(x_m)} \right] + p n_d \left[\frac{1 - G(x_d - p_c(f) + p)}{1 - G(x_d)} \right] \right\}.$$

Provided both channels are active, the first-order condition together with symmetry imply

$$p_c(f) = \frac{\lambda(x_d)\,\lambda(x_m) + \lambda(x_d)\, n_r f}{\lambda(x_d)\, n_r + \lambda(x_m)\, n_d}. \tag{10}$$

Firms' equilibrium profit for a given f is

$$\pi_c(f) = p_c(f) - n_r f = \frac{\lambda(x_d)\,\lambda(x_m) + \triangle_m n_r n_d f}{\lambda(x_d)\, n_r + \lambda(x_m)\, n_d}.$$

On the other hand, in case the direct channel is not active, we pin down the direct channel price using our refinement by allowing a vanishingly small number n'_d of consumers to search directly. The resulting price is $p'_c(f) = \frac{\lambda(x_d)\lambda(x_m)\left(n_r + n'_d\right) + \lambda(x_d) n_r f}{\lambda(x_d) n_r + \lambda(x_m) n'_d}$. Taking the limit as n'_d goes to zero (but holding constant the actual value of n_r), the common price that firms set in both channels is indeed $f + \lambda(x_m)$.

The common price $p_c(f)$, together with the equilibrium price when all consumers search directly ($p_d = \lambda(x_d)$) and the equilibrium price when all consumers search on M ($p_m(f) = f + \lambda(x_m)$), all intersect at $f = \triangle_m$. Moreover, $p_c(f)$ is strictly increasing in f with a slope strictly between zero and one, so for $f > \triangle_m$ it lies strictly between $\lambda(x_d)$ and $f + \lambda(x_m)$. It is helpful to plot these three different candidate equilibrium prices as a function of f, which we do in figure 1. Define $\tilde{f}$ such that $p_c(\tilde{f}) = x_d$. This is the fee at which the common price would leave direct consumers with zero expected surplus from search. The left panel of figure 1 plots the three candidate equilibrium prices for the case in which $\tilde{f} < x_m - \lambda(x_m)$, which we will refer to as case 1. Case 1 arises when competition on the direct channel is not sufficiently intense and the number of direct consumers is not sufficiently large so that the uniform price $p_c(f)$ set by firms exceeds x_d when platform fees are high (but are still low enough that regular consumers want to search on M). Specifically, in case 1, the fee $\tilde{f}$ leaves regular consumers with a positive expected surplus from searching and buying on M when firms set their prices assuming that the direct channel is no longer active (i.e. no one searches or buys in the direct channel). The right panel of figure 1 plots the three candidate equilibrium prices for the alternative

[11] Given a firm's profit is now a weighted average of two separate profit expressions, to ensure second-order conditions hold, we assume its profit is quasi-concave in p. Using that $f \geq \Delta_m$ and $x_m > x_d$, one can show that a sufficient condition for this to hold is that g is log-concave.

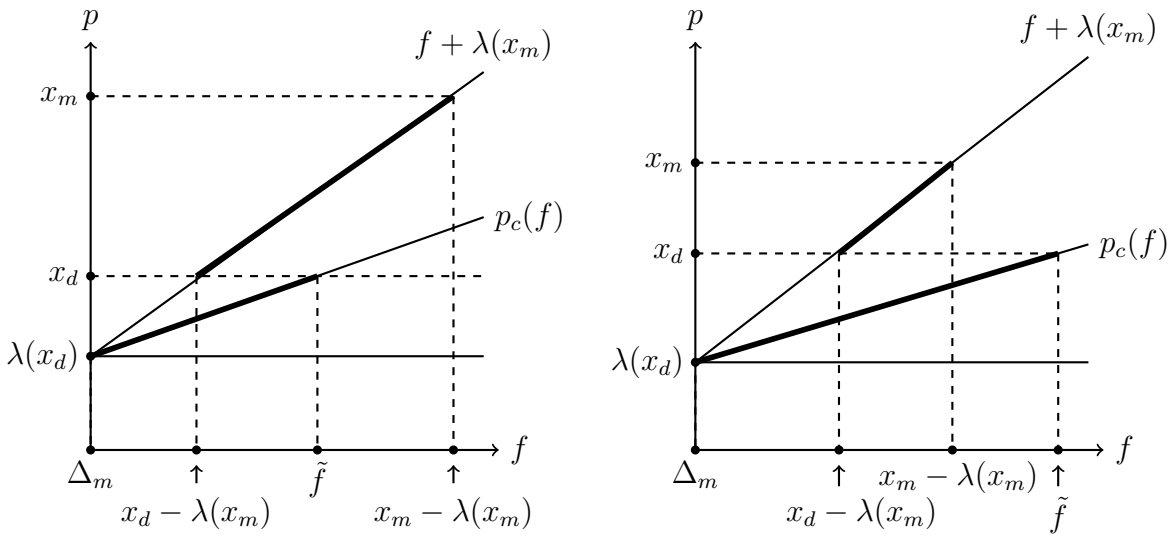

Figure 1: Candidate equilibrium prices as a function of f

case, in which $\tilde{f} \geq x_m - \lambda(x_m)$, which we will refer to as case 2. For each of the two cases, the bold lines represent the possible symmetric pricing equilibria (in the user subgame) for $f \geq \triangle_m$.

The full rationale for these different stage-2 symmetric pricing equilibria is given in the proof of Proposition 3, which is in the appendix. We wish to emphasize two points here.

One is that since M selects f, regardless of the possibility of multiple pricing equilibria for lower levels of f (as can be seen in Figure 1), M will always want to set f as high as possible provided firms continue to list on M. In case 1, the fee $f = x_m - \lambda(x_m)$ is the highest fee possible, since at any higher fee, even regular consumers would not want to search on M since they would expect to get a negative surplus from searching. Moreover, given the fee $f = x_m - \lambda(x_m)$, even if firms priced at $p_c(f)$ (i.e. firms assume that direct consumers still search and buy directly), direct consumers would not want to search on the direct channel. Without direct consumers searching, firms would never make any profit by delisting.[12]

The other point is that sometimes M's fee f will be constrained by the possibility

[12]Given that direct consumers are left unserved in the equilibrium in case 1, an alternative plausible equilibrium selection is that some firms delist and only serve direct consumers. In such an equilibrium, each firm would be indifferent between remaining listed and serving regular consumers, and delisting and serving only direct consumers. We ruled out such an equilibrium by our equilibrium selection criteria, which focused only on symmetric equilibria. In Online Appendix B, we characterize this asymmetric equilibrium, showing that the implications of a PPC for consumers remain similar.

of firms delisting. This can arise in case 2. To see why note that in case 2 regular consumers still want to search on M for a range of fees $f > x_m - \lambda(x_m)$ given that if firms price at $p_c(f)$ in this range, consumers still get a positive surplus from searching on either channel. This reflects that the competition on the direct channel is sufficiently intense and the number of direct consumers is sufficiently large that it constrains the uniform price $p_c(f)$ set by firms to be below x_d even for relatively high platform fees. However, with consumers searching on both channels when fees exceed $x_m - \lambda(x_m)$, this raises the possibility that firms will prefer to delist from M, so as to reoptimize their direct prices.

To analyze what happens when we allow for the possibility of firms delisting, suppose an individual firm i deviates by withdrawing from M. Assuming regular consumers still search on M, firm i's deviating profit is

$$\hat{\pi}_d = \max_{p_d^i} \left\{ p_d^i n_d \left[\frac{1 - G(x_d - p_c(f) + p_d^i)}{1 - G(x_d)} \right] \right\}.$$

If the pricing equilibrium is determined by the price $p_c(f)$, firms have no incentive to deviate if

$$\pi_c(f) \geq \hat{\pi}_d. \tag{11}$$

Define the level of f that equates the left-hand side and right-hand side of (11) as $\overline{f}$. In our proof of Proposition 3 we will show that a unique $\overline{f}$ exists. If $f > \overline{f}$ so that firms delist, the trivial equilibrium arises, in which all consumers search and purchase on the direct channel only. To avoid this, M will therefore set the highest possible fee in which firms still list (i.e. a non-trivial equilibrium still arises).

With these definitions in place, we are now ready to summarize our equilibrium findings.

Proposition 3. (PPC equilibrium)

(i) Direct channel is inactive: *Suppose* $\min\{\tilde{f}, \overline{f}\} < x_m - \lambda(x_m)$. *Then M operates, uses a PPC, and sets* $f^* = x_m - \lambda(x_m)$. *Firms will set their price equal to* x_m *on both channels, direct consumers will not search, and regular consumers will have their expected surplus fully extracted.*

(ii) Both channels are active: *Suppose* $\tilde{f} \geq x_m - \lambda(x_m)$ *and* $\overline{f} > x_m - \lambda(x_m)$. *Then M operates, uses a PPC, and sets a fee* $f^* > x_m - \lambda(x_m)$. *Firms will set the price on both channels equal to* $p_c(f)$ *where* $\lambda(x_d) < p_c(f) \leq x_d$, *with regular consumers being left with positive expected surplus and direct consumers non-negative expected surplus.*

Clearly, regardless of which scenario arises in Proposition 3, M improves its profit by imposing the PPC. Instead of having to set a fee below $\triangle_m = \lambda(x_d) - \lambda(x_m)$ due to the showrooming constraint, M is able to increase its fee to at least $x_m - \lambda(x_m)$ with the PPC.[13] The fee $x_m - \lambda(x_m)$ can be written as $\triangle_s + \triangle_m + x_d - \lambda(x_d)$, which is strictly higher than M's fee without showrooming ($\triangle_s + \triangle_m$). Therefore, a PPC not only prevents showrooming, but it forces the direct consumers to share the burden of the platform fee (if the direct channel remains active), or it shuts down the direct channel (if the direct channel becomes inactive).

A sufficient condition for (i) in Proposition 3 to apply is that case 1 arises (i.e. $\tilde{f} < x_m - \lambda(x_m)$). This happens when

$$x_m > x_d + \lambda(x_m)\left(\frac{x_d}{\lambda(x_d)} - 1\right)\frac{n_d}{1 - n_d}. \tag{12}$$

Thus, provided (12) holds, the imposition of a PPC will lead to the full surplus extraction from regular consumers. Note (12) holds if x_m is sufficiently high, x_d is sufficiently low, and/or n_d is sufficiently low.[14] In this case, the existence of direct consumers searching on the direct channel does not keep the common price low enough to stop M from extracting all of the regular consumers' expected surplus by setting $f = x_m - \lambda(x_m)$, leaving direct consumers no longer willing to search at all.[15] This could capture the example of hotels, in which the platform reduces search costs a lot, and in which there are not many consumers who just search hotels directly.

If instead the direct channel alternative is more important in terms of there being a relatively large number of direct consumers and direct search is not too much more costly than search on M, then M can set its fee higher than $x_m - \lambda(x_m)$ without causing the common price to lead direct consumers to want to stop searching because the common price will not increase too much above $\lambda(x_d)$. Essentially, the common price is constrained by the need for firms to compete for direct consumers, which is why both regular and direct consumers may be left with a positive expected surplus from search.[16] Moreover, in such a scenario, the platform's fee may be constrained

[13] The proposition does not cover the special case in which $\tilde{f} \geq x_m - \lambda(x_m)$ and $\overline{f} = x_m - \lambda(x_m)$. In this case there are two non-trivial pricing equilibria. As a result, either the properties in (i) of Proposition 3 apply since the equilibrium in which firms price at x_m is selected or the properties in (ii) of Proposition 3 apply since the equilibrium in which firms price at $p_c(f)$ is selected, where $\lambda(x_d) < p_c(f) \leq x_d$. In either case, M sets $f = x_m - \lambda(x_m)$.

[14] If x_m is high it implies $\lambda(x_m)$ is low which also helps ensure (12) holds, and similarly if x_d is low it implies $\lambda(x_d)$ is high which also helps ensure (12) holds.

[15] A special case in which this outcome always arises is when searching directly becomes infeasible (i.e. $s_d \to \infty$) or the number of direct consumers is negligible (i.e. $n_d \to 0$).

[16] A special case in which this outcome always arises is when searching directly becomes almost

by the possibility of firms delisting and just selling on the direct channel. This could capture an example like airlines, where searching directly may not be much more difficult than on the platform, and indeed many consumers just search directly.

Comparing the equilibrium in Proposition 3 with Proposition 2 gives the following implications of allowing M to impose a PPC.

Proposition 4. (The effect of a PPC)

(i) Low platform costs: *Suppose $c \leq n_r\tau$. Imposing a PPC increases prices (on both channels), decreases surplus for both types of consumers, increases M's profit, and either leaves welfare unchanged or lowers it. Firms' profit increases if both channels remain active, but otherwise decreases.*

(ii) High platform costs: *Suppose $c > n_r\tau$. Imposing a PPC makes M viable (and so increases its profit), increase prices (on both channels), decreases direct consumers' surplus, total consumer surplus and firms' profit, but has an ambiguous effect on the surplus for regular consumers and on welfare.*

Provided M remains viable under showrooming, so case (i) in Proposition 4 applies, a PPC makes all consumers worse off. Instead of the equilibrium on-platform price being strictly lower than $\lambda(x_d)$, now it is strictly higher. The case in which direct consumers no longer search due to the price being too high results in regular consumers having their surplus fully extracted, welfare being strictly lower, and firms being worse off since they no longer sell in the direct channel.

If instead, direct consumers continue to search in equilibrium, a PPC still lowers consumer surplus, although now firms are better off and total welfare is left unaffected. Consumer surplus is lower for two reasons. The main reason is M's higher fee, which is above $x_m - \lambda(x_m) > \triangle_m$ rather than being below $\triangle_m$. This results in p_c being above $\lambda(x_d)$, whereas without a PPC the price was only $\lambda(x_d)$ even on the more expensive direct channel. A second reason is that competition between firms is softened under the uniform pricing that results from a PPC whenever both channels remain active. When firms are forced to set a uniform price, they will set a price that is "biased" towards the equilibrium discriminatory price of the more competitive platform channel (i.e. $f + \lambda(x_m)$) instead of a simple weighted average of the two discriminatory prices (i.e. $n_r(f + \lambda(x_m)) + n_d\lambda(x_d)$). This reflects that demand on the platform channel is more sensitive to a firm's price (due to lower search costs). But when $f > \triangle_m$, the discriminatory price of the more competitive

equally efficient as searching on M ($s_d \to s_m$) and the number of regular consumers is negligible ($n_r \to 0$).

platform channel is also the higher of the two discriminatory prices, so this bias causes firms to set a uniform price above the simple weighted average price, thus softening competition. This second effect thus reinforces the first in lowering consumer surplus, and it is why firms' profit increases provided both channels remain active.

On the other hand, if M requires a PPC to remain viable because showrooming constrains its fee too much (case (ii) in Proposition 4), then the effects of a PPC are more ambiguous. While a PPC still increases prices and makes direct consumers worse off (since with a PPC, prices exceed what they would be if firms only competed in the direct channel), the effect on regular consumers is ambiguous given their search costs are lowered by M. This captures the case in which a PPC could be good for regular consumers—it is needed for the viability of M and it doesn't lead to very high prices. The latter arises when the direct channel is sufficiently important, in which case firms will set a common price across the two channels that is not very high, and furthermore, M's fee may be constrained by firms ability to delist. Even in this case, a PPC decreases total consumer surplus. Given $f > x_m - \lambda(x_m)$, total consumer surplus under a PPC is less than $n_r x_m + n_d x_d - p_c(x_m - \lambda(x_m))$. On the other hand, total consumer surplus without a PPC is equal to $x_d - \lambda(x_d)$ given M is not viable. Using that $x_d > \lambda(x_d) > \lambda(x_m)$, the latter expression for total consumer surplus is greater than the former, so total consumer surplus is always lower under a PPC. Moreover, firms are always worse off, even if both channels remain active.[17] Finally, a PPC affects welfare in three ways when it is needed for M's viability. It increases welfare by lowering search costs for regular consumers. It decreases welfare through the additional costs c incurred by M. And it decreases welfare whenever it leads to the shut-down of the direct channel, as was the case before.

Finally, note M will still want to impose PPC even if it can hide the identity of its participating firms so that it does not face any showrooming problem. Comparing the equilibrium in Proposition 3 with Proposition 1 implies that M still increases its fee (and its profit) in this case. Instead of setting a fee equal to $\triangle_s + \triangle_m$ when M faces no showrooming problem, with a PPC it sets a fee at or exceeding $x_m - \lambda(x_m)$, which is strictly higher. This reflects that a PPC not only prevents showrooming, but it raises the "price" regular consumers face if they search in the direct channel, so allowing M to extract more from them on the platform. Direct consumers are

[17] As shown in the proof of Proposition 3, to avoid firms wanting to delist when both channels remain active, M has to set $f < \lambda(x_d)/n_d$, but this also implies firms' profit is lower under a PPC than the $\lambda(x_d)$ they obtain without a PPC.

thus always worse off under a PPC compared to a platform that successfully hides the identity of participating firms. When the direct channel is no longer active under a PPC, regular consumers are also worse off, due to the higher prices they face. Furthermore, a PPC can increase the price regular consumers pay even when M can hide firms' identities and the direct channel remains active. This is indeed the case when $n_d\lambda(x_m)\Delta_s < n_r\lambda(x_d)(x_d - \lambda(x_d))$. However, when this condition does not hold, regular consumers may be better off with a PPC compared to the case in which instead firms' identities are hidden. This possibility arises, since under a PPC, the direct channel can constrain the common price below the level of the intermediated price that would arise when the firms' identity is hidden but no PPC is imposed.

4 Platform competition

In this section we extend our previous model to allow for competition between two platforms M^1 and M^2. Given the platforms face fixed costs of operation, it is natural to consider sequential entry.[18] Suppose M^1 decides whether to enter in stage 1a, incurring a fixed cost c to operate, and M^2 decides whether to enter in stage 1b, incurring the same fixed cost c to operate. After observing whether neither, one or both platforms enter, consumers make their search and purchase decisions.

If only M^1 enters in stage 1a or only M^2 enters in stage 1b, the model is identical to the monopoly model already considered. However, if both M^1 and M^2 enter, we assume that half of the n_r regular consumers incur a cost a if they purchase on M^2 but incur no such cost on M^1, while the other half of regular consumers incur a cost a if they purchase on M^1 but incur no such cost on M^2. We assume each consumer draws a from a common distribution H which is twice continuously differentiable and has a strictly positive density function h over $[0, \overline{a}]$. Thus, consumers treat the platforms as horizontally differentiated. To ensure second-order conditions hold, we also assume that $1 - H$ and $1 + H$ are (weakly) log-concave.

This model formulation is motivated by a situation where after both platforms have entered, consumers try one of the two platforms randomly at first, and then having learnt how to use that platform, consumers may face some inconvenience from using the other platform. In the monopoly case, since there is just one platform to try, there would be no such inconvenience cost. This approach has the modelling advantage of reducing to the monopoly model in case only one platform enters, while

[18] As shown in Online Appendix C, the case with simultaneous entry gives similar results.

ensuring that each consumer's choice of whether to buy from her preferred platform or buy directly is not distorted by the introduction of horizontal differentiation when both platforms enter and share the market equally in equilibrium.

We apply the natural extension of our previous equilibrium selection rule that if there are multiple equilibria in the user subgame for given fees, one being the trivial equilibrium and others involving active participation by firms on one or both platforms, then we will select the equilibrium with active participation by firms on both platforms (if it exists) and otherwise we will select an equilibrium with active participation by firms on one of the platforms (if it exists).

We assume $\frac{n_r}{2h(0)} \geq c$, which ensures that in the benchmark case without showrooming or PPCs, platforms can recover their fixed costs. This parallels our assumption (9) for the monopoly case. For the same benchmark case without showrooming or PPCs, we also assume that $\triangle_m > \frac{1}{h(0)}$, which ensures platform competition is sufficiently effective so that consumer prices are lower than those arising without any platform.

4.1 No free-riding benchmark

We first consider the benchmark case in which the showrooming constraint is not present and PPCs are not allowed. Since showrooming is absent, firms set their on-platform prices and direct prices separately. The equilibrium direct price is $\lambda(x_d)$. Given platform M^j's fee f^j, the equilibrium on-platform price on M^j is $f^j + \lambda(x_m)$, paralleling the firms' pricing in (5) and (7).

To understand why these pricing formulas still apply, consider the case that M^j sets $f^j < f^k$. Then some consumers whose preferred platform is M^k but with small enough a will search on M^j to take advantage of the lower prices. This suggests firms' pricing on M^j will be more complicated because firms face two different types of consumers (those who prefer M^j and do not incur a, and those who prefer M^k and incur a). However, this is not the case since consumers who incur a will buy at a firm i as long as $v^i - p^i - a \geq x_m - p_m - a$ and they will continue searching on M^j otherwise. This reflects that these consumers will incur the same cost a regardless of which firm they buy from on M^j, so the a term cancels out and they behave the same as the consumers who do not incur a. As a result, even though firms on M^j sell to two types of consumers when $f^j < f^k$, our existing pricing formulas still apply.

Consider the symmetric equilibrium fee f^*. In equilibrium, consumers use their

preferred platform. If M^j deviates by setting $f^j < f^*$, consumers whose preferred platform is M^k $(k \neq j)$ and draw $a < f^* - f^j$ will use M^j. In this case, M^j will set f^j to maximize its deviation profit $\left(\frac{n_r}{2}\right) f^j \left(1 + H(f^* - f^j)\right)$. Alternatively, M^j can increase its fee above f^*. In this case, consumers whose preferred platform is M^j and draw $a < f^j - f^*$ will use M^k, so M^j will set f^j to maximize its deviation profit $\left(\frac{n_r}{2}\right) f^j \left(1 - H(f^j - f^*)\right)$. In either case, imposing symmetry on the first order conditions (and using log-concavity of $1 + H$ and $1 - H$ to ensure second-order conditions hold) implies $f^* = \frac{1}{h(0)}$.

We summarize these results in the following proposition, which characterizes the equilibrium outcome for platforms and firms.

Proposition 5. (No free-riding benchmark with differentiated platforms)
Both platforms operate and set the fee $f^* = \frac{1}{h(0)}$. Firms join both platforms, setting the common on-platform price $\frac{1}{h(0)} + \lambda(x_m)$ and the direct price $\lambda(x_d)$.

Without any possibility of showrooming, the equilibrium between competing platforms has each platform setting its fee to reflect the extent of horizontal differentiation between them in a quite standard way. Regular consumers will search and buy on their preferred platform, having access to all firms in either case. Note regular consumers expect the surplus $x_m - \left(\frac{1}{h(0)} + \lambda(x_m)\right)$ from searching on their preferred platform and $x_d - \lambda(x_d)$ from searching directly. Our assumption that $\triangle_m > \frac{1}{h(0)}$ together with $x_d > \lambda(x_d)$ ensures regular consumers will search on their preferred platform in equilibrium rather than directly. Each platform's equilibrium profit is $\frac{n_r}{2h(0)} - c$, which is non-negative by assumption.

4.2 Showrooming

Suppose that having searched a firm on a particular platform, consumers can observe the firm's identity and its prices on all channels, and can switch and buy from the firm directly or through the other platform.

At the benchmark equilibrium prices considered in Section 4.1, consumers would not want to switch to buy directly. This follows from our assumption that platform competition is sufficiently effective (i.e. $\triangle_m > \frac{1}{h(0)}$) so that prices are lower on the platform. However, at the benchmark equilibrium fee f^*, firms may want to lower their direct price to induce regular consumers to switch to buy directly. Then each platform will need to lower its fee to prevent this.

When $f^j < f^k$, a firm's subgame equilibrium on-platform prices are $p_m^j = f^j + \lambda(x_m)$ and $p_m^k = f^k + \lambda(x_m)$. These prices are the best-response prices when all

other firms price in this way. In particular, a firm would not change its on-platform pricing due to its ability to get consumers to switch across platforms (e.g. to induce consumers to buy on the cheaper platform). Note at these equilibrium prices, firms make the same per-consumer margin of $\lambda(x_m)$ on both platforms, even though the fees are different. If a firm reduces its price on M^j to attract consumers who search on M^k to buy on M^j, it will obtain a lower margin for all the purchases on M^j. Moreover, its price is no longer a best response to other firms' prices on M^j so it will obtain less profit from consumers searching on M^j. The only way such a deviation could be profitable is that it boosts demand from consumers who search on M^k. Note, however, these consumers' preferred platform is M^k and they will incur the cost a if they buy on M^j. If the firm wanted to increase demand in this way, it can do so at a lower cost by directly lowering its price on M^k, but we already know its price on M^k is its best-response price, so this is also not profitable.

In the following proposition, which is proven in the appendix, we show the resulting equilibrium fee is exactly the same equilibrium fee as in the monopoly case with showrooming (Section 3.2), as defined by τ. The proposition also characterizes what happens if the platforms are not viable when they set this fee.

Proposition 6. (Showrooming equilibrium with differentiated platforms)
(i) *Suppose* $c \leq \frac{n_r\tau}{2}$. *Both platforms operate and set the fee* $f^* = \min\left\{\frac{1}{h(0)}, \tau\right\}$. *Firms join both platforms and set the common on-platform price* $\min\left\{\frac{1}{h(0)}, \tau\right\} + \lambda(x_m)$ *and the direct price* $\lambda(x_d)$.
(ii) *Suppose* $\frac{n_r\tau}{2} < c \leq n_r\tau$. *Only* M^1 *operates and sets the fee* $f^* = \tau$. *Firms join* M^1 *and set the on-platform price* $p_m^* = \tau + \lambda(x_m)$ *and the direct price* $\lambda(x_d)$.
(iii) *Suppose* $c > n_r\tau$. *Neither platform operates. Firms set the direct price* $\lambda(x_d)$.

The proposition shows that provided both platforms are viable, fees are determined by whichever constraint is tighter—the constraint on fees from platform competition or the constraint on fees from showrooming.

4.3 Narrow price parity

Recently, under pressure from competition authorities, the two largest hotel booking platforms in Europe have each removed PPCs with respect to their competitors (wide-PPCs) but, in countries that have not banned the practice, have kept a PPC with respect to hotels selling directly online. The following proposition de-

scribes the equilibrium that arises in case platforms are allowed to use narrow but not wide PPCs.

Proposition 7. (Competing platforms and narrow-PPC)

Platforms operate. They adopt narrow-PPCs if $\tau < \frac{1}{h(0)}$. *Both platforms set* $f^* = \frac{1}{h(0)}$. *Firms join both platforms and set the common on-platform price* $\frac{1}{h(0)} + \lambda(x_m)$ *and direct price* $\lambda(x_d)$.

If $\frac{1}{h(0)} \leq \tau$, where τ is defined in the same way as for the monopoly case in Section 3.2, then narrow-PPCs have no impact since fees are anyway not constrained by showrooming. On the other hand, if $\tau < \frac{1}{h(0)}$, narrow-PPCs remove the constraint on fees imposed by the showrooming possibility, allowing platforms to set their fees above τ. Each platform is strictly better off imposing a narrow-PPC and slightly increasing its fee when $\tau < 1/h(0)$. The narrow-PPC removes the showrooming constraint and allows this platform to better respond to the rival platform's fee given that $\frac{1}{h(0)}$ is the equilibrium fee level when regular consumers cannot switch to buy directly. Thus, both platforms will adopt narrow-PPCs if they are allowed. As a result, narrow-PPCs replicate the outcome in the benchmark case (i.e. Proposition 5). Moreover, recall that the on-platform price is lower than the direct price in the benchmark case given our assumption that $\frac{1}{h(0)} < \triangle_m$, so there is no violation of narrow-PPCs in equilibrium. Moreover, a firm cannot be better off delisting from one or both platforms in order to re-optimize its direct price since in the equilibrium in Proposition 7, direct prices are already equal to the unconstrained equilibrium level $\lambda(x_d)$.

Proposition 8. (Implication of narrow-PPCs)

(i) Competition constraint is the relevant constraint: *Suppose* $\frac{1}{h(0)} \leq \tau$. *Narrow-PPCs are irrelevant.*

(ii) Showrooming constraint is the relevant constraint: *Suppose* $\tau < \frac{1}{h(0)}$. *The use of narrow-PPCs has no affect on direct consumers.*

(ii-a) *When* $c \leq \frac{n_r \tau}{2}$, *the use of narrow-PPCs increases prices paid by regular consumers, decreases consumer surplus, leaves firms' profit unchanged, increases the platforms' profit, and leaves total welfare unchanged.*

(ii-b) *When* $\frac{n_r \tau}{2} < c \leq n_r \tau$, *the use of narrow-PPCs increases prices paid by regular consumers, decreases consumer surplus, leaves firms' profit unchanged, decreases total welfare, and has an ambiguous effect on total platform profit.*

(ii-c) *When $c > n_r\tau$, the use of narrow-PPCs decreases prices paid by regular consumers, increases consumer surplus, decreases firms' profit, increases the platforms' profit, and increases total welfare if $n_r\triangle_s > 2c$.*

Proposition 8 shows the implications of platforms introducing narrow-PPCs. In case platform competition rather than showrooming is the binding constraint on fees, then narrow-PPC will have no effect (case (i) in Proposition 8). However, if showrooming rather than platform competition is the binding constraint on fees, and if both platforms are viable with showrooming (case (ii-a) in Proposition 8), narrow-PPCs remove the showrooming constraint on fees, resulting in higher fees, higher platform profits, higher prices and lower consumer surplus.

If instead only one platform is viable with showrooming (case (ii-b) in Proposition 8), then by eliminating the possibility of showrooming, narrow-PPCs will result in both platforms becoming viable. In comparing consumer outcomes, note this involves comparing the case in which the fee is constrained by platform competition with the monopoly case in which the fee is constrained by showrooming. But since the showrooming constraint is the tighter constraint (by assumption in case (ii)), then fees and prices will be higher even though narrow-PPC supports platform competition. The effect on the platforms' profit is more subtle. While each platform would want to impose a narrow-PPC to remove the showrooming constraint, when both do so, because both platforms become viable, total platform profit becomes $\frac{n_r}{h(0)} - 2c$. This profit can be higher or lower than profit without a narrow-PPC, which is $n_r\tau - c$. The duplication of the fixed cost c also explains why total welfare is lower with narrow-PPCs.

Finally, we have the case in which neither platform is viable under showrooming (case (ii-c) in Proposition 8). The use of narrow-PPCs will restore the viability of both platforms, which is good for the platforms and is good for consumers (given we assumed platform competition was sufficiently strong to lower prices compared to the case consumers can only search directly). Firms are worse off due to intensified competition for regular consumers on the platforms compared to when they sell to all consumers directly. Moreover, total welfare is higher provided the surplus differentiation created by platforms exceeds the costs of the two platforms. Thus, we find narrow-PPCs are only beneficial for consumers (and sometimes overall welfare) if without them, platforms would not be viable due to showrooming.

4.4 Wide price parity

Now consider what happens when platforms use wide-PPCs, which they will want to do so as to remove the constraint on their fees from platform competition and from showrooming. Provided we add one additional assumption, we show that wide-PPCs undermine competition between the platforms and result in the same fees, prices and outcomes for consumers and firms as the monopoly outcome with a PPC (i.e. Proposition 3). Each platform obtains half of the corresponding monopoly platform's profit.

The additional assumption is that

$$\frac{x_m - \lambda(x_m)}{2} > \triangle_m. \tag{13}$$

Note this assumption, which is very much a sufficient condition for our result, holds if G is the uniform distribution given that $x_d > \lambda(x_d)$.

To understand how platform competition works and the role of the above assumption, recall that we analyzed two cases with a monopoly platform, namely case 1 and case 2 in Section 3. These corresponded to the left-hand side and right-hand side of Figure 1, respectively. Corresponding to case 1, consider the proposed equilibrium in which both platforms set $f = x_m - \lambda(x_m)$ and firms price at $p_m(f) = x_m$ on the platform as well as on the direct channel. At these prices, direct consumers will not search. As a result, firms cannot do better delisting. Each platform obtains revenue of $\frac{n_r(x_m - \lambda(x_m))}{2}$. Clearly, if either platform increases its fee above $x_m - \lambda(x_m)$, regular consumers would no longer search and the deviating platform would obtain no revenue. As shown in the proof of Proposition 9, which is given in the appendix, neither platform can do better by instead lowering its fee in an attempt to obtain more business (i.e. by inducing firms to list on it exclusively, and so more consumers to use it for searching and buying) given their best deviation profit can be bounded above by $n_r \triangle_m$ and given our assumption in (13). For a platform to convince an individual firm to join it exclusively when all other firms are listed on both platforms, it has to lower its fee by at least one half to offset the fact that half of the consumers are expected to be searching on the other platform. Furthermore, even if its fee is low enough to convince consumers to only search on it since it will attract all firms exclusively, half of the consumers "dislike" transacting on the deviating platform and so most of these consumers will want to switch and buy directly after searching on it unless it sets its fees well below $\triangle_m$. Under our assumption (13), this will not be profitable.

A similar logic applies for the different scenarios corresponding to case 2 for a monopoly platform, implying that the competing platforms obtain equilibrium revenue that is equal to $\frac{n_r(x_m-\lambda(x_m))}{2}$ or higher. If either platform increases its fee from the corresponding monopoly level, it will either cause consumers to no longer search on the platform or firms to delist from the platform. Moreover, by lowering its fee in an attempt to induce firms to list exclusively, the best a deviating platform can obtain is again bounded above by $n_r\triangle_m$, for the same reasons as above, so that our assumption (13) rules out such a deviation.

Proposition 9. (Competing platforms and wide-PPC equilibrium)
Platforms operate and they adopt wide-PPCs. Under the additional assumption that (13) holds, there is a symmetric equilibrium in which both platforms set their fees at the same level as in the monopoly case (Proposition 3), with the firms' prices determined in the same way as well.

Proposition 9 establishes that the equilibrium outcome in the case of a monopoly platform remains unchanged when platforms compete but can use wide-PPCs. This reflects the sense in which wide-PPCs can eliminate platform competition. Note the assumption (13) which is used to prove Proposition 9 is very much a sufficient condition. In practice, a platform's deviation profits from trying to attract firms and regular consumers exclusively will be substantially below $n_r\triangle_m$. This is because the deviating platform would need to set a fee significantly below $\triangle_m$ in order to induce a sufficiently large number of the half of regular consumers who prefer the other platform to be willing to buy through it. Note that unless the deviating platform's fee is low enough, these consumers prefer to search on the deviating platform where all firms are listed, and then switch to buy directly to avoid the inconvenience cost of buying on the deviating platform. Moreover, in scenario (ii) in Proposition 3, the equilibrium fee is higher than $x_m-\lambda(x_m)$. As a result, the competing platforms, who each get half of the corresponding monopoly revenue, will have even less incentive to deviate.

Without any PPC, each platform's fee is $\min\left\{\frac{1}{h(0)},\tau\right\}$, if it is indeed viable. With competing platforms that impose wide-PPCs, we know from Proposition 3 that each platform can set a fee at least equal to $x_m-\lambda\left(x_m\right)$. But note $x_m-\lambda\left(x_m\right)>\triangle_m>\tau$ given $x_m>x_d$ and the definition of τ. Thus, under wide-PPCs, each platform can increase its fee relative to the equilibrium with showrooming. With wide-PPCs, each platform also increases its fee above the equilibrium fee arising with narrow-PPC, which is $f^*=\frac{1}{h(0)}$. This reflects our assumption that $\triangle_m>\frac{1}{h(0)}$.

Thus, wide-PPCs have the same qualitative implications as imposing a PPC in the monopoly case, meaning the main implications of Proposition 4 continue to apply here.

Finally, wide-PPCs lead to higher fees, higher prices (on all channels), and lower consumer surplus than under narrow-PPCs. Welfare is either the same or lower under wide-PPCs compared to narrow-PPCs, with lower welfare arising whenever wide-PPCs result in direct consumers no longer searching on the direct channel. In short, our results imply that allowing wide-PPCs as opposed to just narrow-PPCs is never good for consumers or overall welfare, and is sometimes bad.

5 Policy implications

Based on our analysis, PPCs have two main anticompetitive effects. To the extent that firms compete with platforms to complete consumer transactions (e.g. complete a booking), then both narrow and wide PPCs suppress or distort competition for this service. This is because either type of PPC shuts down the ability of firms to undercut the platform in attracting consumers to complete purchases if the fee the platform charges is too high. The second anticompetitive effect arises when a wide-PPC applies across all channels which suppresses and distorts platform competition. It eliminates the incentive competing platforms would otherwise have to lower their fees to firms, as lower fees by one platform cannot be passed on to consumers by way of lower prices. We find that under wide-PPCs, the equilibrium fees replicate the monopoly outcome under a PPC. While these fees may be constrained by the ability of firms to delist, the level of fees and prices is still always higher than without wide-PPCs. Thus, wide-PPCs can be viewed as anticompetitive, with the onus on platforms to justify what efficiency-enhancing benefits wide-PPCs deliver that can't be delivered with less restrictive alternatives. Showrooming with respect to direct sales is not a valid justification. In the face of showrooming, narrow-PPCs are a less restrictive alternative, and we find consumers are always better off under narrow-PPCs compared to wide-PPCs.

This view that narrow-PPCs are a less restrictive alternative than wide-PPCs to deal with showrooming is reflected in the approach of several competition authorities that have allowed platforms to continue with narrow-PPCs on the grounds they removed wide-PPCs, such as that of the European Commission and Australia's ACCC (with respect to Booking.com and Expedia), and the CMA in the U.K. (with respect to price comparison websites for motor insurance).

If platforms are viable under showrooming, then banning narrow-PPC as well also increases consumer surplus since it ensures the showrooming constraint lowers platform fees. Even if platform viability due to showrooming is a concern, there may be less restrictive alternatives than narrow-PPCs to address showrooming. For example, in the absence of PPCs, a monopoly platform could use registration fees to help recover costs without inducing any showrooming. This assumes users continue to expect others to use the platform. These expectations may be put into question if a platform tries to extract too much surplus via registration fees. In settings in which users are heterogeneous and platforms do not know each user's expected surplus, registration fees would cause some users to no longer join, which through cross-group network effects, could lead to a downward spiral of reduced consumer and firm participation. The coordination problems caused by the use of registration fees are likely even more severe when it comes to competing platforms. This motivates the consideration of other types of fees, such as per-click fees or referral fees.

In our context, a per-click fee is a fee a firm incurs each time a consumer clicks on its "page" on the platform to view its details. Because a per-click fee is incurred regardless of whether the firm makes a sale on the platform, this would lead to a similar outcome to the use of a registration fee. In contrast, a referral fee is a fee that is only charged when consumers click on the firm's page on the platform and then purchase from the firm directly. The use of cookies and other technologies may make this feasible. In our model, a referral fee would eliminate the incentive of firms to set a lower direct price to encourage consumers to showroom. Thus, a referral fee may be a less restrictive way for platforms to rule out showrooming while preserving the constraint on fees implied by the direct alternative (i.e. consumers considering whether to search directly instead) as well as competition between platforms. The existence of such alternative fee mechanisms further pushes the case for banning even narrow-PPCs.

6 Conclusion

This paper has shown that the fees charged by a platform are constrained by several forces: (i) consumers' willingness to search directly rather than on the platform, (ii) showrooming, (iii) competition with other platforms, and (iv) firms willingness to delist from the platform. Narrow-PPCs remove the first two constraints, and leads to the third constraint (platform competition) determining fees. This may be an acceptable outcome if platform viability under showrooming is in question, and

platform competition is sufficiently effective. In contrast, wide-PPCs remove the platform competition constraint as well, leaving just the fourth constraint (the possibility of firms delisting), resulting in higher fees and prices, and indeed the same outcome that arises with a monopoly platform under a PPC. Thus, our findings support banning wide-PPCs but whether narrow-PPCs should be banned as well depends on whether platforms would remain viable without them.

There are several interesting ways in which our theory can be extended. In this paper, the search cost on platforms is assumed to be exogenously lower than in the direct market. It would be interesting to study what happens instead if platforms can determine how much to invest in lowering their search costs, which provides them with another way to attract more consumers to search on their particular platform. In such a setting, showrooming could arise across platforms, with consumers searching on the most efficient platform but not necessarily completing their purchase on the platform, something wide-PPCs would eliminate but narrow-PPCs would not. In a follow-up paper (Wang and Wright, 2019), we explore this possibility.

We have assumed platforms lower consumers' search costs but that they do not influence consumers choices in other more direct ways, such as providing recommendations or rankings. In practice, if consumers are steered away from firms that are expected to generate less revenue for the platform, then firms may be reluctant to induce consumers to use alternative channels (including buying directly with discounted direct prices). Teh and Wright (2019) indeed find that such steering can reduce or eliminate showrooming, suggesting steering provides another arguably less restrictive way platforms can address showrooming than price parity clauses.

Finally, instead of directly steering consumers, platforms may affect traffic by choosing the accuracy of matching system. Both de Cornière (2016) and Zhong (2016) consider platforms that can use a "broad match" technology to match consumers's queries to firms. A more precise matching system yields a higher match value to consumers on average. If the initial matching quality is poor, improving accuracy of the matching system intensifies the competition among firms on the platform and reduces on-platforms prices. Therefore, in this case, a platform with a better matching system works in a similar way to a platform with a low search cost in that it increases consumers' willingness to use platforms to search. This suggests PPCs should work in a similar way in such a setting.

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Appendix: Proof of Propositions

Proof of Proposition 2. In the proposed equilibrium, firms all join M, setting the direct price $p_d = \lambda(x_d)$ and on-platform price $p_m(f) = f + \lambda(x_m)$, where $p_d \geq p_m(f)$ so that regular consumers do not want to switch to buy directly in equilibrium. A firm's profit in this candidate equilibrium is $\pi^* = n_r\lambda(x_m) + n_d\lambda(x_d)$. For this to be an equilibrium, firm i must not be able to do better by changing p_d^i and/or p_m^i to induce regular consumers to showroom.

Consider such a deviation by firm i. Since firm i can always set p_m^i high enough to make buying directly from firm i better than buying through M, firm i's demand from regular consumers is those who encounter it on M and prefer to switch and buy at p_d^i rather than continuing to search on M. That is, consumers with $v^i - p_d^i \geq x_m - p_m(f) = x_m - (f + \lambda(x_m))$ will switch. Firm i's deviating profit is

$$\pi_d'\left(p_d^i\right) = n_r p_d^i \left[\frac{1 - G(x_m - p_m(f) + p_d^i)}{1 - G(x_m)}\right] + n_d p_d^i \left[\frac{1 - G(x_d - p_d + p_d^i)}{1 - G(x_d)}\right]. \tag{14}$$

Since p_d^i cannot exceed $\overline{v}$ and the deviating profit above is continuously differentiable in p_d^i, a maximum deviating profit exists. Denote the maximum deviating profit $\hat{\pi}_d(f)$, so $\hat{\pi}_d(f) \equiv \max_{p_d^i}\left\{\pi_d'\left(p_d^i\right)\right\}$.

Then M wants to set f as high as possible (to maximize fee revenue) subject to the no-showrooming constraint[19]

$$n_r\lambda(x_m) + n_d\lambda(x_d) \geq \hat{\pi}_d(f). \tag{15}$$

We want to show that the highest possible f satisfying (15) is positive but strictly less than $\triangle_m$.

Suppose $f = 0$. Then the maximum deviating profit is

$$\begin{aligned}
\hat{\pi}_d(0) &= \max_{p_d^i}\left\{n_r p_d^i \left[\frac{1 - G(x_m - \lambda(x_m) + p_d^i)}{1 - G(x_m)}\right] + n_d p_d^i \left[\frac{1 - G(x_d - p_d + p_d^i)}{1 - G(x_d)}\right]\right\} \\
&< n_r \max_{p_m^i}\left\{p_m^i \left[\frac{1 - G(x_m - \lambda(x_m) + p_m^i)}{1 - G(x_m)}\right]\right\} + n_d \max_{p_d^i}\left\{p_d^i \left[\frac{1 - G(x_d - \lambda(x_d) + p_d^i)}{1 - G(x_d)}\right]\right\} \\
&= n_r\lambda(x_m) + n_d\lambda(x_d),
\end{aligned}$$

implying the constraint holds strictly, and M can always do better by increasing f. The strict inequality follows from the assumption that $\lambda'(z) < 0$.

[19] Note the constraint in (8), which ensures regular consumers want to search on M, will be satisfied if the no-showrooming constraint (15) is satisfied.

Suppose instead that $f = \triangle_m$. Then

$$\begin{aligned}\hat{\pi}_d(\triangle_m) &= \max_{p_d^i}\left\{n_r p_d^i\left[\frac{1-G(x_m-\lambda(x_d)+p_d^i)}{1-G(x_m)}\right]+n_d p_d^i\left[\frac{1-G(x_d-p_d+p_d^i)}{1-G(x_d)}\right]\right\}\\ &\geq n_r p_d + n_d p_d\\ &> n_r\lambda(x_m)+n_d\lambda(x_d),\end{aligned}$$

where the weak inequality comes from the fact we can always set $p_d^i = \lambda(x_d)$ in the maximization, and the strict inequality comes from the property that $\lambda(x_d) > \lambda(x_m)$. This means M would never want to set f this high.

Since (14) is strictly increasing in f for a given p_d^i and using the envelope theorem, we know $\hat{\pi}_d$ is strictly increasing in f. Since we have shown that this maximum deviating profit is strictly greater than the equilibrium profit when $f = \triangle_m$ and strictly smaller than the equilibrium profit when $f = 0$, by the intermediate value theorem there must exist a unique f within $(0, \triangle_m)$ which makes (15) bind. This is τ in the proposition. □

Proof of Proposition 3. We start by characterizing equilibria in users' subgames ignoring the possibility that firms may delist, thus explaining the potential pricing equilibria displayed in bold in figure 1. We will later relax that firms all join M.

Consider first case 1, as illustrated in the left-hand panel of figure 1. Recall this arises when $\tilde{f} < x_m - \lambda(x_m)$, so we want to show the results in (i) in Proposition 3 hold. If $\triangle_m \leq f < x_d - \lambda(x_m)$, there is a unique (non-trivial) pricing equilibrium in which prices on both channels equal $p_c(f)$. This reflects that if instead prices are set at $p_m(f) = f + \lambda(x_m) < x_d$ so that firms price as if no one is searching directly, direct consumers would still want to search, thus implying that $p_m(f)$ cannot be the equilibrium price. If $x_d - \lambda(x_m) \leq f \leq \tilde{f}$, there are two possible (non-trivial) pricing equilibria. If direct consumers are expected to search, there is an equilibrium in which firms price at $p_c(f) < x_d$, such that direct consumers indeed want to search. Alternatively, if direct consumers are not expected to search, there is an equilibrium in which firms price at $p_m(f) = f + \lambda(x_m) > x_d$, such that direct consumers indeed do not want to search. Finally, if $\tilde{f} < f \leq x_m - \lambda(x_m)$ then there is a unique pricing equilibrium in which firms price on both channels at $p_m(f) = f + \lambda(x_m) > x_d$. Since $p_c(f) > x_d$ for this range of f, there is no equilibrium in which direct consumers search. On the other hand, regular consumers continue to search on the platform since $p_m(f) \leq x_m$ for this range of f. For yet higher f, consumers would not want to search on M.

Stepping back to M's stage 1 choice of f, clearly M does best by setting f at the highest possible level at which regular consumers still search using the platform. This implies $f = x_m - \lambda(x_m)$. At this fee, the unique non-trivial equilibrium in the user subgame is that direct consumers will not search and firms will price on both channels at

$p_m(f) = f + \lambda(x_m) = x_m$. Given consumers don't search directly, any firm that delists from M would end up strictly worse off as it would not attract any demand. Thus, our assumption that firms continue to join M at the equilibrium f holds true. Note the selection of equilibria in the user subgames with a lower f is immaterial, since M will be strictly worse off with a lower f.

Next consider case 2 in which $\tilde{f} \geq x_m - \lambda(x_m)$ (as illustrated in the right-hand panel of figure 1), again assuming initially that firms continue to list on M. If $\triangle_m \leq f < x_d - \lambda(x_m)$, then for the same reason as in case 1 above, there is a unique (non-trivial) pricing equilibrium in which prices on both channels equal $p_c(f)$. If $x_d - \lambda(x_m) \leq f \leq x_m - \lambda(x_m)$, then the same two pricing equilibria discussed above in case 1 apply, for the identical reason. Finally, if $x_m - \lambda(x_m) < f \leq \tilde{f}$ then there is a unique (non-trivial) pricing equilibrium in which firms price on both channels at $p_c(f) \leq x_d$. Since $f + \lambda(x_m) > x_m$ for this range of f, there is no equilibrium in which only regular consumers search. For yet higher f, even direct consumers would not want to search given the expected price $p_c(f) > x_d$.

Stepping back to M's stage 1 choice of f, clearly M does best by setting f at the highest possible level at which regular consumers still search using the platform and firms still list. Delisting can constrain f in this case since M can set $f > x_m - \lambda(x_m)$ and direct consumers will still search directly, meaning delisting may be profitable. Recall (11) was the condition for a firm to have no incentive to delist. Note the L.H.S. of (11) is increasing in f at the rate

$$\frac{\triangle_m n_r n_d}{\lambda(x_d)\, n_r + \lambda(x_m)\, n_d}. \tag{16}$$

We next show that the R.H.S. of (11) is increasing faster than the L.H.S. of (11) in f for $f > \triangle_m$. From the envelope theorem, the derivative of the R.H.S. with respect to f is

$$\frac{\partial \hat{\pi}_d}{\partial p_c(f)} \frac{\partial p_c(f)}{\partial f} = p'_d n_d \left(\frac{g\left(x_d - p_c(f) + p'_d\right)}{1 - G\left(x_d\right)} \right) \left(\frac{\frac{n_r}{\lambda(x_m)}}{\frac{n_r}{\lambda(x_m)} + \frac{n_d}{\lambda(x_d)}} \right),$$

where $p'_d = \arg\max_{p_d^i} \left\{ p_d^i n_d \left(\frac{1 - G\left(x_d - p_c(f) + p_d^i\right)}{1 - G(x_d)} \right) \right\}$. Using the solution for $p'_d = \frac{1 - G\left(x_d - p_c(f) + p'_d\right)}{g\left(x_d - p_c(f) + p'_d\right)}$, this can be written as

$$\frac{\partial \hat{\pi}_d}{\partial p_c(f)} \frac{\partial p_c(f)}{\partial f} = \frac{\left(1 - G\left(x_d - p_c(f) + p'_d\right)\right)}{1 - G\left(x_d\right)} \frac{\lambda(x_d)\, n_r n_d}{\lambda(x_d)\, n_r + \lambda(x_m)\, n_d}. \tag{17}$$

Compare (17) with (16). Provided $p_c(f) \geq p'_d = \lambda\left(x_d - p_c(f) + p'_d\right)$ when $f > \triangle_m$, then the R.H.S. will be increasing faster with f than the L.H.S. for $f > \triangle_m$. Note we have $p_c\left(f = \triangle_m\right) = \lambda\left(x_d\right)$, and therefore $\lambda\left(x_d\right) < p_c(f)$ for $f > \triangle_m$. So suppose $p_c(f) \leq p'_d$. This implies $x_d - p_c(f) + p'_d \geq x_d$, so $\lambda\left(x_d - p_c(f) + p'_d\right) \leq \lambda\left(x_d\right)$ given $\lambda'(z) < 0$, and $p'_d = \lambda\left(x_d - p_c(f) + p'_d\right) \leq \lambda\left(x_d\right) < p_c(f)$ when $f > \triangle_m$, which is a contradiction.

Therefore, it must be that $p_c(f) > p'_d$, so (17) exceeds (16).

Since the R.H.S. of (11) is increasing faster than the L.H.S. in f for $f > \triangle_m$, there can at most be a single level of $f > \triangle_m$ where the two sides are equal, denoted $\overline{f}$. We next show that $\overline{f}$ exists.

Consider $f = \triangle_m$ so that $p_c(f) = \lambda(x_d)$ according to (10). The L.H.S. of (11) is equal to $n_d\lambda(x_d) + n_r\lambda(x_m)$, which is strictly higher than the R.H.S. of (11), which is equal to $n_d\lambda(x_d)$. Consider $f = \frac{\lambda(x_d)}{n_d} > \lambda(x_d)$ so that $p_c\left(\frac{\lambda(x_d)}{n_d}\right) = \frac{\lambda(x_d)}{n_d}$ according to (10), and $\pi_c\left(\frac{\lambda(x_d)}{n_d}\right) = \lambda(x_d)$. Then we can show

$$\hat{\pi}_d = \max_{p_d^i} \left\{ p_d^i n_d \left[\frac{1 - G(x_d - \frac{\lambda(x_d)}{n_d} + p_d^i)}{1 - G(x_d)} \right] \right\} > \lambda(x_d). \tag{18}$$

To see this, note by setting $p_d^i = \frac{\lambda(x_d)}{n_d}$, the objective function in (18) equals $\lambda(x_d)$. Furthermore, the derivative of the objective function in (18) evaluated at $\frac{\lambda(x_d)}{n_d}$ equals $n_d - 1 < 0$, so $\hat{\pi}_d > \lambda(x_d)$ when p_d^i is set a sufficiently small amount below $\frac{\lambda_d}{n_d}$. Thus, with respect to (11), the L.H.S. exceeds the R.H.S. at $f = \Delta_m$, the R.H.S. exceeds the L.H.S. at $f = \lambda(x_d)/n_d$, and the R.H.S. increases faster than the L.H.S. for $f > \Delta_m$, so by the intermediate value theorem there must exist a unique $\overline{f}$ such that L.H.S. equals the R.H.S.

To avoid firms delisting, M will set the highest possible f in which firms still list (i.e. a non-trivial equilibrium still arises). There are thus three possibilities for case 2:

1. Suppose $\overline{f} > \tilde{f} \geq x_m - \lambda(x_m)$. Then M will set $f = \tilde{f}$. Firms price at $p_c(\tilde{f}) = x_d$, so direct consumers get no surplus but are just willing to search. If M were to set any higher f, then direct consumers would not want to search. If firms price on M assuming only regular consumers search, they would price at $f + \lambda(x_m)$, which would mean regular consumers would also not want to search. Therefore, if M were to set any higher f, it would obtain no revenue.
2. Suppose $\tilde{f} > \overline{f} \geq x_m - \lambda(x_m)$. Then M will set $f = \overline{f}$. Firms price at $p_c\left(\overline{f}\right) < x_d$, so even direct consumers get some positive surplus from search. If M were to set any higher f, then firms would delist and M would obtain no revenue.
3. Finally, suppose $\overline{f} \leq x_m - \lambda(x_m) \leq \tilde{f}$. Then M will set $f = x_m - \lambda(x_m)$. Firms price at $p_m = f + \lambda(x_m) = x_m$ on both channels, so direct consumers do not search. Firms will not want to delist in the proposed equilibrium given consumers are not searching directly. If M were to set any higher f, then firms would delist and M would obtain no revenue. Note if $f = x_m - \lambda(x_m)$ and $\overline{f} < x_m - \lambda(x_m)$ then the candidate equilibrium in the user subgame based on direct consumers searching and firms pricing at $p_c(f)$ does not apply since firms would want to delist. Based on our equilibrium selection rule, we therefore select the equilibrium in the user subgame in which firms list on M and firms price at $p_m = f + \lambda(x_m) = x_m$ on both channels rather than the trivial

equilibrium in which firms do not list on M. In the special case that $\overline{f} = x_m - \lambda(x_m)$ and $f = x_m - \lambda(x_m)$, the candidate equilibrium in the user subgame based on direct consumers searching and firms pricing at $p_c(f)$ cannot be ruled out by firms delisting, which is why this case is also included in 2. above (as stated in footnote 11).

□

Proof of Proposition 6. Given M^1 and M^2 set the same fee f^* in the proposed equilibrium, and $f^* \leq \tau$, consumers will search and buy on their preferred platform. An individual firm i can lower its direct price to induce regular consumers to switch to buy directly. The corresponding deviating profit is the same as in the monopoly case (however, here, the deviation attracts half of the regular consumers to switch from M_1 and half of the regular consumers to switch from M_2). Since $f^* \leq \tau$, such a deviation is not profitable. If $\frac{1}{h(0)} < \tau$, then the analysis of Section 4.1 applies given platform competition is the binding constraint, so $f^* = \frac{1}{h(0)}$. If $\tau \leq \frac{1}{h(0)}$, then $f^* = \tau$. Recall that in the monopoly case, when $f^* = \tau$, an individual firm i is indifferent about deviating in that the highest deviating profit from charging $\hat{p}_d^i$ to attracting some regular consumers (those with $v > x_m - (\tau + \lambda(x_m)) + \hat{p}_d^i$) to switch to buy directly is the same as its equilibrium profit.

Now suppose platform k sets a higher f than platform j, which continues to set $f^j = \tau$. Consider firm i setting the same deviation price (denoted here as $\hat{p}_d^i$) to induce showrooming as in the case of a monopoly platform (see equation (14)) and consider consumers who draw $v > x_m - (\tau + \lambda(x_m)) + \hat{p}_d^i$. Then firm i will attract the same number of these consumers (half from M^j and half from M^k) to switch to buy directly as in (14). But at this direct price $\hat{p}_d^i$, consumers with $v \in (x_m - (f^k + \lambda(x_m)) + \hat{p}_d^i, x_m - (\tau + \lambda(x_m)) + \hat{p}_d^i)$ will also switch from M^k to buy directly, which implies such a deviation is strictly profitable for firm i. This means platform k will end up with no revenue. Thus, neither platform has an incentive to increase f above $f^* = \tau$.

At the same time, lowering f below $f^* = \tau$ is not a profitable deviation either, given $\tau \leq \frac{1}{h(0)}$. This is true from Proposition 5 if $\tau = \frac{1}{h(0)}$. In case $\tau < \frac{1}{h(0)}$, it is true because here M^k's profit is increasing in f^k for all $f^k < \tau$ when M^j sets $f^j = \tau$. To prove this statement, suppose to the contrary that there exists an interior $f^{k*} \in (0, \tau)$ that maximizes M^k's profit. The log-concavity of $1 + H$ implies that f^{k*} is pinned down by $f^{k*} = (1 + H(\tau - f^{k*}))/h(\tau - f^{k*})$. The log-concavity of $1 + H$ and $f^{k*} < \tau$ further imply that $f^{k*} \geq 1/h(0) > \tau$, a contradiction to the initial supposition that $f^{k*} < \tau$. We therefore conclude that M^k has no incentive to reduce its fee below τ.

In this equilibrium, each platform makes $f^* \left(\frac{n_r}{2}\right) - c$, so will enter provided this is non-negative. Since we have already assumed that $c \leq \frac{n_r}{2h(0)}$, this will be true provided we also have $c \leq \frac{n_r \tau}{2}$. □

Proof of Proposition 9. Consider the proposed equilibrium corresponding to (i) in Proposition 3 in which each platform sets $f = x_m - \lambda(x_m)$. If one platform (say M^j) lowers its fee f^j below that of the other platform (say M^k) to try to attract more demand, it will not attract any additional business unless it sets f^j low enough so that the price firms set on M^j (which we denote as $\tilde{p}_m$) is strictly lower than the price firms set on the direct channel (i.e. p_d). To see this, note that if $\tilde{p}_m \geq p_d$, regular consumers that previously preferred to buy from M^k in equilibrium will all prefer to complete their transaction with firms directly (so as to avoid the transaction cost a) rather than on M^j, and so M^j will not attract any additional consumers by lowering f^j. Therefore, suppose f^j is set so $\tilde{p}_m < p_d$. If all firms exclusively join M^j, all regular consumers, including those whose preferred platform is M^k, will use M^j to search. However, among the regular consumers whose preferred platform is M^k, only those with small enough cost a will buy on M^j after searching on M^j, with the rest switching to buy direc0tly from firms. Specifically, they will buy on M^j provided $a < p_d - \tilde{p}_m$, so M^j's revenue will be $n_r \left(\frac{1}{2} + \frac{H(p_d - \tilde{p}_m)}{2}\right) f^j$.

We claim the maximum deviation revenue that M^j can achieve following the above deviation arises if we set $\overline{a} = 0$. This puts the least restriction on the extent to which $\tilde{p}_m$ must be less than p_d in order that M^j attract the consumers who prefer M^k, and therefore the least restriction on M^j's choice of f^j such that it attracts all these consumers. To show this formally, define M^j's deviating revenue as a function of $\overline{a}$. The claim is it will be at its maximum at $\overline{a} = 0$. Moreover, the corresponding maximum possible deviating revenue is $n_r \triangle_m$, since the equilibrium prices $\tilde{p}_m = f^j + \lambda(x_m)$ and $p_d = \lambda(x_d)$ satisfy $\tilde{p}_m \leq p_d$, so M^j can set $f^j \leq \lambda(x_d) - \lambda(x_m) = \triangle_m$ to attract all consumers to buy on M^j. Suppose this is not true and the maximum deviating revenue is achieved for some $\overline{a} > 0$. Then f^j has to be strictly greater than $\triangle_m$ as the demand cannot exceed n_r. In this case there must be some regular consumers switching to buy directly after searching in M^j. (If there were no consumers switching, then the equilibrium prices must be respectively $\tilde{p}_m = f^j + \lambda(x_m)$ and $p_d = \lambda(x_d)$, but then no switching implies $f^j \leq \triangle_m$, which contradicts our presumption that $f^j > \triangle_m$.) Fix this $f^j > \triangle_m$ and reduce $\overline{a}$. This must increase demand to M^j as less regular consumers will want to switch if $\overline{a}$ is sufficiently close to zero. Therefore, M^j's deviating revenue increases. Since such property holds for an arbitrary $\overline{a} > 0$, it contradicts the claim that some $\overline{a} > 0$ maximizes M^j's deviating revenue. Thus M^j's deviating revenue is maximized at $\overline{a} = 0$.

We have shown that an upper bound on the maximum deviating revenue that M^j can achieve (which arises by setting $\overline{a} = 0$) is $n_r \triangle_m$. Thus, a sufficient condition to rule out such undercutting being profitable is that the platforms each get revenue exceeding $n_r \triangle_m$ in the proposed equilibrium. Since they share the market evenly in the proposed equilibrium, this requires $\frac{f}{2} > \triangle_m$ where $f = x_m - \lambda(x_m)$ is the proposed equilibrium fee, or equivalently (13) holds.

Now consider the proposed equilibrium corresponding to (ii) in Proposition 3. From the proof of Proposition 3, we know that there are two subcases to consider, whereby both platforms either set $\tilde{f}$ or $\overline{f}$:

1. In equilibrium, both platforms set their fee equal to $\tilde{f}$. By construction, firms will not want to delist from one or both platforms at this equilibrium fee. If either platform increases its fee, it will induce delisting for the same reason as explained in the case of a monopoly platform. If a single platform lowers its fee in order to try to induce delisting from the other platform, then the logic of case 1 applies, given $\triangle_m < \frac{x_m - \lambda(x_m)}{2} < \frac{\tilde{f}}{2}$.
2. In equilibrium, both platforms set their fee equal to $\overline{f}$ where $x_m - \lambda(x_m) \leq \overline{f} < \tilde{f}$. If either platform increases its fee, it will induce delisting for the same reason as explained in the case of a monopoly platform. Moreover, a platform will not reduce its fee since the logic of case 1 applies, given $\triangle_m < \frac{x_m - \lambda(x_m)}{2} < \frac{\overline{f}}{2}$.

Thus, the monopoly equilibrium fee remains an equilibrium fee with competing platforms. □

Online Appendix

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This online appendix provides formal details for some claims in the main text.

A Unobserved fees

In this section we reconsider the analysis of Section 3 in the main text in case consumers cannot observe M's fee f. Then M may find it profitable to slightly increase its fee above the level consumers expect, thus leading to a hold-up problem.[1] To explore this possibility, suppose regular consumers, once searching on M, believe that any deviation in the observed price from the equilibrium price is due to M setting a fee f different from the expected one, with firms optimally setting their prices based on this f in equilibrium. Under these beliefs, if M does deviate in setting a higher or lower f compared to the equilibrium level of f, consumers will correctly infer f after sampling the first firm on M.

Given consumers can switch to buy directly in case PPC is not imposed, consumers do not need to buy on M and therefore will switch if they find the actual fee is too high. In particular, they will switch to buy directly if they find the fee is greater than τ as defined in Section 3.2. Thus M will still not want to set a fee higher than τ, so our equilibrium characterization is still valid under showrooming even if f is not observed by consumers.

Now suppose M imposes a PPC. In this case, if regular consumers choose to search on M, they will either buy from M or not buy at all. Following the analysis in Section 3.3, there are two types of equilibrium. Consider first the equilibrium in which regular consumers are left with positive surplus, so f is constrained by the possibility that firms delist from M and only sell directly. If M sets f higher than this proposed equilibrium level, firms will delist and therefore M does not want to set a higher fee, so our equilibrium characterization is still valid in this case even if f is not observed by consumers.

Finally, consider the equilibrium in which a PPC leads to full surplus extraction. In this case, M's equilibrium profit as characterized in Section 3.3 is $n_r(x_m - \lambda(x_m))$. If M deviates to set a higher fee, regular consumers will stop searching after inspecting the first firm searched on M since they can infer the true fee level from observing the price after their first search and expect negative surplus from continuing search. Only those consumers who find a match value higher than the observed price will buy at the

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[1]Note the hold-up problem can also apply to the direct consumers under a PPC since the price they pay in the direct market is also determined by the same fee.

first searched firm. The firm, acting like a standard monopolist, will choose p to maximize $(p-f)(1-G(p))$. Call this solution $p^M(f)$. Given this pricing, if M chooses f greater than $x_m-\lambda(x_m)$, it will set f to maximize $n_r f(1-G(p^M(f)))$. Our equilibrium characterization continues to apply provided $\max_f\{n_r f(1-G(p^M(f)))\}$ is no higher than $n_r(x_m-\lambda(x_m))$, thus ruling out the hold-up problem.

To illustrate this possibility, suppose v is uniformly distributed on $[0,1]$ so that $G(v)=v$. M's equilibrium profit under full-surplus extraction is thus $n_r(x_m-\lambda(x_m))=n_r(2x_m-1)$. Now suppose M raises its fee above $x_m-\lambda(x_m)$. Each firm prices as a monopolist and consumers do not search further after searching the first firm. Each firm's optimal price is then $p^M(f)=\frac{1+f}{2}$ and the resulting total demand is $\frac{n_r(1-f)}{2}$. Without considering the constraint $f>x_m-\lambda(x_m)$, M's optimal fee given that firms set the monopoly price can be derived by solving $\max_f\{f(\frac{n_r(1-f)}{2})\}$. The solution is $f=\frac{1}{2}$ and the resulting profit is $\frac{n_r}{8}$. Clearly, a sufficient condition for our equilibrium results to continue holding is $2x_m-1\geq\frac{1}{8}$, or equivalently, $x_m\geq\frac{9}{16}$. This requirement is not much stronger than the minimum requirement for active search that we assumed throughout the paper; i.e. $x_m\geq\frac{1}{2}$ for this particular G.

B Asymmetric equilibrium

In the main text, we always select the symmetric equilibrium in which the *ex ante* homogeneous firms make the same decisions. Under this selection rule, as shown in case 1 of Proposition 3, the direct channel may be shut down by a PPC even though consumers and firms could be jointly better off by exclusively trading on the direct channel. Therefore, an alternative equilibrium selection rule that may be more plausible in this case is that some firms may only sell directly ("direct firms") in order to serve direct consumers, while other firms still sell on both channels ("listing firms") but are subject to PPC. In this extension, we explore this alternative equilibrium selection rule. Specifically, we select this asymmetric equilibrium (if it exists) whenever the symmetric equilibrium we characterize in the main text implies an inactive direct market.

Let the fraction of listing firms be denoted $\eta\in(0,1)$, with the remaining $1-\eta$ of firms being direct firms. For simplicity, we assume that direct consumers can distinguish the two types of firms when searching so they can search among the direct firms only. This will be more efficient for them given in the equilibrium characterized below, the direct firms set lower prices than the listing firms. In the main text we showed the direct channel will only be shut down when a PPC is imposed and we are in case 1 of Proposition 3, so we only need to consider this case. In the equilibrium in which direct consumers only search and buy from the direct firms, direct firms will set the price $p_d=\lambda(x_d)$. In addition, given that they will not sell to direct consumers, the listing firms set the benchmark equilibrium

price $p_m = f + \lambda(x_m)$. Since regular consumers can get an expected surplus $x_d - \lambda(x_d)$ by searching directly, M needs to set a fee which ensures that consumers are willing to search using M, i.e.

$$x_m - (f + \lambda(x_m)) \geq x_d - \lambda(x_d) \quad \Leftrightarrow \quad f \leq \Delta_s + \Delta_m.$$

Furthermore, firms must be indifferent between listing on M and only selling directly, i.e.

$$\frac{n_r \lambda(x_m)}{\eta} = \frac{(1 - n_r)\lambda(x_d)}{1 - \eta}.$$

This condition pins down the fraction of listing firms, $\eta^* = \frac{n_r \lambda(x_m)}{n_r \lambda(x_m) + (1-n_r)\lambda(x_d)}$. Then M optimally sets $f = \Delta_s + \Delta_m$ to extract maximal surplus while keeping regular consumers searching on M.

Now we can compare this outcome to the case without a PPC. If M can operate without imposing a PPC ($c \leq n_r \tau$), the price for regular consumers is $\tau + \lambda(x_m)$ and the direct price is $\lambda(x_d)$. Under a PPC, the price for regular consumers is $\Delta_s + \lambda(x_d)$ and the direct price is $\lambda(x_d)$. Since $\tau < \Delta_m$, a PPC increases both the regular consumer price and the weighted average price, which in turn leads to lower consumer surplus. Firms' total profit and the total welfare are not changed by PPCs.

If M cannot operate without imposing a PPC ($c > n_r \tau$), all consumers search and buy directly and firms set direct price $\lambda(x_d)$ without a PPC. Without a PPC, consumer surplus is $x_d - \lambda(x_d)$. With a PPC, consumer surplus is $n_r[x_m - (\Delta_s + \lambda(x_d))] + (1 - n_r)(x_d - \lambda(x_d)) = x_d - \lambda(x_d)$. Imposing a PPC leads to a higher price for regular consumers and a higher average price, but no effect on direct consumers.

C Simultaneous platform entry

Suppose $\frac{n_r \tau}{2} < c \leq n_r \tau$, so that it is only profitable for one platform to enter under the case with showrooming. If each platform has to decide whether to enter at the same time, then we can characterize a symmetric mixed strategy equilibrium in which each platform enters with probability α. The equilibrium entry condition for a platform is then

$$\alpha \frac{n_r \tau}{2} + (1 - \alpha) n_r \tau - c = 0,$$

which implies in equilibrium each platform enters with the probability $\alpha^* = 2(1 - \frac{c}{n_r \tau})$. So the revised version of Proposition 6 that takes into account simultaneous platform entry is as follows:

Proposition 10. (Showrooming equilibrium with differentiated platforms)

(i) *Suppose* $c \leq \frac{n_r\tau}{2}$. *Both platforms operate and set the fee* $f^* = \min\left\{\frac{1}{h(0)}, \tau\right\}$. *Firms join both platforms and set the common on-platform price* $\min\left\{\frac{1}{h(0)}, \tau\right\} + \lambda(x_m)$ *and the direct price* $\lambda(x_d)$.

(ii) *Suppose* $\frac{n_r\tau}{2} < c \leq n_r\tau$. *Each platform operates with probability* $2(1 - \frac{c}{n_r\tau})$. *If both platforms turn out to operate, fees and prices are characterized as in (i). If only one platform turns out to operate, it sets the fee* $f^* = \tau$. *Firms join the operating platform and set the on-platform price* $p_m^* = \tau + \lambda(x_m)$ *and the direct price* $\lambda(x_d)$. *In case no platform turns out to operate, firms set the direct price* $\lambda(x_d)$.

(iii) *Suppose* $c > n_r\tau$. *Neither platform operates. Firms set the direct price* $\lambda(x_d)$.

Given this result, consider the implications of a narrow-PPC. In the range of $c \leq \frac{n_r\tau}{2}$ and $c > n_r\tau$, the equilibrium characterizations in Proposition 6 continue to apply.

In the range $\frac{n_r\tau}{2} < c \leq n_r\tau$, we know each platform enters with probability $\alpha = 2(1 - \frac{c}{n_r\tau})$. Then on-platform prices with narrow-PPC are $\frac{1}{h(0)} + \lambda(x_m)$, while without narrow-PPC the price is $\lambda(x_d)$ with probability $(1-\alpha)^2$ in case no platform enters and $\tau + \lambda(x_m)$ with probability $1-(1-\alpha)^2$ in case one or both platforms enter. So comparing the expected prices, the prices paid by regular consumers will be higher (in expectation) under narrow-PPC whenever

$$\frac{1}{h(0)} + \lambda(x_m) > (1-\alpha)^2 \lambda(x_d) + \left(1 - (1-\alpha)^2\right)(\tau + \lambda(x_m)).$$

Rearranging and substituting in the solution $\alpha = 2(1 - \frac{c}{n_r\tau})$, we have that the prices paid by regular consumers are higher (in expected value) under narrow-PPC if

$$c < \frac{n_r\tau}{2}\left(1 + \sqrt{\frac{\frac{1}{h(0)} - \tau}{\Delta_m - \tau}}\right),$$

and lower if the inequality is reversed.

Next compare consumer surplus for regular consumers in case $\tau < \frac{1}{h(0)}$ and $\frac{n_r\tau}{2} < c \leq n_r\tau$. Then regular consumers' surplus with narrow-PPC is $x_m - \left(\frac{1}{h(0)} + \lambda(x_m)\right)$, while without narrow-PPC it is $x_d - \lambda(x_d)$ with probability $(1-\alpha)^2$ and $x_m - (\tau + \lambda(x_m))$ with probability $1-(1-\alpha)^2$. So comparing expected surplus, this is lower under narrow-PPC whenever

$$x_m - \left(\frac{1}{h(0)} + \lambda(x_m)\right) > (1-\alpha)^2(x_d - \lambda(x_d)) + \left(1 - (1-\alpha)^2\right)(x_m - (\tau + \lambda(x_m))).$$

Rearranging and substituting in the solution $\alpha = 2(1 - \frac{c}{n_r\tau})$, we have that regular con-

sumers' surplus is lower (in expectation) under narrow-PPC if

$$c < \frac{n_r \tau}{2}\left(1 + \sqrt{\frac{\frac{1}{h(0)} - \tau}{\Delta_m + \Delta_s - \tau}}\right),$$

and higher if the inequality is reversed.

Next compare firms' profit in case $\tau < \frac{1}{h(0)}$ and $\frac{n_r \tau}{2} < c \leq n_r \tau$. Firms' profit with narrow-PPC is $\lambda(x_m)$, while without narrow-PPC it is $\lambda(x_d)$ with probability $(1-\alpha)^2$ and $\lambda(x_m)$ with probability $1-(1-\alpha)^2$. Thus, firms' expected profit is always lower as a result of narrow-PPC in this range.

Platform's expected profit without narrow-PPC in case $\tau < \frac{1}{h(0)}$ and $\frac{n_r \tau}{2} < c \leq n_r \tau$ is zero, by construction, so narrow-PPC unambiguously increases the platform's profit in this range.

Finally, compare total welfare in case $\tau < \frac{1}{h(0)}$ and $\frac{n_r \tau}{2} < c \leq n_r \tau$. Total welfare with narrow-PPC is $n_r x_m + n_d x_d - 2c$, while without narrow-PPC it is x_d with probability $(1-\alpha)^2$, $n_r x_m + n_d x_d - c$ with probability $2\alpha(1-\alpha)$, and $n_r x_m + n_d x_d - 2c$ with probability α^2 without narrow-PPC. So comparing expected welfare, this is lower under narrow-PPC whenever

$$n_r x_m + n_d x_d - 2c < (1-\alpha)^2 x_d + 2\alpha(1-\alpha)(n_r x_m + n_d x_d - c) + \alpha^2 (n_r x_m + n_d x_d - 2c),$$

or equivalently,

$$\alpha > 1 - \frac{2c}{n_r \Delta_s}.$$

Substituting in the solution $\alpha = 2(1 - \frac{c}{n_r \tau})$, we have that expected welfare is lower under narrow-PPC if

$$c < \frac{n_r}{2\left(\frac{1}{\tau} - \frac{1}{\Delta_s}\right)}$$

and is higher if the inequality is reversed.

Proposition 11. (Implication of narrow-PPCs)

(i) Competition constraint is the relevant constraint: *Suppose* $\frac{1}{h(0)} \leq \tau$. *Narrow-PPCs are irrelevant.*

(ii) Showrooming constraint is the relevant constraint: *Suppose* $\tau < \frac{1}{h(0)}$.

(ii-a) *When* $c \leq \frac{n_r \tau}{2}$, *the use of narrow-PPCs increases consumer prices, decreases regular consumers' surplus, leaves firms' profit unchanged, increases the platforms' profit, and leaves total welfare unchanged.*

(ii-b) *When* $\frac{n_r \tau}{2} < c \leq n_r \tau$, *the use of narrow-PPCs increases expected consumer prices*

iff $c < \frac{n_r\tau}{2}\left(1+\sqrt{\frac{\frac{1}{h(0)}-\tau}{\Delta_m-\tau}}\right)$, *decreases regular consumers' expected surplus iff* $c < \frac{n_r\tau}{2}\left(1+\sqrt{\frac{\frac{1}{h(0)}-\tau}{\Delta_m+\Delta_s-\tau}}\right)$, *lowers firms' expected profits, increases the platforms' expected profit, and decreases total welfare iff* $c < \frac{n_r}{2\left(\frac{1}{\tau}-\frac{1}{\Delta_s}\right)}$

(ii-c) *When* $c > n_r\tau$, *the use of narrow-PPCs decreases consumer prices, increases consumer surplus, decreases firms' profit, increases the platforms' profit, and increases total welfare if* $n_r\triangle_s > 2c$.