Competitive bottlenecks and platform spillovers*

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Abstract

The classic competitive bottleneck setting of Armstrong (2006) provides a useful way to understand how despite apparent competition between platforms to attract buyers who only join one of the platforms, they may not compete at all for sellers on the other side who potentially want to reach buyers on all platforms. We extend this classic insight to more general settings which take into account the pass-through of platform fees through seller pricing and allow for a wide range of platform design choices. An equivalence result between the equilibrium outcome and a "seller-excluded" welfare benchmark shows how such platforms may tend to make design choices that are distorted against sellers' interests. We apply this equivalence result to show that, compared to choices that would maximize total welfare, platforms set excessive commission fee levels, excessive first-party entry and self-preferencing, insufficient platform investment, and excessively stringent policies aimed at limiting sellers from monetizing from buyers outside the platform. A key condition for the equivalence result is the absence of spillovers across platforms with respect to their design choices. In settings where such spillovers arise, we identify conditions under which our welfare results continue to apply. Several features of mobile app platforms fit our framework, suggesting that such platforms' design choices may be biased against sellers' interests in a way that is harmful to overall welfare.

1 Introduction

Concerns around the gatekeeper role of big-tech platforms in controlling the access of developers, suppliers and advertisers to end consumers has motivated major new legislation around the world. In Europe, the Digital Markets Act will come into force in 2024, in China the Anti-Monopoly commission released new Anti-Monopoly Guidelines for the Platform Economy in 2021, and in the U.S. fives acts were proposed in 2021, including the Open App Markets Act and the American Innovation and Choice Online Act.

Given the digital services in question are sometimes dominated by a single firm (Google for search and for browsers, Meta for social networks, and Amazon for ecommerce, in certain regions), this is perhaps not surprising. But in other cases these issues have been raised despite there being more than one large platform offering a similar service. A case in point is mobile app platforms, which enable millions of developers to distribute their apps to the billions of consumers that use

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mobile devices globally. In this case there are two major players: Apple, with iOS and the App Store, and Google, with Android and the Play Store. Yet these platforms are often cited as the main examples of gatekeepers, with the DMA and other proposed laws applying to them. This raises important questions: in what way are these apparently competing platforms, gatekeepers still? If they are, what types of regulations may be most effective for these types of platforms? Should the new laws also apply to them?

One economic theory that may be usefully applied to such situations is that of "competitive bottlenecks", introduced in Armstrong (2006), and further developed in Armstrong and Wright (2007) and Belleflamme and Peitz (2019a). It considers the classic competitive bottleneck setting of two competing platforms where all buyers have to singlehome (they can only join one of the two platforms) and all sellers (e.g., suppliers or developers) are free to multihome (they can join either or both platforms). In such settings, each seller's decision to join a given platform is strategically independent of its decision to join the other platform. As a consequence of such independence, despite the fact platforms may compete for buyers, they act as gatekeepers in selling access to these buyers, not competing at all to attract sellers. This results in an equilibrium outcome where each platform's fees charged to sellers maximize buyers' surplus and platform profit, ignoring the surplus of the sellers, leading platforms to set fees to sellers as if each platform is a monopoly on the seller side.

The competitive bottleneck insight has been appreciated and reproduced in various policy documents (e.g., Belleflamme and Peitz (2019a) and the references therein), as well as being applied to different market sectors and used in various empirical models (as surveyed in the next section). But surprisingly, apart from in Armstrong (2006) initial setting, there are few welfare results, and most of the analysis of this setting has been developed based on certain key assumptions, assumptions that don't fit well marketplace applications such as mobile app platforms. For instance, Armstrong (2006) and most of the subsequent literature assumes that the fees charged on both sides are membership fees, there is no pricing by sellers to buyers, and as a result there is no pass through of the fees platforms charge sellers back to buyers. With the exception of Armstrong (2006), these existing works focus on predicting the equilibrium structure of platform membership fees across the two sides rather than welfare implications. Moreover, policymaker interest goes beyond fee setting, with policymakers raising concerns about other platform choices as well (Crémer et al., 2019; Scott Morton et al., 2019; Furman et al., 2019).[1]

To address these gaps, in this paper, we (i) consider a general framework of a competitive setting, and (ii) develop welfare results in this general setting. The framework takes into account the nature of buyer-seller interactions and that platforms make multi-dimensional choices over a range of design choices in addition to seller-side fees. We illustrate this with examples where the platforms' instruments include commission fees, investments, choosing whether to enter to compete with sellers by offering their own products and whether to steer buyers towards such products, policies that limit the ability of buyers and sellers to bypass the platform (i.e., disintermediation), and app tracking policies in the case of mobile app platforms. The framework can also incorporate monopolistic or competing sellers setting prices to buyers, and allows for pass-through of platform

[1] For instance, policymakers are concerned platforms enter with their own products and steer buyers towards them, and that they prevent sellers from directing their customers to cheaper channels.

fees from sellers to buyers. We illustrate the ability of the framework to capture such settings with some micro-founded applications.

Using this framework, we provide a simple way to operationalize the welfare effects of competitive bottleneck settings. We define a benchmark based on the joint surplus of buyers and the profit of the platforms without considering any surplus obtained by sellers. We call this the "seller-excluded" welfare benchmark. In our baseline setting, the equilibrium outcome is equivalent to the outcome maximizing this seller-excluded welfare. This equivalence result allows us to readily obtain welfare properties based on how various platform instruments affect seller surplus. Specifically, if sellers are worse off as a result of higher levels of some platform instrument, then the equilibrium that arises from platform "competition" will exhibit excessive levels of that instrument from a welfare perspective. Thus, we find in the competitive bottleneck settings, platforms choose excessive commission fee levels, excessive first-party entry, excessive self-preferencing, insufficient investment, excessive limitations on disintermediation by sellers, and excessively stringent app tracking policies.

We then extend the framework to consider situations where the equivalence result breaks down. A key reason why platforms' choice of instruments can deviate from the seller-excluded outcome is if there are spillovers across platforms. We decompose these into two fundamental types: spillovers from a platform's choice of seller-side instruments on the utility buyers obtain on the other platform and on the other platform's revenue (holding buyer-side market shares constant). We explain how such spillovers naturally arise when there are within-seller economies of scale (e.g., sellers face fixed entry costs), within-seller network effects (e.g., sellers' products enjoy network effects), sellers set uniform prices across the platforms (e.g., if platforms impose price-parity clauses), and when sellers can shift transactions to a direct channel. Such utility and revenue spillovers break down the link between the equilibrium levels and the seller-excluded outcome. This is because each individual platform does not take into account the externalities of its choices on the rival platform's revenue and buyer utility, and indeed wants to distort its seller-side instruments to lower the rival platform's buyer utility so as to shift more demand to its own platform. We show how negative utility and revenue spillovers can lead to an additional source of distortion of platforms' choices of instruments away from efficient levels. In addition to such spillovers, we show how imposing constraints on buyer-side monetization and allowing for myopic buyers can also cause the equivalence result to break down, and provide conditions to sign welfare effects in these cases too.

We apply the lessons of our framework to the case of mobile app platforms.[2] Even though in this context the fees charged by platforms to developers can be constrained to some extent by each platform taking into account how its fees may get partially passed through to consumers and thus consumers' choice of which platform to adopt, our result implies the platforms still do not compete for developers or take into account their interests. This implies their choices of commissions, first-party entry, self-preferencing, investment, prevention of disintemediation and limitations on app tracking are distorted away from efficient levels, as discussed above. Moreover, the economics of mobile app platforms suggest several negative utility and revenue spillovers arise, thus suggesting

[2] In addition to this application and the many existing applications of competitive bottleneck theory noted in the next section, our framework can also be usefully applied to local Internet-providers, where consumers typically only subscribe to one network but developers will pay for prioritized service across each of the local networks (see Greenstein et al. (2016)).

an additional source of distortion of their choices of these instruments away from efficient levels. These distortions may be alleviated by stopping platforms from banning (or otherwise limiting) alternative app stores and the direct downloading of apps on their operating systems. Doing so would open up alternative ways for app developers to get around the bottleneck they face in accessing consumers who are unique to a given platform. Such policy changes will be introduced as part of Europe's Digital Markets Act (DMA) and have been proposed in other jurisdictions as well.

1.1 Related literature

A number of other works have built on Armstrong's competitive bottleneck setting. Sticking to the same structure of pricing as Armstrong (2006), with membership fees on both sides, Armstrong and Wright (2007) formalize the condition for singlehoming to arise on one side and multihoming on the other, rather than just assuming all users on one side (e.g., buyers) singlehome.[3] They also analyze the case that platforms can impose exclusivity on the side that would otherwise multihome to explore how it can change results compared to the competitive bottleneck outcome. Hagiu (2009) more explicitly takes into account seller competition but focuses on the case with membership fees still.[4] Other works have also examined how homing configurations affect the equilibrium outcome. Belleflamme and Peitz (2019a) compare the surplus implications (on each side of users as well as the total welfare) of the equilibrium in the two-sided singlehoming benchmark setting with what happens in the competitive bottleneck setting, while allowing for partial multihoming on the seller-side (i.e., not all sellers multihome) on the equilibrium path. Likewise, Bakos and Halaburda (2020) start from the competitive bottleneck and two-sided singlehoming settings, showing that multihoming on both sides weakens platforms' incentive to cross-subsidize across the two sides of the market. Reisinger (2014) focuses on an exogenous competitive-bottleneck homing specification (singlehoming buyers and competing sellers) but allows for platforms to charge two-part tariffs on each side. Tremblay et al. (2023) explores alternative homing assumptions including a competitive bottleneck setting in a Cournot competition model.

Compared to this existing literature, we provide a more general framework to reconsider competitive bottlenecks in. This framework allows for richer microfoundations such as sellers' pricing, pass-through of fees and entry decisions, and allows for platforms to choose multiple types of seller-side fees and other platform design instruments. In addition, unlike this earlier literature, we explicitly characterize how equilibrium platform fees and design choices are distorted away from welfare maximizing outcomes. Some other recent works also explore distortions in platform design. Teh (2022) considers how these distortions relate to the business model of a monopoly marketplace platform, while Choi and Jeon (2023) consider distortions caused by ad-funded platforms.

In considering welfare results arising in the context of mobile app platforms and app developers, our paper is related to the recent contributions by Etro (2023) and Jeon and Rey (2023). Etro

[3] This line of work contrasts with Rochet and Tirole (2003)'s framework of competing platforms. They focus on platforms that charge transaction fees to both sides, and assume both buyers and sellers are free to multihome. Recently, Teh et al. (2023), show that when multihoming buyers have strong enough preferences for using a particular platform to complete a transaction, a conclusion similar to the classic competitive bottleneck insight can still emerge.

[4] Karle et al. (2020) go further to explain how competitive conditions among sellers shape market structure among homogeneous competing platforms. In their setting, two competing sellers would never both multihome under their equilibrium selection, so the competitive bottleneck insights that we focus on does not arise in their setting.

(2023) considers a setup where platforms compete for singlehoming buyers via device prices and charge sellers (i.e., developers) ad-valorem commissions, where sellers are free to multihome and their participation decisions are independent across platforms. In his setting, sellers set their prices under monopolistic competition, allowing for the pass-through of seller-side fees back to buyers. He shows that competition through device prices results in the redistribution of commission revenues back to buyers such that the equilibrium seller-side commissions turn out to maximize buyer surplus.

Jeon and Rey (2023) consider an alternative setup where sellers need to incur a fixed cost to develop apps before joining any platform, so that seller participation decisions across platform are generally interdependent. They show that when the cost of porting apps to a new platform is low (so that most sellers never singlehome — they either join no platform or multihoming on both platforms), the platforms set commissions above the buyer-surplus maximizing benchmark.[5] Intuitively, platforms fail to internalize the negative externality generated by a higher commission on the entry of sellers on the rival platform. Our results are consistent with theirs. However, like Etro, they do not characterize their outcome in terms of the seller-excluded outcome. Doing so helps to understand more generally the drivers of fees (and other instrument choices) relative to a total welfare benchmark, and to identify the more general utility and platform revenue spillover conditions that explain distortions in equilibrium platform fees and other instruments.

The competitive bottleneck setting has been used to study many other applied settings in recent years. Some of the more notable applications include Choi (2010) and Choi and Jeon (2021) on tying; Hagiu and Hałaburda (2014) and Belleflamme and Peitz (2019b) on pricing disclosure; Edelman and Wright (2015) on price coherence and price parity restrictions; Anderson and Coate (2005) and the many subsequent works that build on their framework of media markets, and Hagiu and Lee (2011) and Carroni et al. (2023) for exclusive content. Empirical applications include Yellow Pages (Rysman, 2004), magazines (Kaiser and Wright, 2006; Song, 2021), and video game consoles (Lee, 2013).

2 Model setup

Our environment generalizes Armstrong (2006)'s competitive bottleneck setting. There are $m \geq 2$ platforms. To fix ideas, suppose buyers are the ones to singlehome. Buyers choose one of m platforms to join (i.e. are restricted to singlehome) while sellers can choose to join any number of platforms (e.g., none, one, two, ... , all m platforms) reflecting that they are free to multihome.[6]

Each platform $i = \{1, ..., m\}$ charges a lump-sum membership fee to buyers, P_i^B, and also chooses an n-dimensional "instrument vector" $a_i \in \mathcal{A} \subseteq \mathbb{R}^n$. We allow the instrument vector to have a general interpretation which allows for monetary fees, investment, and (possibly discrete) platform design choices that affect buyers and/or sellers. For example, a_i could be a three-dimensional vector indicating the level of ad-valorem transaction fees (r_i) and lump-sum membership fee (P_i^S) charged to sellers, and the level of platform investment (I_i) on its marketplace that benefit buyers

[5] Hagiu (2009) also allows for seller economies of scale in joining platforms, but does not explore the welfare implications of this feature.

[6] In Section 4.2, we discuss how our framework can accomodate situations where some sellers are restricted to singlehoming.

(e.g., increases the value to buyers of making a transaction) and/or sellers (e.g., by increasing the value of transactions to buyers, it allows sellers to charge more); i.e.,

$$a_i = (r_i, P_i^S, I_i) \in \mathcal{A} \subseteq [0,1] \times \mathbb{R}_+ \times \mathbb{R}_+. \tag{1}$$

To ensure well-defined maximization problems, we assume set $\mathcal{A}$ is compact.[7] Restrictions on what vector a_i could capture will be clear below when we introduce additional functional assumptions. Let $\boldsymbol{a} = (a_1, ..., a_m) \in \mathcal{A}^m$ and $\boldsymbol{P}^B = (P_1^B, ..., P_m^B)$ denote the profile of platform instrument vectors and membership fees to buyers, where we use the bold form throughout the paper to denote profiles of objects involving all m platforms.

□ **Buyers.** Let $\boldsymbol{s} = (s_1, ..., s_m) \in [0,1]^m$ denote platforms' buyer-side market share profile, with each entry $s_i \in [0,1]$ being platform i's market share of buyers, which is endogenously determined. Let $\boldsymbol{\epsilon} = (\epsilon_1, ..., \epsilon_m)$ denote the idiosyncratic match values of a buyer with m platforms, which measures buyer horizontal preference for platforms. Following the literature, we assume that there is a continuum of heterogenous buyers (of measure one) and each buyer knows her vector of matching values $\boldsymbol{\epsilon}$. From the perspective of the platforms, $\boldsymbol{\epsilon}$ can be viewed as following some joint cumulative distribution function (CDF) $F(\cdot)$.

Without imposing any particular microfoundations, we posit that each platform i's net buyer utility for each buyer is

$$U_i(\boldsymbol{a}; \boldsymbol{s}) - P_i^B + \epsilon_i. \tag{2}$$

Here, U_i is the gross utility buyers get from participating on platform i and interacting with sellers. Typically, U_i depends on the mass of participating sellers on platform i and sellers' decisions (e.g., their pricing and investment on platform i), both of which depend on the fee levels and design choice of platforms i (as captured by a_i in the instrument vector profile $\boldsymbol{a}$) and the mass of buyers on platform i due to cross-group network externalities (as measured by s_i in the market share profile $\boldsymbol{s}$). The presence of s_i in U_i also captures any potential same-side network effects on the buyer side. These effects are captured by writing U_i as a function of $(\boldsymbol{a}, \boldsymbol{s})$ in general, rather than just being a function of the measure of sellers joining platform i. This allows us to also capture the possibility of U_i depending on a_{-i} (instrument vectors of all platforms $j \neq i$) and s_{-i} (market shares of all platforms $j \neq i$) so as to capture possible spillovers across platforms, which we illustrate with applications in Section 4.

On the buyer side, we assume single-homing and full market coverage: each buyer participates in one and only one platform, so $\sum_{i=1}^m s_i = 1$. Then, the measure of buyers joining platform i is expressed as

$$s_i = \Pr\left(U_i - P_i^B + \epsilon_i \geq \max_{j \neq i} \left\{U_j - P_j^B + \epsilon_j\right\}\right), \tag{3}$$

where the probability is based on the distribution $F(\cdot)$. CDF $F(\cdot)$ is continuously differentiable and symmetric across the m platforms. This formulation is general enough to permit several specifications commonly used in the literature, including the case of independent and identically distributed (IID) shocks across platforms (Perloff and Salop, 1985), as well as alternative correlation

[7] Note that a_i can include seller-side fee decisions of platforms, which are unbounded in principle. However, one can bound the set of feasible fees by either introducing a "choke price" (above which no seller participates) or defining a monopoly fee level (above which the platform profit is strictly decreasing). See, e.g., Quint (2014).

structures such as the circular city model (Salop, 1979), the spokes model (Chen and Riordan, 2007), and the Hotelling model in case $m = 2$.

☐ **Platforms**. Given our general approach for modelling platform i's instrument vector a_i, we do not explicitly model each of the platform's sources of profit. Instead, we express each platform i's profit as

$$\Pi_i = \left(P_i^B - c\right) s_i + R_i\left(\boldsymbol{a}, \boldsymbol{s}\right), \tag{4}$$

where $c \geq 0$ is the per-buyer platform marginal cost and R_i is platform i's "net revenue" or "residual profit", which captures everything else that is unrelated to profits from the buyer-side membership fees.[8] As an illustration, continuing from the example in (1), we have

$$R_i = r_i \times (\text{Transaction revenue}) + P_i^S \times (\text{Measure of sellers joining}) - \text{Investment Cost}, \tag{5}$$

where each of the items in the parentheses can, in general, depend on $(\boldsymbol{a}, \boldsymbol{s})$, i.e., fee levels, design and investment choices, and the mass of buyers on each platform. Throughout, we assume functions U_i and R_i are symmetric across all m platforms.

Notice that we have so far left the seller side payoffs and behavior unspecified. As will be illustrated in Section 2.2, the framework based on functions U_i and R_i can accommodate a wide range of specifications on the seller side. We assume both functions U_i and R_i are continuous in $(\boldsymbol{a}, \boldsymbol{s})$. We will provide some microfounded applications in Section 2.2 to unpack the corresponding U_i and R_i functions in settings without platform spillovers, and will extend some of these applications to settings with platform spillovers in Section 4.1. These applications show how the general forms of U_i and R_i here can capture different microfounded settings of interest, and how a variety of different platform instruments can be handled.

☐ **Timing**. We adopt the canonical timing in the two-sided market literature: (i) The platforms set their buyer fee P_i^B and instrument vector a_i simultaneously. (ii) Observing these fees and instrument choices, buyers and sellers make their platform participation decisions; (iii) The on-platform interaction between buyers and sellers unfolds according to the specified micro-foundation, captured by the functions U_i and R_i.

The solution concept is symmetric Subgame Perfect Equilibrium. In particular, we assume the outcomes in the equilibrium, in the seller-excluded benchmark, and in the total welfare benchmark (to be defined below) all involve symmetric solutions with all platforms choosing the same P_i^B and a_i.

2.1 Discussion of modelling features

Before illustrating the framework with several applications, which we will do in the next subsection, it is useful to first discuss our modelling assumptions, and some potential limitations.

So far, we have made five simplifying assumptions on the buyer side of the market: (i) full coverage; (ii) buyers' net utility (2) is decreasing one-for-one with platform's margin P_i^B; (iii) buyers are restricted to singlehoming; (iv) horizontal differentiation between platforms is sufficiently

[8] Our setup can easily accommodate heterogeneous per-buyer cost c_i for each platform i provided buyers obtain a standalone participating benefit b_i on each platform and $b_i - c_i$ is constant across all platforms $i = 1, ..., m$.

large so that the demand system faced by the platforms, which we derive in the next section, is well-defined; (v) the gross utility function U_i is homogenous across buyers. In Section 5.1 and Section A of the Online Appendix, we show that the first two assumptions can be relaxed and our baseline results can still be obtained in certain cases. The latter three assumptions are more critical. Specifically, assumption (iii) is at the heart of our no-spillover condition, and rules out platform instruments that influence buyers' homing behavior such as buyer-side exclusive contracts or investments in technologies that facilitate multihoming. The existence of multihoming buyers would invalidate the discrete-choice-based buyer participation behavior in (3). We discuss how allowing for some multihoming buyers can be interpreted in terms of cross-platform spillovers in Section 4.2. Assumption (iv) ensures equilibrium uniqueness in the buyer-seller participation subgame. It rules out "tipping equilibria" considered in models of homogenous platforms by Caillaud and Jullien (2003) and Karle et al. (2020), in which users may coordinate to all join only one of the platforms. Finally, assumption (v) rules out models with heterogeneous buyer interaction benefits such as (Rochet and Tirole, 2003, 2006), though it is important to note we impose no such restriction on the seller side.

As will be illustrated in the next subsection, the framework can accommodate a wide range of specifications on the seller side. It can easily handle multiple product categories each occupied by a monopoly seller or atomistic competing sellers. The framework can also allow for non-atomistic oligopolistic sellers within each product category, regardless of whether the number of potential sellers is fixed (as in Teh, 2022), endogenously determined by free-entry condition (as in Nocke et al., 2007)), or depends on the platform's first-party entry decisions (as in the special cases of Hagiu et al. (2022) and Anderson and Bedre-Defolie (2023)). However, a notable restriction on the seller side is that sellers are unable to strategically influence buyers' participation behavior, e.g., non-atomistic sellers making participation decisions or signing exclusivity contracts prior to buyer participation decisions. In such cases, the functions U_i and R_i would have to explicitly account for individual seller's participation in their arguments. Our timing of simultaneous participation by buyers and sellers rules out these possibilities.

2.2 Examples of applications

In this subsection, we show how different applications easily fit within the general framework presented above by introducing five microfounded models of different platform design choices. The same applications will be used later in illustrating our general welfare results. While the applications here do not have platform spillovers built into them, the same applications are extended in Section 4.1 to settings where platform spillovers arise.

For all applications below, we assume there is a mass-one continuum of product categories, each involving one monopolist seller facing the same downward-sloping demand function from buyers. Each product category is indexed by the fixed cost k_i the seller faces to join each platform $i = 1, ..., m$, where $k_i \in [k_{\min}, k_{\max}]$ is distributed according to a log-concave CDF G, where $k_{\min} \geq 0$.[9] It doesn't matter whether k_i for a given seller is correlated across platforms or not.

[9] This setup extends to product categories varying in terms of their draw of demand rather than the variation in fixed participation costs considered here. It also extends to the case with multiple competing sellers that can enter in each product category. In Online Appendix B we provide an example with both of these features and use it to fully characterize the welfare loss associated with the seller-excluded outcome.

The case where k_i and k_j are not perfectly correlated allows us to accommodate the possibility of some sellers singlehoming and others multihoming in equilibrium. For simplicity, we assume sellers do not face any marginal cost of production, an assumption which is reasonable for some digital settings. For convenience, every buyer is assumed to want to buy from each category.

□ **Application 1 (Two-part tariffs and pass-through).** Each platform i chooses $a_i = \left(f_i, P_i^S\right)$, where $f_i \geq 0$ is a per-unit transaction fee and $P_i^S \geq 0$ is a lump-sum membership fee charged to sellers. Facing the price from a seller on platform i of p_i, each buyer chooses the number of units to purchase q to maximize their net utility; i.e., $\arg\max_q \{u(q) - p_i q\}$. As a result, each seller faces the resulting per-buyer demand $q(p_i)$. Facing the per-unit fee f_i, a seller's optimal price on platform i is then

$$p^*(f_i) = \arg\max_{p_i} \{(p_i - f_i)q(p_i)\}.$$

We assume that $p^*(f_i)$ is unique and well-defined for the relevant range of $f_i \geq 0$ (e.g., if $q(.)$ is strictly log-concave) and that the pass-through rate $\partial p^*/\partial f_i \in (0,1)$. Denote $q^*(f_i) \equiv q(p^*(f_i))$. Then the per-buyer profit of each seller is $\pi^*(f_i) = (p^*(f_i) - f_i)q^*(f_i)$ and the per-seller surplus of the buyer is $u^*(f_i) = u(q^*(f_i)) - p^*(f_i)q^*(f_i)$, both of which are decreasing in f_i.

Each seller participates on i iff

$$k_i \leq \pi^*(f_i)s_i - P_i^S \equiv \bar{k}_i,$$

meaning the mass of participating sellers on platform i is $G(\bar{k}_i)$, which is decreasing in f_i. Notice this is independent of decisions by platform j (when holding s_i fixed), reflecting that each seller's decision to join i is strategically independent of its decision to join j, as is the case in the classic competitive bottleneck setting.

We are now ready to define the key functions U_i and R_i in (2) and (4). We have

$$U_i = u^*(f_i)G\left(\bar{k}_i\right).$$

Note that U_i depends on $\left(f_i, P_i^S\right)$ and s_i as indicated in the general setup. Here f_i affects buyer utility $u^*(f_i)$ through the positive pass-through in sellers' pricing, while f_i, P_i^S and s_i all affect how many sellers participate and so buyers' utility via cross-side network effects (as captured by $G\left(\bar{k}_i\right)$). And

$$R_i = \left(f_i q^*(f_i)s_i + P_i^S\right)G\left(\bar{k}_i\right).$$

□ **Application 2 (Platform investment).** The identical setup can apply to other platform instrument choices. Suppose each platform chooses $a_i = (r_i, -I_i)$, where $r_i \in [0,1]$ is now a commission rate and I_i is platform i's level of investment[10] with associated convex cost $C(I_i)$. The platform's investment I_i is assumed to scale up the buyer's gross utility obtained from transacting with any seller, so this now equals $u(q_i)I_i$. Defining the seller's quality-adjusted price $\hat{p}_i = \frac{p_i}{I_i}$, each seller sets $\hat{p}_i$ to maximize $(1-r_i)I_i\hat{p}_i q_i(\hat{p}_i)$. Let the resulting profit maximizing price be denoted $\hat{p}^*$, which note doesn't depend on either r_i or I_i. The per-buyer profit of each seller is $(1-r_i)I_i\pi^*$ and the per-seller surplus of the buyer is $I_i u^*$, where $\pi^* = \hat{p}^* q(\hat{p}^*)$ and $u^* = u(q(\hat{p}^*)) - \hat{p}^* q(\hat{p}^*)$.

[10] It will become clear in Section 3 why we define a_i in terms of $-I_i$ rather than I_i.

Following the same steps in Application 1, we have $\bar{k}_i \equiv (1 - r_i) I_i \pi^* s_i$ and

$$\begin{aligned} U_i &= I_i u^* G\left(\bar{k}_i\right) \\ R_i &= r_i I_i \pi^* s_i G\left(\bar{k}_i\right) - C\left(I_i\right). \end{aligned} \tag{6}$$

□ **Application 3 (First-party entry and self-preferencing).** Suppose now each platform chooses $a_i = (r_i, e_i, l_i)$, where $e_i \in \{0, 1\}$ indicates whether platform i operates as a dual-mode marketplace or not and $l_i \in \{0, 1\}$ indicates whether platform i engages in self-preferencing or not.[11] When it operates in dual mode, it introduces a first-party product whenever a third-party seller has entered in any product category. With probability $1 - \alpha$, the first-party entry is unsuccessful (e.g., the first-party product is poorly received) and the seller (in each category) is in a monopoly position as in the previous applications (with corresponding gross profit π^* and buyer utility u^* from Application 2). With probability α, the first-party entry is successful. The resulting duopolistic competition results in two possible outcomes. When the platform doesn't engage in self-preferencing, the first-party profit is π^{fp} and the third-party seller profit is $(1 - r_i)\pi^d$, where $0 < \pi^d < \pi^*$, while the corresponding buyer utility is $u^d > u^*$. When the platform engages in self-preferencing, the first-party profit is $\pi^{sp} > \pi^{fp}$ and, for expositional simplicity, the third-party seller profit is normalized to zero, while the corresponding buyer utility is u^{sp}, where $u^{sp} < u^d$. We assume that first-party products do not "cross-list" on rival platforms, which ensures that the no-spillover condition holds.

Following the same steps in Application 1, we have $\bar{k}_i \equiv (1 - r_i)(\pi^* - \alpha e_i(\pi^* - (1 - l_i)\pi^d))s_i$,

$$\begin{aligned} U_i &= (u^* + \alpha e_i(l_i u^{sp} + (1 - l_i) u^d - u^*))G\left(\bar{k}_i\right) \\ R_i &= (r_i \pi^* + \alpha e_i(l_i \pi^{sp} + (1 - l_i)\left(r_i \pi^d + \pi^{fp}\right) - r_i \pi^*))G\left(\bar{k}_i\right) s_i. \end{aligned}$$

Here, e_i and l_i directly affect buyers' utility on platform i, as well as indirectly via how many sellers participate on platform i.

□ **Application 4 (Leakage prevention).** Suppose that buyers have to join at least one platform before they can transact with sellers. However, a fraction $\beta > 0$ of sellers have direct sales channels (e.g., their own websites), which allows each of them to avoid the platform fees if buyers switch to purchase from the seller through their direct channel.[12] A fraction $\lambda_i \geq 0$ of buyers are unaware of this option to buy from the seller directly, with the remaining fraction $1 - \lambda_i$ aware and able to switch costlessly (and so buy from whichever channel is cheapest). Buyers realize which situation they are in after participating on a platform. Each platform chooses $a_i = (r_i, \lambda_i)$, where $\lambda_i \in [\lambda_{\min}, \lambda_{\max}]$ reflects that the platform can influence the probability any given buyer will be aware of a seller's direct-channel option via its governance design. For example, a platform could take steps to prevent communication by sellers which informs buyers of their direct channel.

Participating sellers set prices p_i (on platforms $i = 1, ..., m$) and p_d (their price when selling directly if they have a direct channel). Buyers who are informed on platform i would buy directly if and only if $p_i \geq p_d$. Moreover, given $r_i \geq 0$, each seller would always want to induce leakage.

[11] A literature has recently emerged to address whether the choice of dual-mode marketplace is desirable in the context of a single platform, either absent the possibility of self-preferencing (see, for example, Etro (2021)) or also allowing for the possibility of self-preferencing (see, for example, Hagiu et al. (2022) and Anderson and Bedre-Defolie (2023)).

[12] Hagiu and Wright (2023) study leakage in the case of a monopoly platform.

Therefore, if a seller who has a direct channel joins a non-empty set of platform(s) $\phi \subseteq \{1, 2, ..., m\}$, then it sets its prices to maximize

$$\sum_{i \in \phi} \left((1 - r_i) \, p_i q(p_i) \lambda_i + p_d q(p_d)(1 - \lambda_i) \right) s_i$$
$$\text{subject to } p_d \leq p_i, i \in \phi.$$

Given the pricing problem across channels is additively separable, the optimal price is $p_d = p_i = \arg\max_{p_i} \{p_i q(p_i)\}$ for all $i \in \phi$, so the standard profit and utility terms π^* and u^* still apply in this case. The same pricing decision also applies to sellers without a direct channel: they set $p_i = \arg\max_{p_i} \{p_i q(p_i)\}$ for all $i \in \phi$.

Each seller with a direct channel would participate on platform i if and only if $k_i \leq (1 - \lambda_i r_i) \, \pi^* s_i \equiv \bar{k}_i$, while those without direct channels participate if and only if $k_i \leq (1 - r_i) \, \pi^* s_i \equiv \bar{k}_i^n$. Reflecting these two types of sellers, functions U_i and R_i are written as

$$\begin{array}{l} U_i = \left(\beta G\left(\bar{k}_i \right) + (1 - \beta) G\left(\bar{k}_i^n \right) \right) u^* \\ R_i = r_i \left(\beta \lambda_i G\left(\bar{k}_i \right) + (1 - \beta) G\left(\bar{k}_i^n \right) \right) \pi^* s_i \end{array}.$$

□ **Application 5 (App tracking restrictions).** Similar to Application 4, suppose that buyers have to join at least one platform before they can transact with sellers. Buyers on platform i can obtain (e.g., unlock) q units of content from sellers by either: (i) paying the seller price p_i per unit; or (ii) watching ads, which results in ad disutility z per unit to buyers and generates per-unit ad revenue $\pi_a (1 - \tau_i) > 0$ to sellers, where $\tau_i \in [0, \tau_{\max}]$ is how restrictive platform i's app tracking policy is (which can influence the ad revenue of sellers) and $\tau_{\max} < 1$. Suppose seller's revenue from (i) can be taxed by the platform through its commission r_i, while its ad revenue in (ii) cannot. We assume $z \geq 0$ is i.i.d. across buyers and sellers, drawn from the weakly log-concave CDF H.

Each platform choose $a_i = (r_i, \tau_i)$. Then, a typical seller that joins a non-empty set of platform(s) $\phi \subseteq \{1, 2, ..., m\}$ sets its prices to maximize its profit

$$\sum_{i \in \phi} \left((1 - r_i) \, p_i q(p_i)(1 - H(p_i)) + \pi_a (1 - \tau_i) \int_0^{p_i} q(z) dH(z) \right) s_i.$$

We assume its profit is strictly quasiconcave, a sufficient condition for which is that $q(p_i)$ has an elasticity (in magnitude) that is non-decreasing and is no lower than one over the relevant range. Observe that the pricing problems are separable, and so each seller's optimal price p_i^* on platform i is independent of the (r_j, τ_j) (when holding s_i) constant.

Each seller would participate on i if and only if

$$k_i \leq \left((1 - r_i) \, p_i^* q(p_i^*)(1 - H(p_i^*)) + \pi_a (1 - \tau_i) \int_0^{p_i^*} q(z) dH(z) \right) s_i \equiv \bar{k}_i,$$

and so

$$\begin{array}{l} U_i = \left(\int_0^\infty u(q(\min(p_i^*, z)) - \min(p_i^*, z) q(\min(p_i^*, z)) dH(z) \right) G\left(\bar{k}_i \right) \\ R_i = r_i p_i^* q(p_i^*)(1 - H(p_i^*)) s_i G\left(\bar{k}_i \right) \end{array}.$$

3 Equilibrium and seller-excluded outcomes

Denote the symmetric equilibrium buyer fee and platform instrument vector for each platform as P^{B*} and $a^* \in \mathcal{A}$, and let the equilibrium buyer-side market share profile be

$$\boldsymbol{s}^* = \mathbf{1}/m \equiv (1/m, ..., 1/m),$$

where $\mathbf{1}$ is a m-dimension vector of ones.

To pin down the equilibrium, a useful technique is to consider the following "semi-symmetric" participation equilibrium when one of the platforms (say platform i) deviates from the equilibrium and sets $\left(a_i, P_i^B\right) \neq (a^*, P^{B*})$, resulting in an off-equilibrium path instrument vector profile

$$\hat{\boldsymbol{a}} = (a_i, a^*, ..., a^*) \in \mathcal{A}^m,$$

buyer fee profile $(P_i^B, P^{B*}, ..., P^{B*})$ and buyer-side market share profile:

$$\hat{\boldsymbol{s}} = \left(s_i, \frac{1-s_i}{m-1}, \cdots, \frac{1-s_i}{m-1}\right).$$

That is, the deviating platform i's choices result in it having a market share $s_i \neq 1/m$ while all other $m-1$ platforms equally absorb the resulting change in market share (due to the market being covered and symmetry). Then, given that $U_j(\hat{a};\hat{s})$ is symmetric across platform $j \neq i$, we can explicitly rewrite the fixed-point definition of market share s_i in (3) as

$$s_i = \Phi\left(U_i(\hat{\boldsymbol{a}};\hat{\boldsymbol{s}}) - U_{-i}(\hat{\boldsymbol{a}};\hat{\boldsymbol{s}}) - P_i^B + P^{B*}\right), \tag{7}$$

where $U_{-i}(\hat{a};\hat{s}) = U_j(\hat{a};\hat{s})$ and $\Phi\left(.\right)$ is the cumulative distribution function of $\max_{j\neq i}\{\epsilon_j\} - \epsilon_i$.

We assume functional forms are such that a unique fixed-point in (7) always exists. This requires the right-hand side of (7) has a slope less than one with respect to s_i, which holds if the extent of platform horizontal differentiation (measured by $1/\Phi'$) is large relative to the cross-group network effects (measured by the rate at which $U_i - U_{-i}$ changes with s_i). Under this condition, the resulting demand system is analogous to standard discrete choice models.

Platform i chooses $\left(a_i, P_i^B\right)$ to maximize profit Π_i, taking as given (a^*, P^{B*}) set by each other platform. To solve this maximization problem, a useful technique is to reframe the problem as platform i directly choosing the target market share s_i implementable by its fee P_i^B, i.e., maximization with respect to (a_i, s_i). Formally, this can be done by inverting (7), so that $P_i^B(a_i, s_i)$ becomes a function of (a_i, s_i) satisfying

$$P_i^B = U_i(\hat{\boldsymbol{a}};\hat{\boldsymbol{s}}) - U_{-i}(\hat{\boldsymbol{a}};\hat{\boldsymbol{s}}) + P^{B*} - \Phi^{-1}\left(s_i\right). \tag{8}$$

Then, platform i's problem is to choose (a_i, s_i) to maximize

$$\begin{aligned}\Pi_i &= \left(P_i^B - c\right)s_i + R_i \\ &= (U_i - U_{-i} + P^{B*} - \Phi^{-1}\left(s_i\right) - c)s_i + R_i.\end{aligned}$$

By continuity of profit functions, a solution to this maximization problem exists for each platform i (which can be non-interior and non-unique). By the envelope theorem, each platform's optimal choice of $a_i \in \mathcal{A}$ can be obtained by maximizing Π_i while holding s_i constant at the equilibrium value $1/m$. Then, in any symmetric equilibrium with each platform setting its n-dimensional instrument vector $a^* \in \mathcal{A}$, we must have a^* satisfying the fixed-point relation

$$a^* \in \arg\max_{a_i \in \mathcal{A}} \left\{ \frac{1}{m} \left(U_i(\hat{\boldsymbol{a}}; \mathbf{1}/m) - U_{-i}(\hat{\boldsymbol{a}}; \mathbf{1}/m) \right) + R_i(\hat{\boldsymbol{a}}; \mathbf{1}/m) \right\}. \tag{9}$$

Taking into account equilibrium multiplicity, we denote

$$\mathcal{A}^* = \{a^* \in \mathcal{A} : a^* \text{ satisfies (9)}\}.$$

as the set of all symmetric equilibrium platform instrument vectors. If the equilibrium is unique, then $\mathcal{A}^*$ is a singleton set containing a single vector a^* satisfying (9).

Intuitively, the platform uses its buyer membership fee to implement its target buyer-side market share, and so its optimal instrument vector focuses on how a_i affects: (i) the membership fee that platform i can charge buyers (while maintaining its market share), which is captured by the extent to which a higher a_i changes the utility difference between the two platforms; and (ii) its other net revenue sources R_i.

3.1 The equivalence result and no spillovers

As Armstrong (2006) showed (Proposition 4), in the classic competitive bottleneck setting (i.e., a homing configuration of multihoming sellers and singlehoming buyers), the equilibrium seller (membership) fees coincide with the fees that maximize the joint buyer surplus and platform profit. In what follows, we show that this property on the platform fee level extends to arbitrary platform instrument vectors in this general environment, so long as a no cross-platform spillover assumption holds, which we will define below. This provides a convenient way to characterize the equilibrium outcome in general, and is the key to obtaining general welfare results.

We label the "seller-excluded" outcome as the profile of platform instrument vectors $(a_1^{SE}, ..., a_m^{SE}) \in \mathcal{A}^m$ that maximizes total welfare less seller profit (i.e., the surplus of buyers and the profit of the platforms):

$$W^{SE}(\boldsymbol{a}) = \sum_{i=1,\dots,m} \left\{ \left(U_i - P_i^B + E_i\right) s_i + (P_i^B - c) s_i + R_i \right\}, \tag{10}$$

where $E_i = E[\epsilon_i | i = \arg\max_{i=1,\dots,m} \{U_i - P_i^B + \epsilon_i\}]$ is the expectation of buyer match value on platform i conditioned on i being chosen. In this definition of the seller-excluded benchmark we allow each platform i to optimally adjust its buyer prices P_i^B in response to changes in $\boldsymbol{a}$, so that the profile of buyer-side market share $\boldsymbol{s} = (s_1, ..., s_m)$ is endogenous.[13] Denote $\boldsymbol{s}^{SE} = \left(s_1^{SE}, ..., s_m^{SE}\right)$ as the corresponding market share of the platforms at the optimum.

We are interested in comparing the seller-excluded benchmark with the equilibrium outcome. One complication for the comparison is that $\boldsymbol{s}$ varies when $\boldsymbol{a}$ varies in the maximization of (10). As

[13] Another approach to defining the seller-excluded outcome is to shut down platforms' response in buyer prices by allowing both $\boldsymbol{a}$ and $\boldsymbol{P}^B$ to be set to maximize (10). It is easily seen that this modification does not affect the characterization in (11) below given the assumptions of symmetric platforms and a fully covered buyer-side market.

such, in general, the resulting buyer-side market share could be different across the two benchmarks. Nonetheless, our assumption of symmetric platforms addresses this issue since it implies $\boldsymbol{s}^* = \boldsymbol{s}^{SE} = \mathbf{1}/m$. Given symmetry, we can omit the terms E_i and c in (10) as they become irrelevant in maximizing W^{SE}. Imposing symmetry $a_i^{SE} = a^{SE} \in \mathcal{A}$ and applying the principle of maximum, we can pin down a^{SE} with a fixed-point relation

$$a^{SE} \in \arg\max_{a_i \in \mathcal{A}} \{\frac{1}{m} \sum_{i=1,...,m} U_i(\hat{\boldsymbol{a}}^{SE}; \mathbf{1}/m) + R_i(\hat{\boldsymbol{a}}^{SE}; \mathbf{1}/m)\}, \tag{11}$$

where $\hat{\boldsymbol{a}}^{SE} = (a_i, a^{SE}, ..., a^{SE})$. Taking into account the possibility of multiple solutions, we denote $A^{SE} \subseteq A$ as the set of all such (symmetric) maximizers a^{SE}:

$$\mathcal{A}^{SE} = \left\{a^{SE} \in \mathcal{A} : a^{SE} \text{ satisfies (11)}\right\}.$$

Our first key result shows that the n-dimensional instrument vectors a^* and a^{SE} defined in (9) and (11) coincide, so that $\mathcal{A}^* = \mathcal{A}^{SE}$, when the following "no cross-platform spillover" condition holds:

- **No cross-platform spillover**: For all platforms i, $U_i(\boldsymbol{a}; \mathbf{1}/m)$ and $R_i(\boldsymbol{a}; \mathbf{1}/m)$ are independent of the vector a_j (for all $j \neq i$).

Intuitively, the no-spillover condition captures situations where: (i) the utility a buyer gets on one platform does not depend, either directly or through sellers' reactions, on the actions taken by any rival platform, and (ii) the revenue a platform generates does not depend, either directly or through sellers' reactions, on the actions taken by any rival platform. In particular, many microfounded models with multihoming sellers and singlehoming buyers in the two-sided platform literature (such as those discussed in Armstrong (2006) and Belleflamme and Peitz (2019a)) satisfy these assumptions. Indeed, inspecting the U_i and R_i functions for the five applications introduced in Section 2.2, it can immediately be seen that all satisfy the no spillover condition.[14]

Proposition 1 *(Equivalence). Suppose that the no cross-platform spillover condition holds. Then,*

$$\mathcal{A}^* = \mathcal{A}^{SE} = \left\{a_i^* \in \mathcal{A} : a_i^* \in \arg\max_{a_i \in \mathcal{A}} \left\{\frac{1}{m} U_i(\hat{\boldsymbol{a}}; \mathbf{1}/m) + R_i(\hat{\boldsymbol{a}}; \mathbf{1}/m)\right\}\right\}, \tag{12}$$

where $\hat{a} = (a_i, a^*, ..., a^*)$. That is, the set of equilibrium instrument vectors coincides with the set of seller-excluded instrument vectors.

The power of equivalence result is that it implies the platforms' equilibrium instrument vector is distorted in the direction of outcomes with a lower seller profit compared to the total welfare benchmark. Our result based on an arbitrary platform instrument vector allows us to comment not just on platform fees (e.g., Apple's App Store commission level) but also on platform design

[14]Note the condition does not impose restriction on how s_i and s_{-i} affect U_i and R_i. Moreover, even if U_i or R_i somehow depend on how s_{-i} is distributed among the other $m-1$ platforms, it does not create a spillover that would affect our results due to the symmetric setup.

choices (e.g., Apple's investments in its App Store, its decision to sell its own apps and to promote these over third-party apps, and various in-app transaction policies it might adopt such as its no-steering rule which prevents developers from directing users within their iOS apps to make purchases outside the App Store). For instance, in Application 1, seller profit is decreasing in the platform transaction fee and membership fee. Then, the seller-excluded outcome would imply excessive seller transaction and membership fees, relative to total welfare maximization. Note the characterization in (12) holds for any value of m, suggesting that any such distortion, if it exists, is not necessarily eliminated by having more platforms compete.

Before proceeding to formalize the welfare implications, it is useful to compare our equivalence result with that obtained by Armstrong (2006) (his Proposition 4). He focuses on platforms that just choose seller membership fees (i.e., a single instrument). Adapted to our context, Armstrong (2006)'s equivalence result is obtained by fixing each platform's buyer market share fixed at some arbitrarily given level as part of a market share profile $\boldsymbol{s}$ in both definitions (9) and (11). In a two-sided platform setting, when comparing different platform choices on the seller-side, the question is whether we compare the corresponding outcomes on the seller-side allowing the buyer-side's demand to adjust (our approach) or do we hold the buyer side's demand constant (Armstrong's approach). Our approach of allowing the buyer-side demand to adjust is consistent with a standard welfare analysis in which cross-side network effects are taken into account when considering the effects of different choices of the platforms' seller-side instruments. Indeed, this will allow us to readily derive welfare implications of the equilibrium outcome. Nonetheless, in our environment, the two approaches lead to the same results for all our propositions because the only market share profile on the buyer side that satisfies symmetry and full coverage is $\boldsymbol{s} = \mathbf{1}/m$. Meanwhile, in Section A of the Online Appendix, we show that Proposition 1 continues to hold with Armstrong's approach, with possibly asymmetric platforms and an incompletely covered buyer-side market.

Proposition 1 is also a stronger result than the corresponding part of Armstrong (2006)'s Proposition 4 given we allow for the maximization of W^{SE} without having to hold buyer-side market shares constant. However, to obtain this result we require the setting be symmetric in market shares. Intuitively, when platforms are asymmetric, the maximization of W^{SE} would take into account how changes in the buyer-side market shares $(s_1, ..., s_m)$ affect the asymmetric surpluses generated across the platforms, which is something individual platforms would not take into account (by the envelope theorem). This means that the buyer-side market shares would typically be different in the characterizations of a^* and a^{SE} in (9) and (11). The assumption of symmetric platforms harmonizes this difference in the market shares across the two characterizations.

3.2 Welfare implications

Next, we describe how the seller-excluded outcome (and therefore, given Proposition 1, the equilibrium outcome) is distorted relative to the total welfare benchmark in general. Following the same idea in constructing the seller-excluded welfare (10), we denote total welfare as $W(\boldsymbol{a}) = W^{SE}(\boldsymbol{a}) + SS(\boldsymbol{a})$, where $SS(\boldsymbol{a})$ is the total seller surplus (across all m platforms) and recall $\boldsymbol{a} \in \mathcal{A}^m$. In what follows, we assume that

$$\hat{SS}(a_i) \equiv SS((a_i, a_i, ..., a_i)) \tag{13}$$

is weakly decreasing in $a_i \in \mathcal{A}$. That is, we interpret a uniformly higher platform instrument vector as lowering seller surplus. Given that we can always redefine the sign of the relevant components of vector a_i, this assumption is equivalent to $\hat{SS}(a_i)$ being monotonic in a_i. For example, in our Application 2, seller surplus decreases with a higher platform fee r_i but increases with a higher platform investment I_i. By defining $a_i = (r_i, -I_i)$, seller surplus decreases in all dimensions of a_i.

By symmetry, we can alternatively define the instrument vectors resulting from maximizing the seller-excluded welfare and the total welfare as:

$$a^{SE} \in \mathcal{A}^{SE} \equiv \arg\max_{a_i \in \mathcal{A}} \hat{W}^{SE}(a_i) \equiv \arg\max_{a_i \in \mathcal{A}} W^{SE}((a_i, a_i, ..., a_i)) \quad (14)$$

$$a^{W} \in \mathcal{A}^{W} \equiv \arg\max_{a_i \in \mathcal{A}} \hat{W}(a_i) \equiv \arg\max_{a_i \in \mathcal{A}} \hat{W}^{SE}(a_i) + \hat{SS}(a_i), \quad (15)$$

where recall we do not impose uniqueness of the solutions. To compare sets $\mathcal{A}^W$ and $\mathcal{A}^{SE}$, we adopt the following notion:[15]

- **Strong set order** (Topkis, 1979). A set $\mathcal{A}''$ is higher than set $\mathcal{A}'$ in *strong set order* (denoted as $\mathcal{A}'' \geq_{sso} \mathcal{A}'$) if for any pairs of vectors $a' \in \mathcal{A}'$ and $a'' \in \mathcal{A}''$, we have $a' \vee a'' \in \mathcal{A}''$ and $a' \wedge a'' \in \mathcal{A}'$. Here, $a' \vee a''$ is the dimension-wise maxima of the two vectors and $a' \wedge a''$ is the dimension-wise minima of the two vectors.

In particular, if sets $\mathcal{A}'$ and $\mathcal{A}''$ are singletons, then strong set order is equivalent to the usual vector ordering $a'' \geq a'$, in which each of the n different platform instruments in vector a'' is higher than those in vector a'. If only set $\mathcal{A}'$ is a singleton and contains only an element a', then strong set ordering implies that every $a'' \in \mathcal{A}''$ satisfies $a'' \geq a'$.

One well-known complication of multi-dimensional comparative statics is the cross-dimension effects, whereby distortions in one of the dimensions may reinforce or diminish distortions in other dimensions. For example, platform i's excessive commission (relative to the welfare benchmark) may reinforce or diminish its incentive to charge an excessive seller participation fee and to set an insufficient level of investment. To proceed, we define the following concept:

- **Quasi-supermodularity** (Milgrom and Shannon, 1994). A function $\hat{W}(a_i)$ is *quasi-supermodular* in its argument $a_i \in \mathcal{A}$ if, for any pair of vectors $a'_i \in \mathcal{A}$ and $a''_i \in \mathcal{A}$, we have

$$\hat{W}(a'_i) - \hat{W}(a'_i \wedge a''_i) \geq (>)0 \Rightarrow \hat{W}(a'_i \vee a''_i) - \hat{W}(a''_i) \geq (>)0.$$

Quasi-supermodularity is implied by the standard weak supermodularity condition, and the latter property is preserved by summation.[16] More generally, there are a few easy-to-check sufficient conditions for $\hat{W}(a_i)$ to be quasi-supermodular: (i) $\hat{W}(a_i)$ is monotone in a_i; or (ii) there exists a strictly increasing function $h : \mathbb{R} \to \mathbb{R}$ such that $h(\hat{W}(a_i))$ is supermodular in a_i (e.g., log

[15] Obviously, if sellers always obtain no surplus (e.g., they fully compete away all their surplus), $\mathcal{A}^W$ and $\mathcal{A}^{SE}$ trivially coincide. Hence, our discussion is aimed at the more intersting situation where sellers obtain positive surplus at the welfare maximizing solution.

[16] That is, if we assume continuous choice and differentiability, and let $a_i = (z_1, z_2, ...z_n) \in \mathbb{R}^n$, then this is equivalent to $\partial^2 \hat{W}/\partial z_k \partial z_l \geq 0$ for every pair of dimensions $k \neq l$, $k, l = 1, 2, ...n$.

transformations). Moreover, if a_i is a scalar, then quasi-supermodularity trivially holds; if a_i is two-dimensional, then quasi-supermodularity is equivalent to $\hat{W}(a_i)$ obeying single-crossing difference in a pairwise manner.[17]

Proposition 2 *(Comparing benchmarks). Suppose the total seller surplus function $\hat{SS}(a_i)$ is weakly decreasing in $a_i \in A$ and one of the following conditions holds:*

- *The function $\hat{W}(a_i)$ (or $\hat{W}^{SE}(a_i)$) is quasi-supermodular.*
- *Platform instrument a_i is a scalar, i.e., $\mathcal{A} \subseteq \mathbb{R}$.*

Then, regardless of whether the no cross-platform spillover condition holds or not, $\mathcal{A}^{SE} \geq_{sso} \mathcal{A}^W$. That is, the set of seller-excluded instrument vectors is higher than the set of welfare-maximizing instrument vectors in terms of strong set order.

When the no-spillover conditions hold, we can combine Proposition 1 and Proposition 2:

Corollary 1 *Suppose that the no cross-platform spillover condition and the conditions in Proposition 2 hold. Then*

$$\mathcal{A}^* = \mathcal{A}^{SE} \geq_{sso} \mathcal{A}^W$$

Proposition 2 and Corollary 1 formally establish the previous claim that a seller-excluded outcome implies a market distortion where the platforms' instrument vector is distorted in the direction of outcomes with a lower seller surplus compared to the total welfare benchmark. In particular, for decision variables that reduce seller surplus such as seller fees, then the seller-excluded outcome implies excessive fees; for decision variables that increase seller surplus such as platform investments, then the seller-excluded outcome implies insufficient investment.

The quasi-supermodularity assumption in Proposition 2 is not a very restrictive assumption and holds easily in several classes of examples. First, quasi-supermodularity assumption trivially holds if a_i is single-dimensional, and so it is restrictive only when a_i is multi-dimensional. Second, if vector a_i refers only to some combination of seller-side fees offered by the platforms, then in many standard models including Application 1 in Section 2.2, above-cost pricing generates deadweight losses so that $\hat{W}(a_i)$ is monotone in a_i and so satisfies quasi-supermodularity. Third, in applications where each dimension of a_i is a binary variable, then W and W^{SE} are trivially monotone and hence satisfies quasi-supermodularity.

3.3 Applications continued

In this section, we apply Propositions 1 and 2 to Applications 1-5 from Section 2.2, which we can do given that seller surplus is decreasing in a_i for all five applications. Additional details are provided in Online Appendix C.

[17]That is, if we assume continuous choice and differentiability, and let $n = 2$ so that a platform's instrument vector is $a_i = (z_1, z_2) \in \mathbb{R}^2$, then this is equivalent to $\partial\hat{W}/\partial z_k$ being single-crossing in z_l for each dimension $k \neq l$, $k = 1, 2$. That is, If $\partial\hat{W}/\partial z_k \geq (>)0$ at $z_l = z_l'$, then $\partial\hat{W}/\partial z_k \geq (>)0$ for all $z_l > z_l'$.

□ **Application 1 (Two-part tariffs).** By symmetry, $\hat{W}(a_i)$ is proportional to

$$(u^*(f_i) + \pi^*(f_i) + f_i q^*(f_i)) G(\bar{k}_i) - m \int_{k_{\min}}^{\bar{k}_i} k dG(k),$$

which is decreasing in platform fees $\left(f_i, P_i^S\right)$ by the standard deadweight loss logic. Thus, $\hat{W}(a_i)$ satisfies the quasi-supermodularity condition. By Propositions 1-2, we conclude that in the equilibrium, the equilibrium platform fees coincide with the SE benchmark fees, which are excessive compared to the levels maximizing total welfare.

□ **Application 2 (Platform investment).** By symmetry, $\hat{W}(a_i)$ is proportional to

$$I_i(u^* + \pi^*) G(\bar{k}_i) - m \int_{k_{\min}}^{\bar{k}_i} k dG(k) - C(I_i).$$

In the case of weakly convex G, the function above is weakly supermodular (hence quasi-supermodular) in $(r_i, -I_i)$. By Propositions 1-2, we conclude that in the equilibrium, platform fees are excessive and investment levels are insufficient compared to the levels maximizing total welfare.

□ **Application 3 (First-party entry and self-preferencing).** By symmetry, $\hat{W}(a_i)$ is proportional to

$$\left(u^* + \pi^* + \alpha e_i \left(l_i \Delta^{sp} + (1 - l_i) \Delta^{fp}\right)\right) G(\bar{k}_i) - m \int_{k_{\min}}^{\bar{k}_i} k dG(k),$$

where we define $\Delta^{sp} = \pi^{sp} + u^{sp} - \pi^* - u^*$ and $\Delta^{fp} = \pi^{fp} + \pi^d + u^d - \pi^* - u^*$ as the ex-post efficiency gain from first-party entry with and without self-preferencing. Suppose $\Delta^{fp} > \Delta^{sp}$. Then it can be shown that $\hat{W}$ is decreasing in r_i (given a higher fee decreases seller participation $\bar{k}_i$), decreasing in e_i regardless of l_i provided Δ^{fp} is not too large, and decreasing in l_i, thus satisfying the quasi-supermodularity condition.

By Propositions 1-2, we conclude that in the equilibrium, the platform fees, first-party entry intensity, and self-preferencing intensity (r_i, e_i, l_i) coincide with the SE benchmark, which are excessive compared to the total welfare benchmark. Thus, from a welfare perspective, not only do platforms set their seller-fees too high, but they enter and sell their own competing version of participating sellers' products when sometimes they should not, and moreover, steer buyers to purchase their versions of the sellers' product (i.e., self-preference) when sometimes they should not, whereas the reverse (setting seller-fees too low, not entering when they should, and not steering when they should) is never true.

□ **Application 4 (Leakage prevention).** By symmetry, $\hat{W}(a_i)$ is proportional to

$$(u^* + \pi^*)(\beta G(\bar{k}_i) + (1 - \beta) G(\bar{k}_i^n)) - m\beta \int_{k_{\min}}^{\bar{k}_i} k dG(k) - m(1 - \beta) \int_{k_{\min}}^{\bar{k}_i^n} k dG(k).$$

Given that $\bar{k}_i$ depends only on $\lambda_i r_i$, this setup is formally equivalent to each platform i choosing a "baseline fee" r_i and a "leakage-adjusted effective fee" $\lambda_i r_i$. It then follows that $\hat{W}$ is decreasing in r_i and λ_i, thus satisfying the quasi-supermodularity condition. By Propositions 1-2, we conclude that the equilibrium (r_i, λ_i) coincides with the SE benchmark. Hence, platforms charge excessive fees and engage in excessive leakage prevention compared to the levels maximizing total welfare.

□ **Application 5 (App tracking).** It turns out that directly verifying quasi-supermodularity is not straightforward for this application. However, one technique is to note that each seller's optimal price on each platform i, p_i^*, is a strictly increasing function of $\frac{1-\tau_i}{1-r_i}$. Hence, for the purpose of establishing quasi-supermodularity, we can alternatively reframe each platform's choice of instrument vector as choosing a commission rate r_i and a target seller price p_i^* (which is implemented via app tracking policy $\tau_i \in [0, \tau_{\max}]$). Given this reformulation, it can be shown that seller surplus is increasing in p_i^*, and that $\hat{W}$ is quasi-supermodular in $(r_i, -p_i^*)$ when distribution G has a constant and sufficiently small elasticity (so that seller participation becomes relatively unresponsive to changes in post-participation profits). The latter implies $(r_i, -p_i^*)$ in the equilibrium is higher than its counterpart in the welfare-maximizing outcome. Given that p_i^* is decreasing in τ_i, the lower price in the equilibrium is necessarily driven by $\tau_i^* \geq \tau_i^W$. Thus, we conclude that in the equilibrium, platform fees are too high, while their app tracking policies are too restrictive, compared to those maximizing total welfare.[18]

4 Cross-platform spillovers

When there are cross-platform spillovers in the instrument vector a_i, the equilibrium outcome will generally be different from the seller-excluded outcome, and we can no longer rely on the equivalence result of Proposition 1 to obtain our welfare results. Our goal in this section is to sign the difference between the equilibrium and seller-excluded outcomes when spillovers have well-defined structures, and to use this to extend our previous welfare results.

- **Negative (Positive) cross-platform spillovers:** For all platforms i, $U_i(\boldsymbol{a}; \mathbf{1}/m)$ and $R_i(\boldsymbol{a}; \mathbf{1}/m)$ are weakly decreasing (increasing) in each dimension of each individual rival platform's vector a_j (for every $j \neq i$).

Note that the spillover definition above requires the same sign for each dimension of the rival platforms' instrument vector a_j. Recall that we have defined vectors a_i such that seller surplus $\hat{SS}(a_i)$ defined in (13) is decreasing with respect to each dimension of a_i. Hence, the spillovers definition above essentially requires U_i and R_i to be monotone in the same direction when vectors a_i are defined in the same way. For example, in the next subsection we consider an amended version of our Application 2 whereby U_i is decreasing in each rival's commission r_j but increasing in each rival's investment I_j. By defining $a_j = (r_j, -I_j)$, as we did in that application, U_i is decreasing in all dimensions of a_j, and $\hat{SS}$ is also decreasing in each of the platform instruments as well.

In the presence of spillovers, we can sign the difference between the equilibrium outcome and the seller-excluded outcome, i.e., (9) and (11), as follow:

Proposition 3 *(Spillovers). Suppose the negative (positive) cross-platform spillovers condition holds and one of the following conditions holds:*

[18] Note this analysis is only meant to highlight the distortions in the welfare of the buyers, sellers and platforms that arise from the seller-excluded outcome. Specifically, we don't consider the possible efficiency benefits of targeted advertising for the advertising firms, as well as the privacy costs that buyers may incur from advertisers being able to better track their activities across other apps and websites.

- *The function $\hat{W}^{SE}(a_i)$ is quasi-supermodular.*[19]

- *Platform instrument a_i is a scalar, i.e., $\mathcal{A} \subseteq \mathbb{R}$.*

Then, $\mathcal{A}^* \geq_{sso} (\leq_{sso}) \mathcal{A}^{SE}$. That is, the set of equilibrium instrument vectors is higher (lower) than the set of seller-excluded instrument vectors in strong set order. If, in addition, the total seller surplus function $\hat{SS}(a_i)$ is weakly decreasing in a_i, then $\mathcal{A}^* \geq_{sso} \mathcal{A}^W$.

Proposition 3 says that negative spillovers create an additional distortion (in the same direction) to the one identified in Proposition 2 (provided the stated conditions hold). That is, the conclusion in Section 3 on distortions in the direction of a lower seller surplus continues to hold. Meanwhile, positive spillovers can potentially mitigate the distortions identified in Proposition 2, leading to an ambiguous welfare implication in this case.

Quasi-supermodularity of W^{SE} is a key condition in multi-dimensional comparative statics exercises, but beyond the case of scalar instruments it can be hard to verify in some applications. An alternative and perhaps more widely-applicable approach to sign the difference between a^{SE} and a^* is to utilize the idea that in many applications the spillovers are generated through seller participation behavior, which depends on how decisions on platform i affects the overall participation profit of each seller. As such, seller participation profit can be a sufficient proxy variable to describe the spillover pattern.

For example, in Section 4.1 below, we consider Application 2 with $a_i = (r_i, -I_i)$, but assume the fixed cost sellers incur to partipicate on a platform are perfectly correlated and only have to be incurred once, so that there is no additional fixed cost to participate on additional platforms once a seller has participated on one platform. Then, a seller participates if and only if its common draw of participation cost k is lower than $\sum_{i=1,...,m} \bar{k}_i$ (where $\bar{k}_i \equiv (1 - r_i) I_i \pi^* s_i$), which is the sum of the profits it earns on all platforms. In this case, the effect of platform i's choice of a_i on platforms $j \neq i$ occurs only via $(r_i - 1) I_i$, which can be understood as a "proxy" instrument on how platform i generates spillovers to other platforms. Therefore, instead of identifying the sign of distortions in r_i and I_i separately, one can instead directly look at $(r_i - 1) I_i$ and the implications on the total seller participation.

More formally, define:

- **Negative (Positive) proxied spillovers:** There exists a weakly increasing "proxy instrument" function $b : \mathcal{A} \to \mathbb{R}$ such that the following holds. For all platforms i, (i) U_i and R_i depend on the vector a_j (for all $j \neq i$) only through scalar $b_j = b(a_j)$, that is, $U_i = U_i(a_i, \boldsymbol{b}; \boldsymbol{s})$ and $R_i = R_i(a_i, \boldsymbol{b}; \boldsymbol{s})$ where $\boldsymbol{b} = (b_1, b_2, ..., b_m)$; and (ii) U_i and R_i are weakly decreasing (increasing) in b_j (for all $j \neq i$) when holding $\boldsymbol{s} = \boldsymbol{1}/m$ fixed.

Corollary 2 *(Proxied spillovers). Suppose the negative (positive) cross-platform proxied spillovers condition holds. Then, the proxy instrument satisfies $b(a^*) \geq (\leq) b(a^{SE})$ for any equilibrium instrument vector a^* and seller-excluded instrument vector a^{SE}. If, in addition, the conditions in Proposition 2 hold, then negative cross-platform proxied spillovers imply $b(a^*) \geq b(a^W)$.*

[19] To be more technically precise, we only need quasi-supermodularity to hold for $a_i \geq a^* \wedge a^{SE}$ i.e., for vectors a_i higher than the dimension-wise minima of arbitrary vectors $a^* \in \mathcal{A}^*$ and $a^{SE} \in \mathcal{A}^{SE}$.

We will use both Proposition 3 and Corollary 2 in the following section in order to sign some welfare results.

4.1 Sources of spillovers

In what follows, we discuss some sources of cross-platform spillovers (and whether they are negative or positive). For each source of cross-plaform spillover, one can potentially consider how they may work with respect to various different instruments. Below we illustrate the three spillovers highlighted with the sets of instruments considered in Applications 1, 2 and 4.[20] Additional derivation details are given in Section D of the Online Appendix.

□ **Within-seller economies of scale.** Spillovers naturally arise in situations where sellers make decisions that affect their profits across multiple platforms. One example is when sellers only need to incur a common participation cost once (e.g., app development cost) to participate and sell on all platforms. In this case, sellers' participation decision depends on the total post-participation profit they earn on all platforms, so that any platform instruments that decrease seller net profit (e.g., fees) generate negative cross-platform spillovers.

More generally, a common participation cost can be interpreted as a special case of sellers investing in their product or app (or in better marketing them), whereby higher investment increases demand for their product on all platforms, and so anything (e.g., fees) that decreases sellers' marginal return from their investments, would create a negative cross-platform spillover.[21]

As an illustration of how spillovers arise through seller participation, we modify Application 2 as follows (a similar spillover structure can also be constructed for the other applications we considered in Section 2.2). Suppose the fixed cost sellers incur to partipicate on a platform are perfectly correlated but only have to be incurred once, so that there is no additional fixed cost to participate on additional platforms once a seller has participated on one platform. Denote $k = k_i$ for all $i = 1, ..., m$, where k follows the CDF G. Thus, in the equilibrium, a type-k seller either joins no platforms or joins all platforms. The latter occurs if and only if

$$k \leq \sum_{i=1}^{m} (1 - r_i) I_i s_i \pi^* \equiv \bar{k}.$$

Since the post-participation behaviors of sellers remain the same, we have the same expressions for functions U_i and R_i in (6), except that a seller's participation threshold is now $\bar{k}$, which is increasing in I_j and decreasing in r_j for $j \neq i$.

Thus, within-seller economies of scale in seller participation gives rise to negative spillovers in

[20] In the context of media platforms, Anderson and Peitz (2023) also uncovers an alternative source of spillovers: advertising congestion across platforms. That is, if a platform invites more advertisers it reduces the conversion rate on competing platforms. Without advertising congestion, their model results in a seller-excluded outcome similar to ours, whereby platforms do not compete directly for advertisers.

[21] A conceptually similar source of spillover is within-seller network effects. Apps like online multiplayer games, dating networks, and social networks provide positive network effects between users. Thus, the more users who adopt a seller's app, the more value all other users get. This implies any platform instrument that reduces the demand for an individual seller's app (e.g., a higher fee that induces a higher seller price or the seller not to participate on the platform) would lead to negative cross-platform spillovers.

platform fees, and positive spillovers in platform investments.[22] If the platforms' instrument is single-dimensional (i.e., one of these two choices are held fixed), then we can immediately conclude from Proposition 2 and Proposition 3 that $r^* \geq r^{SE} \geq r^W$ or $I^* \leq I^{SE} \leq I^W$.

More generally, if G is linear or non-linear but has a constant elasticity that is above 1, then $\hat{W}^{SE}(a_i)$ is quasi-supermodularity for all $r_i \geq r^{SE}$ and I_i, which is sufficient for applying Proposition 3, so that the same conclusion applies to multi-dimensional instruments. Alternatively, using the idea of negative proxied spillovers via the proxy instrument $(r_i - 1) I_i$ in seller participation threshold $\bar{k}$, Corollary 2 implies $(r^* - 1) I^* \geq \left(r^{SE} - 1\right) I^{SE} \geq \left(r^W - 1\right) I^W$, which then implies $\bar{k}^* \leq \bar{k}^{SE} \leq \bar{k}^W$. That is, the mass of participating sellers is smaller in the equilibrium than in the seller-excluded outcome, which in turn is smaller than in the total welfare benchmark.

□ **Price coherence.** Price coherence (Edelman and Wright, 2015) refers to situations where sellers set the same price across multiple platforms, even when these platforms charge different transaction fees. This can reflect explicit price-parity contracts that the platform might use to enforce this, incentives sellers may face to avoid undercutting on one channel (e.g., if they lower their price on one platform, they will be demoted in rankings by other platforms), or some more behavioral-type factors on the part of buyers which mean within the range of relevant fees, sellers prefer to set uniform prices. Assuming there is a positive fee pass-through in seller pricing, multihoming sellers would then set prices that depend on the average transaction fees across platforms, thus generating negative cross-platform spillovers via fees.

As an illustration, consider a modification to Application 1 where there is an ex-ante probability $\beta > 0$ that any given product category is subjected to price coherence across all platforms. Given that spillovers via seller participation has been discussed above, in what follows we assume that all sellers have zero fixed costs and zero participation costs $k_i = 0$ (i.e., distribution G is degenerate) and $P_i^S = 0$. This simplification means that all sellers will always join at least one of the platforms.

Facing platform fees of f_i (for $i = 1, ..., m$), a seller that joins platform i only or multihomes but is not subjected to price coherence would therefore set its price at $p^*(f_i) = \arg\max \{(p - f_i) q(p)\}$ and obtain a profit of $\pi^*(f_i)$ for each buyer on platform i. The corresponding transaction quantity and buyer surplus are $q^*(f_i)$ and $u^*(f_i)$. For sellers that multihome on a subset $\phi \subseteq \{1, 2, ..., m\}$ of at least two platforms and are subject to price coherence, their effective marginal cost is the average fee

$$f^{avg} = \sum\nolimits_{i \in \phi} s_i f_i.$$

Thus, they set price at $p^*(f^{avg})$ and obtain a profit of $\pi^*(f^{avg})$ for each buyer on each platform. The corresponding transaction quantity and buyer surplus are $q^*(f^{avg})$ and $u^*(f^{avg})$.

All sellers will multihome on all platforms as long as the fee difference $\max_{i,j} |f_i - f_j|$ is not too large, and that no platform has incentive to deviate and induce large fee differences if β is small enough. Then given that only a fraction β of sellers are subjected to price coherence, we have

$$\begin{aligned} U_i &= \beta u^*(f^{avg}) + (1-\beta) u^*(f_i) \\ R_i &= f_i \left(\beta q^*(f^{avg}) + (1-\beta) q^*(f_i)\right) s_i. \end{aligned}$$

[22] In Online Appendix D.1, we construct an alternative version of Application 2 with spillovers based on seller investment decisions, and show how it is essentially equivalent to the application considered here.

Observe that both U_i and R_i are decreasing in f^{avg}: a higher fee would increase the common price by sellers that are subjected to price coherence. Thus, there is a negative spillover, and we conclude from Proposition 2 and Proposition 3 that the equilibrium transaction fee is above the seller-excluded benchmark which in turn is above the welfare-maximizing benchmark: $f^* \geq f^{SE} \geq f^W$.

□ **Building a direct channel.** Whenever a platform increases its transaction fees (be it ad-valorem or per-unit based), it induces sellers to be more willing to shift transactions onto a direct channel to avoid the higher fee. This could involve building out a direct channel, after which fewer transactions would be made on both the platform that increases its fees as well as rival platforms as some buyers shift transactions onto the direct channel. This, therefore, can create negative cross-platform spillovers via fees.

As an illustration, we modify Application 4 by endogenizing the fraction of sellers who own direct channels.[23] Specifically, suppose sellers face heterogenous cost κ to set up their direct channel, where $\kappa \in [0, \kappa_{\max}]$ is distributed according to CDF H with a strictly positive density. For expositional simplicity, we again assume that all sellers have zero fixed costs and zero participation costs $k_i = 0$ (i.e., the distribution G is degenerate). Then, all sellers will always choose to multihome on all platforms due to the fact that sellers do not face any restrictions in setting the on-platform prices, face no other costs, and still keep a fraction of their revenues.

The post-participation pricing problem remains the same as the original Application 4. Therefore, $U_i = u^*$. Meanwhile, a type-κ seller's total profit is $\left(1 - \sum_{i=1}^m \lambda_i r_i s_i\right) \pi^* - \kappa$ if it has a direct channel, and $\left(1 - \sum_{i=1}^m r_i s_i\right) \pi^*$ if it does not have a direct channel. Therefore, comparing payoffs, a type-κ seller sets up a direct channel if and only if

$$\kappa \leq \sum\nolimits_{i=1}^m (1 - \lambda_i) r_i s_i \pi^* \equiv \bar{\kappa},$$

and so

$$R_i = r_i \left(1 - (1 - \lambda_i) H(\bar{\kappa})\right) \pi^* s_i.$$

Observe that R_i decreases when the "cross-channel effective fee difference" $(1 - \lambda_j) r_j$ between platform j and the direct channel increases, because a greater fee difference induces more sellers to set up direct channels, i.e., a higher $\bar{\kappa}$. There is negative spillovers in platform fees r_j, and positive spillovers in leakage prevention efforts λ_j. The latter reflects that when one platform puts more effort into preventing leakage, less sellers will want to set up direct channels, which benefits rival platforms.

Moreover, $\hat{W}^{SE}(a_i)$ is quasi-supermodular if H is weakly convex. Hence, we conclude from Proposition 2 and Proposition 3 that $r^* \geq r^{SE} \geq r^W$ (the equilibrium commission is above the seller-excluded benchmark) and $\lambda^* \leq \lambda^{SE}$ (the equilibrium level of leakage prevention is below the seller-excluded benchmark). Meanwhile, given that seller surplus is decreasing in λ_i, we know $\lambda^W \leq \lambda^{SE}$, and so the comparison between λ^* and λ^W is in general ambiguous after allowing for spillovers. Here, higher leakage prevention by each platform reduces the number of sellers willing to invest in building direct channels. The resulting harm to sellers is not taken into account by

[23] A similar mechanism could arise in the case of Application 5 with mobile apps. When facing higher fees on one platform, a developer may be more likely to broaden its business model from just relying on in-app purchases to introduce third-party ads for users to unlock content, which could then be made available on other platforms as well, shifting transactions away from in-app purchases on those platforms.

platforms, which is a force leading to exessive leakage prevention. On the other hand, each platform doesn't take into account that their leakage prevention efforts benefit rival platforms by reducing the number of sellers investing in direct channels, making the overall effect ambiguous.

Nonetheless, using the idea of negative proxied spillovers via the "cross-channel effective fee difference", Corollary 2 implies $(1-\lambda^*)r^* > (1-\lambda^{SE})r^{SE}$ and so $\bar{\kappa}^* \geq \bar{\kappa}^{SE}$. Then, by changing platforms' decision variables and applying Proposition 2, we have that $\bar{\kappa}^{SE} \geq \bar{\kappa}^W$, implying $\bar{\kappa}^* \geq \bar{\kappa}^W$. That is, investment in direct channels by sellers is higher in the equilibrium than in the total welfare benchmark.

4.2 Mixed homing configurations

Up till now we have focused on situations of competing platforms where all users on one side (say buyers) only singlehome and all users on the other side (say sellers) are free to multihome. This is because, strictly speaking, any changes in homing possibilities would constitute a departure from the competitive bottleneck setting. Nonetheless, it may be useful to illustrate how alternative homing possibilities can be interpreted in our framework with cross-platform spillovers. To do so, we focus on the case the platform instrument is $a_i = f_i$, or some other type of transaction fee charged to sellers.

□ **Some sellers are unable to multihome.** If some sellers face frictions to multihome (e.g., because of contracts that mandate exclusivity) or otherwise are incentivised to singlehome (e.g., because of contracts they reward exclusivity such as market share discounts), then they may face a choice between participating on one platform only or none at all. In other words, sellers view each platform as a substitute: joining platform i would preclude the possibility of (or at least, reduce the payoff from) joining platforms $j \neq i$.

In this case, a higher f_i can not only make these sellers prefer to leave platform i, but also make them more likely to join some other platform j given that if they don't join platform i, then joining platform j now becomes possible (under exclusive contracts) or more attractive (if they would then have access to lower fees for having a high share of business on platform j). This would in turn increase U_j and R_j, and so Proposition 3 predicts $f^* \leq f^{SE}$, consistent with each platform's fee increase exerting a positive utility and revenue spillover on rival platforms.

It is important to note though, this mechanism is not simply driven by sellers voluntarily choosing to singlehome. Sellers may singlehome because they can only make a profit on one of the platforms, something we allowed for in our applications in case sellers face participation costs on each platform that are not perfectly correlated. But that doesn't mean the unprofitable option imposes a competitive constraint on the platform they choose to join. The exception is if sellers choose to singlehome because enough buyers multihome, the case we turn to next.

□ **Some multihoming buyers.** If enough buyers are free to multihome, and do so in equilibrium, then sellers can sometimes be better off only joining the lowest-fee platform even if they could make some incremental revenue from participating as well on higher fee platforms from buyers who singlehome on such platforms. This could make sense if doing so would divert sufficient multihoming buyers to switch their transactions to the cheapest platform (i.e., the platform with the lowest seller fee). Alternatively, sellers could remain on the higher-fee platforms but adjust their prices in a way that diverts the multihoming buyers to use the cheapest platform.

In this case, a higher f_i induces more sellers to engage in such diversion strategies through participation and pricing decisions, and so would tend to increase R_j, suggesting from Proposition 3 that $f^* \leq f^{SE}$. However, depending on how sellers adjust their prices on platform j when engaging in diversion pricing, U_j could increase or decrease, making the overall prediction in this case ambiguous in general.

5 Other sources of non-equivalence

Aside from cross-platform spillovers discussed in Section 4, other factors could also lead to a divergence between equilibrium outcomes and the seller-excluded outcome. We discuss two such factors in this section. We assume that the no-spillover condition holds, i.e., $U_i(\boldsymbol{a}; \boldsymbol{s}) = U_i(a_i; \boldsymbol{s})$ and $R_i(\boldsymbol{a}; \boldsymbol{s}) = R_i(a_i; \boldsymbol{s})$.

5.1 Monetizing via other buyer-side instruments

Our setup can easily accommodate the case of platforms charging a transaction-based fee to buyers on top of buyer membership fees (as well as transaction fees to sellers). In such cases, provided sellers are free to set prices to buyers, one would get neutrality of the platforms' transaction-based fees, so this would be equivalent to normalizing the buyer-side transaction fee to zero.

A more challenging case is that without a membership fee on the buyer side. Suppose instead the platform gets a payoff per subscriber A_i on the buyer side, reflecting for instance "advertising" revenue per subscriber. To establish this formally, we build upon the baseline model in Section 2. Suppose platform profit and buyer net utility functions become $\Pi_i = (A_i - c)\, s_i + R_i$ and

$$U_i - P(A_i) + \epsilon_i, \tag{16}$$

where $P(A_i)$ is the disutility faced by buyers given revenue extraction per buyer A_i by the platform. As we will show below, this setup is equivalent to our current framework if $P' = 1$, i.e., the revenue extraction technology has the same (marginal) efficiency as a membership fee — extracting one dollar of extra revenue from a buyer results in buyers giving up one dollar's worth of utility. More generally though, extracting revenues through advertising or in other ways may be more efficient than using membership fees (i.e. one dollar of extra revenue can be extracted from a buyer with less than a one dollar reduction in utility, so $P' < 1$), or less efficient ($P' > 1$). In such cases, this can affect the platforms' optimal choices of instruments a_i.

For simplicity, we assume a linear extraction technology as in Jullien and Bouvard (2022), so that P' is constant, and focus on the case where each platform i's instrument a_i is a continuous scalar. Suppose further that functions R_i and U_i are differentiable. In this case, each platform i's equilibrium instrument a^* satisfies the first-order condition

$$\frac{1}{mP'} \frac{\partial U_i(a_i; \mathbf{1}/m)}{\partial a_i} + \frac{\partial R_i(a_i; \mathbf{1}/m)}{\partial a_i} = 0. \tag{17}$$

Observe that if $P' > 1$ (equivalent discussions apply to the case of $P' < 1$, and hence are omitted below), platform i's equilibrium instrument choice assigns a smaller weight on the buyer surplus U_i, compared to the baseline model ($P' = 1$). This is intuitive. When the alternative monetization is less efficient, if a platform increases U_i by one unit, it can extract less than one unit of revenue A_i while keeping buyer net utility $U_i - P(A_i)$ constant, and so it optimally chooses a_i to implement a lower U_i than in the baseline model.

Meanwhile, from (14), in the seller-excluded benchmark, the outcome a^{SE} satisfies the first-order condition

$$\frac{d\hat{W}^{SE}(a_i)}{da_i} = \frac{\partial U_i(a_i; \mathbf{1}/m)}{\partial a_i} + m\frac{\partial R_i(a_i; \mathbf{1}/m)}{\partial a_i} + \left(1 - P'\right)\frac{dA^*}{da_i} = 0, \tag{18}$$

where A^* is the equilibrium platform monetization for a given symmetric profile of platform instrument $(a_i, ..., a_i) \in \mathcal{A}^m$ and market shares are given by $\boldsymbol{s} = \mathbf{1}/m$. Compared to the baseline model $(P' = 1)$, $P' > 1$ implies that the seller-excluded benchmark now additionally assigns a weight on inducing platforms to choose a lower level of monetization on the buyer side, reflecting such monetization is surplus-reducing. The magnitude of this new weight depends on the sensitivity of the platforms' monetization response dA^*/da_i in the equilibrium. Meanwhile, in the total welfare benchmark, the outcome a^W satisfies the first-order condition

$$\frac{d\hat{W}(a_i)}{da_i} = \frac{d\hat{W}^{SE}(a_i)}{da_i} + \frac{\partial \hat{SS}(a_i)}{\partial a_i},$$

where the only difference with (18) in the seller surplus term. As such, the conclusion from Proposition 2 continues to hold.

In sum, in this setting, the divergence between a^* and a^{SE} now depends on which of the two effects of $P' \neq 1$ above dominates, while the comparison between a^{SE} and a^W remain the same as in the benchmark setting. Substituting (17) into (18) we obtain (19) in the following Proposition:

Proposition 4 *Suppose that each platform i's instrument a_i is a continuous scalar, functions R_i and U_i are differentiable, and $\hat{W}^{SE}(a_i)$ is strictly quasiconcave. If $P' = 1$, then $a^* = a^{SE}$. Otherwise, $a^* \geq a^{SE}$ if and only if*

$$(1 - P')\left(\frac{dA^*}{da_i} - \frac{1}{P'}\frac{\partial U_i(a^*; \mathbf{1}/m)}{\partial a_i}\right) \leq 0, \tag{19}$$

where

$$\frac{dA^*}{da_i} = -\frac{1}{P'}\frac{\partial^2 U_i(a^*; \mathbf{1}/m)}{\partial a_i \partial s_i} + \frac{\partial^2 R_i(a^*; \mathbf{1}/m)}{\partial a_i \partial s_i}.$$

Moreover, if (19) holds and the total seller surplus function $\hat{SS}(a_i)$ is weakly decreasing in a_i, then $a^* \geq a^W$.

Under the condition in (19), allowing for the possibility that platforms prefer to monetize on the buyer side via advertising rather than membership fees implies equilibrium instrument choices a^* will, if anything, be higher than the corresponding seller-excluded outcome.

In Online Appendix E we show that for most of our applications, $dA_i^*/da_i = 0$ in the equilibrium when the distribution of seller fixed participation cost, G, is assumed to be the uniform distribution on $[0, k_{\max}]$. In such cases, condition (19) just depends on whether $(1 - P')\frac{\partial U_i}{\partial a_i} \geq 0$ holds. In particular, for platform instruments that reduce buyer surplus $(\partial U_i/\partial a_i \leq 0)$ such as transaction fees, we have $a^* \geq a^{SE}$ (and so $a^* \geq a^W$) when such monetization is inefficient ($P' > 1$), and $a^* \leq a^{SE}$ (making the comparison between a^* and a^W ambiguous) when such monetization is efficient ($P' < 1$).[24] Meanwhile, when G is non-linear but with a constant elasticity, if we take the elasticity becoming sufficiently small so that seller participation becomes unresponsive to changes in post-participation profits, then we show that $dA_i^*/da_i \to (1/P')\,\partial U_i/\partial a_i$ in the corresponding equilibrium, and so (19) holds in the limit, meaning $a^* \to a^{SE} \geq a^W$.

5.2 Myopic buyers

Suppose that buyers are "myopic". Specifically, when deciding which platform to join they incorrectly account for their post-participation utility, for example not accounting for it at all or only partially. To establish this formally, we build upon the baseline model in Section 2 by assuming that the measure of buyers joining platform i, i.e., the counterpart of (3), is given by

$$s_i = \Pr\left(\delta U_i - P_i^B + \epsilon_i \geq \max_{j \neq i}\left\{\delta U_j - P_j^B + \epsilon_j\right\}\right),$$

where $0 \leq \delta < 1$ is a discount factor and so $\delta U_i - P_i^B$ is buyer's (myopically) "perceived" net utility on each platform i. Meanwhile, U_i captures their "true" utility, which enters the seller-excluded benchmark and is relevant for a welfare analysis.

Following the same analysis as in Section 3 and given the no-spillover condition, the characterizations of the equilibrium and the seller-excluded outcomes are, respectively, given by

$$\begin{aligned} a^* &\in \arg\max\nolimits_{a_i \in \mathcal{A}} \left\{\tfrac{\delta}{m} U_i(a_i; \mathbf{1}/m) + R_i(a_i; \mathbf{1}/m)\right\} \\ a^{SE} &\in \arg\max\nolimits_{a_i \in \mathcal{A}} \left\{\tfrac{1}{m} U_i(a_i; \mathbf{1}/m) + R_i(a_i; \mathbf{1}/m)\right\}. \end{aligned} \tag{20}$$

Observe that U_i is under-represented in the choice of equilibrium a^* relative to the seller-excluded benchmark a^{SE}. Following the same idea as Proposition 2, we have:

Proposition 5 *(Myopic buyers). Suppose $U_i(a_i; \mathbf{1}/m)$ is weakly decreasing (increasing) in $a_i \in \mathcal{A}$ and one of the following conditions holds:*

- *The function $\hat{W}^{SE}(a_i)$ is quasi-supermodular in $a_i \in \mathcal{A}$.*
- *Platform instrument a_i is a scalar, i.e., $\mathcal{A} \subseteq \mathbb{R}$.*

Then, $\mathcal{A}^* \geq_{sso} (\leq_{sso}) \mathcal{A}^{SE}$. That is, the set of equilibrium instrument vectors is higher (lower) than the set of seller-excluded instrument vectors in strong set order. If, in addition, the total seller surplus function $\hat{SS}(a_i)$ is weakly decreasing in a_i, then $\mathcal{A}^* \geq_{sso}^W \mathcal{A}$.

Recall that we interpret a higher a_i as corresponding to a lower total seller surplus SS. Therefore, the corollary says that for instruments that decrease U_i and seller surplus (e.g., platforms fees), then a^* is above a^{SE}. Together with Proposition 2 (which is unaffected by the factor δ), we get $a^* \geq a^{SE} \geq a^W$. Likewise, the reverse is true for instruments that increase U_i and seller

[24] In Online Appendix B we show (19) holds with equality for any $P' > 0$ in the additional application with demand-side heterogeneity and competing sellers, so that $a^* = a^{SE} \geq a^W$ in that case.

surplus (e.g., investments). Intuitively, when buyers are myopic, platforms would not sufficiently take into account buyer true utility in the equilibrium.[25]

6 Policy discussion

Even if there is strong competition between platforms to sign up buyers, our results show that platform fees and other design choices such as first-party entry, self-preferencing, and leakage prevention will be distorted from a welfare perspective, in a way that shifts surplus from sellers to the platforms (and buyers). Whether such distortions are considered a policy problem depends on the objectives the policymaker holds. If the ultimate objective is only buyer-surplus (or buyer and platform surplus), then as Etro (2023) has shown in the context of fee setting, there may be no concern. However, sellers (e.g., the app developers, merchants, creators, and advertisers that rely on platforms to reach end-consumers) are platform customers too, and it is often their concerns as much as those of end-users that regulators engage with. As such, our focus on the standard benchmark adopted by economists, total welfare, would seem more appropriate.[26]

Beyond the many examples of distortions we've studied, the result that platforms ignore the concerns of sellers other than to the extent they translate into benefits for singlehoming buyers suggests other harms that may arise in competitive bottleneck settings. For instance, customer support and other types of platform investments may be biased towards the buyer-side and away from the seller-side of such platforms. The results also suggests any policy that only promotes more competition for buyers (e.g., reduced switching costs between platforms on the buyer-side) may not help address the underlying distortions. Similarly, a policy that attempts to induce more platforms to compete, will not necessarily reduce the distortions found here. In the case without spillovers, adding more platforms does not necessarily change the distortion between the equilibrium and welfare-maximizing level of platform instrument choices in one direction or the other. With spillovers, adding more platforms can make the distortion worse reflecting that each platform tends to internalize less the effect of their choices on sellers' overall participation choices.[27]

Given our results, an obvious policy solution to consider is regulating platform fees. Several recent works have studied how this can be done in the context of a monopoly platform setting (Gomes and Mantovani, 2022; Bisceglia and Tirole, 2022; Wang and Wright, 2023). However, while such regulations may indeed increase welfare if done correctly, one of the points of our multi-dimensional setting is to note the distortions are not limited to just platform fees. Thus, regulating lower platform fees does not directly address other types of distortions, and indeed in some cases could make them worse.

This suggests a superior approach may be to provide ways for sellers to side-step the bottleneck problem. There are two ways that could be done. One way is to promote buyer multihoming by reducing the cost of buyers participating on multiple platforms, thereby providing sellers with

[25]Relatedly, Etro (2023) focuses on buyer surplus in a specialized model on competition between mobile device platforms, and shows that the existence of myopic buyers imply that the equilibrium may not maximize the actual buyer surplus.

[26]Another approach is to focus on total user surplus (that of buyers and sellers), thus ignoring the profit of platforms. We provide some analysis of this alternative in Online Appendix F, where for our various applications we find similar results to those found for total welfare.

[27]We illustrate these results in Section G of the Online Appendix focusing on the choice of commissions in Application 2.

more than one platform through which they can reach the same buyers. If the platforms engineer barriers or additional costs to multihoming, making sure such practices are ruled out would help (Athey and Morton, 2022). However, sometimes there are inherent cost for buyers to multihome (e.g., purchasing a second mobile device is costly), so achieving widespread multihoming on the buyer side may be unrealistic.

Alternatively, or perhaps in addition, regulators can focus on making sure sellers are not denied other ways to reach and transact with a platform's unique buyers. In the context of mobile app platforms this can be done by making it illegal for platforms to ban (or otherwise limit) alternative app stores from being downloaded and installed by device users. This makes it more likely an app developer could steer its users through an app store that is better for both its users and itself. Similarly, it could become illegal for platforms to take actions that prevent or limit leakage, either via the sellers' own direct channels or via other cheaper platforms. This would increase the feasibility of bypassing a platform that doesn't offer sellers sufficient value. In the case of mobile app platforms, this would mean making Apple's and Google's anti-steering provisions illegal, so app developers would be able to provide customers with links to their other cheaper channels. It would also involve allowing direct downloading of the developer's app in the case of iOS, and allowing digital content purchased elsewhere from a developer to be used inside the app on iOS regardless of whether the app developer sells the same content via the App Store. Similarly, Apple's and Google's tying of their payment solutions to their app stores could be made illegal, thus enabling developers to have a direct relationship with their customers with respect to in-app purchases of digital content (via either their own or a third-party payment solution). Notably, Articles 5(4), 5(5), 5(7) and 6(4) of the Digital Markets Act (DMA), which comes into force in Europe in 2024, will enact all of these prohibitions and obligations.

7 Conclusion

We have provided a general framework to analyze competitive bottleneck settings, allowing for a full range of pricing instruments on the seller-side and for platforms to make non-pricing design choices as well (e.g., investment, first-party entry, self-preferencing, leakage prevention etc). We also allowed for a quite general payoff structure for buyers and sellers, which captures a wide range of microfoundations including settings in which platform fees get passed through to buyers.

We highlighted several sources of divergence between equilibrium choices and those in the seller-excluded benchmark, in each case, providing conditions to help sign the direction of the divergence, and the overall welfare effects of the equilibrium choices. These include spillovers (both in buyer utility and platform revenue), the case where platforms use alternative monetization on the buyer side such as advertising, and the case where buyers do not fully internalize their post-participation surplus when deciding which platform to join. We also briefly explained how other homing configurations such as partial multihoming on the buyer side can be understood in terms of our spillover results.

Here we briefly mention some possible directions for further generalizing our framework and to sign how the distortions in instrument choices may be affected as a result.

The current framework relies on buyers being ex-ante identical except for their taste for each platform. It would be interesting to see in what ways one can generalize results to allow for other forms of heterogeneity among buyers such as in Rochet and Tirole (2003, 2006). However, one potential complication of that setting is it would introduce new types of welfare distortions (e.g., the Spence-type distortions in Weyl (2010), and the displacement and scale distortions in Tan and Wright (2021)) that may be orthogonal to the effects arising from classic competitive bottleneck settings.

On the seller side, it would be interesting to analyze what happens when sellers are "strategic" and can commit to their participation decisions, such that each is large enough that it might internalize how its joining decision impacts buyers' decision about which platform to join. We abstracted from this by assuming buyers and sellers made their joining decisions simultaneously.

Finally, there are other interesting applications of this framework that remain to be explored. Even sticking to our existing applications, there are many combinations of sets of platform instruments (we considered seven) and sources of spillovers (we considered three) that are left to analyze. And once one considers applications to other verticals such as newspapers, video game consoles, and prioritized internet service providers, there may be new instruments and new sources of spillovers that are particularly relevant and which can be usefully analyzed in our framework.

8 Appendix

8.1 Proofs in Section 2

Proof. (Proposition 2). Suppose $\hat{SS}(a_i)$ is weakly decreasing in a_i (the weakly increasing case can be proven similarly). We want to prove the sets of maximizers are such that

$$\mathcal{A}^{SE} \equiv \arg\max_{a_i \in \mathcal{A}} \hat{W}^{SE}(a_i) \geq \arg\max_{a_i \in \mathcal{A}} \hat{W}(a_i) \equiv \mathcal{A}^W \tag{21}$$

in strong set order sense. Specifically, for any $a^{SE} \in \mathcal{A}^{SE}$ and $a^W \in \mathcal{A}^W$, denote $a^{\max} = a^{SE} \vee a^W$ and $a^{\min} = a^{SE} \wedge a^W$ (by construction, $a^{\max} \geq a^{SE}, a^W \geq a^{\min}$), then we want to prove $a^{\max} \in \mathcal{A}^{SE}$ and $a^{\min} \in \mathcal{A}^W$. Suppose $\hat{W}(a_i)$ is quasi-supermodular (note if a_i is a scalar then this is trivially true). By definition of a^W,

$$\begin{aligned}
& \hat{W}(a^W) - \hat{W}(a^{\min}) \geq 0 \\
\Rightarrow\ & \hat{W}(a^{\max}) - \hat{W}(a^{SE}) \geq 0 \quad \text{(quasi-supermodularity of } \hat{W}\text{)} \\
\Rightarrow\ & \hat{W}^{SE}(a^{\max}) - \hat{W}^{SE}(a^{SE}) + \underbrace{\hat{SS}(a^{\max}) - \hat{SS}(a^{SE})}_{\leq 0 \text{ because } \hat{SS} \text{ is weakly decreasing}} \geq 0 \quad \text{(definition of } \hat{W}\text{)} \\
\Rightarrow\ & \hat{W}^{SE}(a^{\max}) - \hat{W}^{SE}(a^{SE}) \geq 0,
\end{aligned}$$

which implies $a^{\max} \in \mathcal{A}^{SE}$. Suppose $\hat{W}^{SE}(a_i)$ is quasi-supermodular instead of $\hat{W}(a_i)$, then we can simply reorder the steps of the proof above:

$$\begin{aligned}
& \hat{W}(a^W) - \hat{W}(a^{\min}) \geq 0 \\
\Rightarrow\ & \hat{W}^{SE}(a^W) - \hat{W}^{SE}(a^{\min}) + \underbrace{\hat{SS}(a^W) - \hat{SS}(a^{\min})}_{\leq 0 \text{ because } SS \text{ is weakly decreasing}} \geq 0 \quad \text{(definition of } \hat{W}\text{)} \\
\Rightarrow\ & \hat{W}^{SE}(a^W) - \hat{W}^{SE}(a^{\min}) \geq 0 \\
\Rightarrow\ & \hat{W}^{SE}(a^{\max}) - \hat{W}^{SE}(a^{SE}) \geq 0 \quad \text{(quasi-supermodularity of } \hat{W}^{SE}\text{)},
\end{aligned}$$

which implies $a^{\max} \in \mathcal{A}^{SE}$. Likewise, by definition of a^{SE},

$$\begin{aligned}
& \hat{W}^{SE}(a^{SE}) - \hat{W}^{SE}(a^{\max}) \geq 0 \\
\Rightarrow \quad & \hat{W}^{SE}(a^{SE}) - \hat{W}^{SE}(a^{\max}) + \underbrace{\hat{SS}(a^{SE}) - \hat{SS}(a^{\max})}_{\geq 0 \text{ because } \hat{SS} \text{ is weakly decreasing}} \geq 0 \\
\Rightarrow \quad & \hat{W}(a^{SE}) - \hat{W}(a^{\max}) \geq 0 \qquad \text{(definition of } \hat{W}) \\
\Rightarrow \quad & \hat{W}(a^{\min}) - \hat{W}(a^{W}) \geq 0 \quad \text{(contrapositive of quasi-supermodularity of } \hat{W}),
\end{aligned}$$

which implies $a^{\min} \in \mathcal{A}^{W}$. If $\hat{W}^{SE}(a)$ is quasi-supermodular instead of $\hat{W}(a)$, then we can again reorder the steps of the proof as shown previously. ■

8.2 Proofs in Section 4

Proof. (Proposition 3). We will focus on the case of negative cross-platform spillovers (the case of positive spillovers can be proven similarly). In what follows we omit the market share profile argument when expressing functions $U_i(\boldsymbol{a};\boldsymbol{s})$ and $R_i(\boldsymbol{a};\boldsymbol{s})$ given that we always set $\boldsymbol{s} = \mathbf{1}/m$. Denote $\hat{\boldsymbol{a}}(a_i;a) \in \mathcal{A}^m$ as a profile such that platform i is choosing instrument vector $a_i \in \mathcal{A}$ while all other platforms $j \neq i$ are choosing the same instrument vector $a \in \mathcal{A}$.

For any given $a_i \in \mathcal{A}$, denote

$$\hat{W}^{SE}(a_i) = U_i(\hat{\boldsymbol{a}}(a_i;a_i)) + mR_i(\hat{\boldsymbol{a}}(a_i;a_i)), \tag{22}$$

which is just the SE objective function (10) after omitting components that are independent of platform instrument vectors when $\boldsymbol{s} = \mathbf{1}/m$. Using notations

$$\begin{aligned}
\hat{Z}(a_i;a) &\equiv U_i\left(\hat{\boldsymbol{a}}(a_i,a)\right) - U_{-i}\left(\hat{\boldsymbol{a}}(a,a_i)\right) + mR_i\left(\hat{\boldsymbol{a}}(a_i,a)\right) \\
\psi(a_i;a) &\equiv U_{-i}\left(\hat{\boldsymbol{a}}(a,a_i)\right) + U_i(\hat{\boldsymbol{a}}(a_i,a_i)) - U_i\left(\hat{\boldsymbol{a}}(a_i,a)\right) + mR_i\left(\hat{\boldsymbol{a}}(a_i,a_i)\right) - mR_i\left(\hat{\boldsymbol{a}}(a_i,a)\right),
\end{aligned}$$

we can expand (22) as

$$\hat{W}^{SE}(a_i) = \hat{Z}(a_i;a) + \psi(a_i;a) \quad \text{for arbitrary } a \in \mathcal{A}, \tag{23}$$

where the negative spillovers condition implies:

$$a_i \geq a \Rightarrow \psi(a_i;a) \leq U_{-i}\left(\hat{\boldsymbol{a}}(a,a_i)\right) \leq U_{-i}\left(\hat{\boldsymbol{a}}(a,a)\right) = \psi(a;a). \tag{24}$$

Using these notations and the definitions in (9) and (11), we get

$$\begin{aligned}
\mathcal{A}^* &= \left\{a^* \in \mathcal{A} | a^* \in \arg\max_{a_i \in \mathcal{A}} \hat{Z}(a_i;a^*)\right\} \\
\mathcal{A}^{SE} &= \left\{a^{SE} \in \mathcal{A} | a^{SE} \in \arg\max_{a_i \in \mathcal{A}} \hat{W}^{SE}(a_i)\right\}.
\end{aligned}$$

We claim that $\mathcal{A}^* \geq_{sso} \mathcal{A}^{SE}$. Specifically, for any $a^{SE} \in \mathcal{A}^{SE}$ and $a^* \in \mathcal{A}^*$, denote $a^{\max} = a^* \vee a^{SE}$, and $a^{\min} = a^* \wedge a^{SE}$ (by construction, $a^{\max} \geq a^*, a^{SE} \geq a^{\min}$), then we want to prove $a^{\max} \in \mathcal{A}^*$ and

$a^{\min} \in \mathcal{A}^{SE}$. By definition of a^{SE},

$$
\begin{aligned}
& \hat{W}^{SE}(a^{SE}) - \hat{W}^{SE}(a^{\min}) \geq 0 \\
\Rightarrow\ & \hat{W}^{SE}(a^{\max}) - \hat{W}^{SE}(a^{*}) \geq 0 \qquad \text{(quasi-supermodularity of } \hat{W}^{SE}) \\
\Rightarrow\ & \hat{Z}(a^{\max}; a^{*}) - \hat{Z}(a^{*}; a^{*}) + \underbrace{\psi(a^{\max}; a^{*}) - \psi(a^{*}; a^{*})}_{\leq 0 \text{ by } (24)} \geq 0 \qquad \text{(by (23))} \\
\Rightarrow\ & \hat{Z}(a^{\max}; a^{*}) - \hat{Z}(a^{*}; a^{*}) \geq 0,
\end{aligned}
$$

which implies $a^{\max} \in \mathcal{A}^{*}$. Likewise, by definition of a^{*},

$$
\begin{aligned}
& \hat{Z}(a^{*}; a^{*}) - \hat{Z}(a^{\max}; a^{*}) \geq 0 \\
\Rightarrow\ & \hat{Z}(a^{*}; a^{*}) - \hat{Z}(a^{\max}; a^{*}) + \underbrace{\psi(a^{*}; a^{*}) - \psi(a^{\max}; a^{*})}_{\geq 0 \text{ by } (24)} \geq 0 \\
\Rightarrow\ & \hat{W}^{SE}(a^{*}) - \hat{W}^{SE}(a^{\max}) \geq 0 \qquad \text{(by (23))} \\
\Rightarrow\ & \hat{W}^{SE}(a^{\min}) - \hat{W}^{SE}(a^{SE}) \geq 0 \quad \text{(contrapositive of quasi-supermodularity of } \hat{W}^{SE}),
\end{aligned}
$$

which implies $a^{\min} \in \mathcal{A}^{SE}$. ■

Proof. (Corollary 2). We will focus on the case of negative cross-platform spillovers (the case of positive spillovers can be proven similarly). In what follows we omit the market share profile argument when expressing functions $U_i(a_i, b_{-i}; \boldsymbol{s})$ and $R_i(a_i, b_{-i}; \boldsymbol{s})$ given that we always set $\boldsymbol{s} = \mathbf{1}/m$. Denote $\hat{\boldsymbol{b}}(b_i; b) \in \mathbb{R}^m$ as a profile such that the proxy instrument equals $b_i = b(a_i)$ for platform i and equals b for all other platforms $j \neq i$.

By contradiction, suppose there exist a pair $a^{*} \in \mathcal{A}^{*}$ and $a^{SE} \in \mathcal{A}^{SE}$ such that $b^{*} = b(a^{*}) < b(a^{SE}) = b^{SE}$. Then

$$
\begin{aligned}
\hat{W}^{SE}(a^{SE}) &= U_i(a^{SE}, \hat{\boldsymbol{b}}(b^{SE}; b^{SE})) + mR_i(a^{SE}, \hat{\boldsymbol{b}}(b^{SE}; b^{SE})) \\
&= U_i(a^{SE}, \hat{\boldsymbol{b}}(b^{SE}; b^{*})) - U_{-i}(a^{*}, \hat{\boldsymbol{b}}(b^{SE}; b^{*})) + mR_i(a^{SE}, \hat{\boldsymbol{b}}(b^{SE}; b^{*})) + U_{-i}(a^{*}, \hat{\boldsymbol{b}}(b^{SE}; b^{*})) \\
&\quad + \underbrace{U_i(a^{SE}, \hat{\boldsymbol{b}}(b^{SE}; b^{SE})) - U_i(a^{SE}, \hat{\boldsymbol{b}}(b^{SE}; b^{*}))}_{<0 \text{ by negative proxied spillovers and } b^{SE} > b^{*}} \\
&\quad + \underbrace{mR_i(a^{SE}, \hat{\boldsymbol{b}}(b^{SE}; b^{SE})) - mR_i(a^{SE}, \hat{\boldsymbol{b}}(b^{SE}; b^{*}))}_{<0 \text{ by negative proxied spillovers and } b^{SE} > b^{*}} \\
&< U_i(a^{*}, \hat{\boldsymbol{b}}(b^{*}; b^{*})) - U_{-i}(a^{*}, \hat{\boldsymbol{b}}(b^{*}; b^{*})) + mR_i(a^{*}, \hat{\boldsymbol{b}}(b^{*}; b^{*})) + U_{-i}(a^{*}, \hat{\boldsymbol{b}}(b^{*}; b^{*})) \\
&= \hat{W}^{SE}(a^{*}),
\end{aligned}
$$

where the inequality is due to negative proxied spillovers and the fixed-point definition of

$$
a^{*} \in \arg\max_{a_i \in \mathcal{A}} \left\{ U_i(a_i, \hat{\boldsymbol{b}}(b(a_i); b^{*})) - U_{-i}(a^{*}, \hat{\boldsymbol{b}}(b(a_i); b^{*})) + mR_i(a_i, \hat{\boldsymbol{b}}(b(a_i); b^{*})) \right\},
$$

thus contradicting the definition of a^{SE} being a maximizer of $\hat{W}^{SE}$. ■

8.3 Proofs in Section 5

Proof. (Proposition 4). Given the linearity assumption, we write $P(A_i) = P' A_i$. On the equilibrium path, assuming all other platforms $j \neq i$ set monetization level A^{*} and instrument vector a^{*}, a deviating platform i's profit function is $(A_i - c)\, s_i + R_i$, where $s_i = \Phi\left(U_i(a_i; \hat{\boldsymbol{s}}) - U_{-i}(a^{*}; \hat{\boldsymbol{s}}) - P' A_i + P' A^{*}\right)$. Applying the inversion technique in Section 3, we express platform i's profit as a function of its choices of s_i and a_i:

$$\Pi(s_i, a_i) = \left(\frac{U_i(a_i; \hat{\boldsymbol{s}}) - U_{-i}(a^*; \hat{\boldsymbol{s}}) - \Phi^{-1}(s_i) + P(A^*)}{P'} - c \right) s_i + R_i.$$

Differentiating and imposing symmetry,

$$\frac{d\Pi_i}{da_i}|_{sym} = 0 \Rightarrow \frac{1}{mP'} \frac{\partial U_i(a^*; \mathbf{1}/m)}{\partial a_i} + \frac{\partial R_i(a^*; \mathbf{1}/m)}{\partial a_i},$$

which satisfies (17), which can be substituted into (18) to yield (19). Meanwhile,

$$\begin{aligned} &\frac{d\Pi_i}{ds_i}|_{sym} = 0 \\ \Rightarrow \quad &A^* = c + \frac{1}{P'} \left(\frac{1/m}{\Phi'(0)} - \frac{\partial U_i(a^*; \mathbf{1}/m)}{\partial s_i} \right) - \frac{\partial R_i(a^*; \mathbf{1}/m)}{\partial s_i}. \end{aligned} \tag{25}$$

Note that this equilibrium pricing equation applies not just at the equilibrium profile $(a^*, ..., a^*)$ but it also applies to arbitrarily symmetric profile of instruments $(a_i, ..., a_i)$. Whenever this profile $(a_i, ..., a_i)$ changes symmetrically (i.e., in the maximization of the SE objective function), we get

$$\frac{dA_i^*}{da_i} = -\frac{1}{P'} \frac{\partial^2 U_i}{\partial s_i \partial a_i} - \frac{\partial^2 R_i}{\partial s_i \partial a_i}. \tag{26}$$

■

Proof. (Proposition 5). The proof of Proposition 3 applies after replacing $U_{-i}(a_i; \mathbf{1}/m)$ with $(1 - \delta)U_i(a_i; \mathbf{1}/m)$ as it represents the divergence between the definitions of a^{SE} and a^* in this case. Then, the case of decreasing $U_i(a_i; \mathbf{1}/m)$ is equivalent to the case of negative spillovers in the proof of Proposition 3 ■

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Online Appendix: Competitive bottlenecks and platform spillovers

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In the following sections we provide additional workings and results referred to but not included in the main paper.

A Asymmetric platforms and incomplete coverage on buyer side

Consider an environment with possibly asymmetric platforms and without necessarily a fully covered buyer-side market. Let U_0 be the exogenous net utility of the buyers' outside option (of not joining any platform). Our goal is to establish variants of Proposition 1 in this extension of our general environment, including the one based on Armstrong (2006)'s approach. To do so, we assume throughout that the no-spillover condition holds, i.e., $U_i = U_i(a_i; \boldsymbol{s})$ and $R_i = R_i(a_i; \boldsymbol{s})$.

The measure of buyers joining platform i is expressed as

$$\begin{aligned} s_i &= \Pr\left(U_i - P_i^B + \epsilon_i \geq \max_{j \neq i} \left\{U_j - P_j^B + \epsilon_j, U_0\right\}\right) \\ &= \Pr\left(\epsilon_i \geq \max_{j \neq i} \left\{U_j - P_j^B - U_i + P_i^B + \epsilon_j, U_0 - U_i + P_i^B\right\}\right) \\ &\equiv \Phi_i\left([U_i - P_i^B - U_j + P_j^B]_{j\neq i}, U_i - P_i^B - U_0\right), \end{aligned}$$

where $[U_i - P_i^B - U_j + P_j^B]_{j\neq i}$ is a $(m-1)$-dimension vector of $U_j - P_j^B - U_i + P_i^B$ for every $j = 2, ..., m$ such that $j \neq i$, while Φ is a function that is increasing in all of its m arguments and it reflects the underlying distribution function $F(\cdot)$ for $\boldsymbol{\epsilon} = (\epsilon_1, ..., \epsilon_m)$. Then, for any given $\boldsymbol{a} = (a_1, ..., a_m) \in \mathcal{A}^m$ and $\boldsymbol{P}^B = (P_1^B, ..., P_m^B)$ chosen, the market share profile $\boldsymbol{s} = (s_1, s_2, ..., s_m)$ is pinned down by the simultaneous fixed-point equation system:

$$s_i = \Phi_i\left([U_i(a_i; \boldsymbol{s}) - P_i^B - U_j(a_j; \boldsymbol{s}) + P_j^B]_{j\neq i}, U_i(a_i; \boldsymbol{s}) - P_i^B - U_0\right) \text{ for } i = 1, ..., m. \tag{27}$$

In what follows, we derive the equilibrium outcome. Denote the equilibrium buyer price profile as $\boldsymbol{P}^{B*} = (P_1^{B*}, ..., P_m^{B*})$, the equilibrium instrument profile as $\boldsymbol{a}^* = (a_1^*, ..., a_m^*) \in \mathcal{A}^m$, and the equilibrium buyer-side market share profile as $\boldsymbol{s}^* = (s_1^*, ..., s_m^*) \in [0,1]^m$. Without loss of generality, consider the maximization problem of, say, platform 1. It chooses $\left(a_1, P_1^B\right)$ to maximize profit

$$\Pi_1 = \left(P_1^B - c\right)s_1 + R_1(a_1; \boldsymbol{s}),$$

taking as given $(a_2^*, ..., a_m^*)$ and $(P_2^{B*}, ..., P_m^{B*})$ chosen by other platforms. We want to reframe the problem as platform 1 directly choosing the target market share s_1 implementable by its fee P_1^B, i.e., maximization with respect to (a_1, s_1). To proceed, note that for any given $(a_1, a_2^* ..., a_m^*)$, $(P_2^{B*}, ..., P_m^{B*})$, and s_1, we can implicitly pin down the implied buyer price by platform 1, denoted as

$$\tilde{P}_1^B(a_1, \boldsymbol{a}_{-1}^*, \boldsymbol{P}_{-1}^{B*}; s_1)$$

and market share $(s_2, ..., s_m)$ of other platforms using (27). Define a residual function $\beta_1 \equiv U_1(a_1; \boldsymbol{s}) - \tilde{P}_1^B$,

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and substitute it into (27) to get

$$\begin{aligned} s_1 &= \Phi_1\left([\beta_1 - U_j(a_j;\boldsymbol{s}) + P_j^B]_{j\neq 1}, \beta_1 - U_0\right) \\ s_i &= \Phi_i\left((U_i(a_i;\boldsymbol{s}) - P_i^B - \beta_1, [U_i(a_i;\boldsymbol{s}) - P_i^B - U_j(a_j;\boldsymbol{s}) + P_j^B]_{j\neq i\neq 1}), U_i(a_i;\boldsymbol{s}) - P_i^B - U_0\right) \text{ for } i = 2,...,m. \end{aligned}$$

This system implicitly pins down β_1 and $(s_2,...,s_m)$ as a function of $(a_1, a_2^*..., a_m^*)$, $(P_2^{B*},...,P_m^{B*})$, and s_1. Crucially, the system is independent of a_1, meaning the residual function β_1 and the market share $(s_2,...,s_m)$ of other platforms are independent of a_1 once s_1 is held fixed. This means that we can write

$$\tilde{P}_1^B(a_1, \boldsymbol{a}_{-1}^*, \boldsymbol{P}_{-1}^{B*}, s_1) = U_1(a_1;\boldsymbol{s}) + \beta_1,$$

where β_1 is independent of a_1.

Then, platform 1's problem is to choose (a_1, s_1) to maximize

$$\begin{aligned} \Pi_1(a_1,s_1) &= \left(\tilde{P}_1^B - c\right)s_1 + R_1(a_1;\boldsymbol{s}) \\ &= (U_1(a_1;\boldsymbol{s}) + \beta_1 - c)s_1 + R_1(a_1;\boldsymbol{s}). \end{aligned} \tag{28}$$

By the envelope theorem, the platform's optimal choice of $a_1 \in \mathcal{A}$ can be obtained by maximizing Π_1 while holding $\boldsymbol{s}$ constant at the equilibrium value $\boldsymbol{s}^*$. Since this analysis applies to all platforms $i = 1,...,m$, we conclude that the equilibrium $\boldsymbol{a}^* = (a_1^*,...,a_m^*)$ satisfies

$$a_i^* \in \arg\max_{a_i\in\mathcal{A}} \{s_i^* U_i(a_i;\boldsymbol{s}^*) + R_i(a_i;\boldsymbol{s}^*)\} \text{ for every } i = 1,...,m, \tag{29}$$

where the set element notation takes into account the possibility of multiple maximizers.

□ **Armstrong (2006)'s approach.** By Armstrong's approach, to formulate the seller-excluded benchmark, we impose an arbitrarily fixed $\boldsymbol{s}'=(s_1',...,s_m')$. Then, consider the seller-excluded welfare objective function from (10):

$$W^{SE}(\boldsymbol{a}) = \sum_{i=1,...,m}\{(U_i - c)s_i' + R_i\} + \sum_{i=1,...,m} E_i s_i' + U_0(1 - \sum_{i=1,...,m} s_i'),$$

where $E_i = E[\epsilon_i | i = \arg\max_{i=1,...,m}\{U_i - P_i^B + \epsilon_i, U_0\}]$ is the expectation of buyer match value on platform i conditioned on i being chosen. Note that fixing $\boldsymbol{s}'$ is equivalent to fixing $(E_1,...,E_m)$ in this environment. Then, by the no-spillover condition, maximizing $W^{SE}(\boldsymbol{a})$ with respect to $\boldsymbol{a}$ gives

$$a_i^{SE} \in \arg\max_{a_i\in\mathcal{A}} \{s_i' U_i(a_i;\boldsymbol{s}') + R_i(a_i;\boldsymbol{s}')\} \text{ for every } i = 1,...,m, \tag{30}$$

where the set element notation takes into account the possibility of multiple maximizers. Observe that (29) and (30) have the same expression if we evaluate both of them at the same market share profile $\boldsymbol{s}' = \boldsymbol{s}^*$. That is, for given $\boldsymbol{s}'$,

$$\mathcal{A}^* = \mathcal{A}^{SE} = \arg\max_{a_i\in\mathcal{A}} \{s_i' U_i(a_i;\boldsymbol{s}') + R_i(a_i;\boldsymbol{s}')\},$$

which corresponds to the result in Proposition 1.

□ **Symmetric but incomplete coverage on buyer side.** Let us return to our approach of defining the seller-excluded benchmark to examine how the market coverage assumption affects Proposition 1. To do so, we impose symmetry in the analysis above. Let $\bar{s} = \sum_{i=1,...,m} s_i$, so (29) becomes

$$a^* \in \arg\max_{a_i\in\mathcal{A}} \left\{ \frac{\bar{s}^*}{m} U_i(a_i; \mathbf{1}\frac{\bar{s}^*}{m}) + R_i(a_i; \mathbf{1}\frac{\bar{s}^*}{m}) \right\}.$$

Meanwhile, imposing symmetry and dropping constant terms, the seller-excluded welfare objective that is relevant for determining a_i^{SE} becomes

$$\hat{W}^{SE}(a_i) = \bar{s}U_i(a_i;\mathbf{1}\frac{\bar{s}}{m})+mR_i(a_i;\mathbf{1}\frac{\bar{s}}{m})+\bar{s}E\left[\epsilon_i|i=\arg\max_{i=1,\dots,m}\left\{U_i(a_i;\mathbf{1}\frac{\bar{s}}{m})-P^B+\epsilon_i,U_0\right\}\right]-c\bar{s}+(1-\bar{s})U_0,$$

where P^B is the symmetric equilibrium level of P_i^B for all platforms that comes out of the choice of P_i^B by each platform i given an arbitrary (symmetrically imposed) instrument a_i, while the total market coverage, given symmetry, is

$$\bar{s} = \Pr\left(U_i(a_i;\mathbf{1}\frac{\bar{s}}{m})+\max_{i=1,\dots,m}\{\epsilon_i\}-P^B\geq U_0\right).$$

That is, $\bar{s}$ is given by the mass of buyers opting for one of the m platforms as opposed to the outside option.

There are two potential source of divergence (relative to the equilibrium outcome). First, the market coverage levels are different, that is, $\bar{s}^{SE}\neq\bar{s}^*$. Second, the last term in the expression of $\hat{W}^{SE}$ means that the SE objective places a weight on raising $U_i(a_i;\mathbf{1}\frac{\bar{s}}{m})-P^B$ and the market coverage $\bar{s}$ (which can be understood as an inverse measure of deadweight losses), while the platforms' choice of a_i does not take into account the market coverage (by the envelope theorem).

Consider the special case where $m=2$, $U_i=\bar{U}(a_i)$ does not depend on the market share profile, and $R_i=\bar{R}(a_i)s_i$. For instance, this is satisfied in the demand heterogeneity example of Section B if we take $m=2$. For such cases, we claim that with an additional assumption noted below on the distribution of ϵ_i, then

$$\mathcal{A}^*=\mathcal{A}^{SE}=\arg\max_{a_i\in\mathcal{A}}\left\{\bar{U}(a_i)+\bar{R}(a_i)\right\}.$$

We next prove this claim. Recall the symmetry assumption implies functions $\Phi_1=\Phi_2=\Phi$. In this case, the market share profile $\boldsymbol{s}=(s_1,s_2)$ is explicitly pinned down by

$$\begin{aligned} s_1 &= \Phi\left(\bar{U}(a_1)-P_1^B-\bar{U}(a_2)+P_2^B,\bar{U}(a_1)-P_1^B-U_0\right)\\ s_2 &= \Phi\left(\bar{U}(a_2)-P_2^B-\bar{U}(a_1)+P_1^B,\bar{U}(a_2)-P_2^B-U_0\right).\end{aligned}$$

Denote $\Phi'=\Phi'_{in}+\Phi'_{out}$ where Φ'_{in} and Φ'_{out} are the derivatives of function Φ with respect to its first and second arguments. Note that if we express $\tilde{P}_1^B$ as a function of s_1, then $\frac{\partial\tilde{P}_1^B}{\partial s_1}=\frac{-1}{\Phi'}$ by total differentiation. In what follows, we assume Φ is log-concave, in the sense that

$$\frac{\Phi(x,y)}{\Phi'(x,y)}\text{ is increasing in its second argument.}$$

By standard results, this assumption is satisfied if e.g., ϵ_1 and ϵ_2 are i.i.d. with the same CDF F and a log-concave density f. It is also satisfied if (ϵ_1,ϵ_2) arise from the standard Hotelling model setup with linear transport costs but with outside buyers also uniformly and symmetrically located outside both ends of the unit interval, possibly facing a different linear transport cost.

Returning to profit maximization problem in (28), the imposed condition on U_i and R_i allows us to simplify it as $\Pi_1(a_1,s_1)=\left(\tilde{P}_1^B-c+\bar{R}(a_1)\right)s_1$. To solve for the equilibrium buyer price, the first-order condition gives

$$\frac{d\Pi_1}{ds_1}|_{symmetry}=0\Rightarrow P^{B*}=c-\bar{R}(a^*)+\frac{\Phi(0,\bar{U}(a^*)-P^{B*}-U_0)}{\Phi'(0,\bar{U}(a^*)-P^{B*}-U_0)}. \quad (31)$$

Meanwhile,

$$\mathcal{A}^*=\arg\max_{a_i\in\mathcal{A}}\left\{\bar{U}(a_i)+\bar{R}(a_i)\right\}$$

is immediate from (29). The resulting equilibrium profit is $\Pi_1^*=\frac{\Phi(0,\bar{U}(a^*)-P^{B*}-U_0)}{\Phi'(0,\bar{U}(a^*)-P^{B*}-U_0)}$. Note this profit expression applies for any arbitrary symmetrically imposed vector a_i.

Moving to the seller-excluded welfare maximization, the pricing equation (31) implies that, for arbitrary (symmetrically imposed) vector a_i, we have

$$\bar{U}(a_i) - P^{B*} = c + \bar{U}(a_i) + \bar{R}(a_i) + \frac{\Phi(0, \bar{U}(a_i) - P^{B*} - U_0)}{\Phi'(0, \bar{U}(a_i) - P^{B*} - U_0)}.$$

Log-concavity implies

$$\frac{d\left(\bar{U}(a_i) - P^{B*}\right)}{d\left(\bar{U}(a_i) + \bar{R}(a_i)\right)} \in (0,1),$$

and so $\bar{s}$ increases with $\bar{U}(a_i)+\bar{R}(a_i)$. Then, rewrite $\hat{W}^{SE}(a_i)$ by splitting platform profit and buyer surplus,

$$\begin{aligned} \hat{W}^{SE}(a_i) &= 2\Pi_i + BS \\ &= \frac{2\Phi(0, \bar{U}(a_i) - P^{B*} - U_0)}{\Phi'(0, \bar{U}(a_i) - P^{B*} - U_0)} + E\left[\max_{i=1,2}\left\{\bar{U}(a_i) - P^{B*} + \epsilon_i, U_0\right\}\right], \end{aligned}$$

which is increasing in $\bar{U}(a_i) - P^{B*}$, which in turn is increasing in $\bar{U}(a_i) + \bar{R}(a_i)$. Therefore, we conclude

$$\mathcal{A}^{SE} = \arg\max_{a_i \in \mathcal{A}} \left\{\bar{U}(a_i) + \bar{R}(a_i)\right\},$$

as required.

B Demand-side heterogeneity and competing sellers

In this section we provide an additional application beyond those provided in Section 2.2. This illustrates how we can accommodate:

1. heterogeneity in demand across product categories;
2. competing sellers within product categories;
3. positive pass-through from platform fees into seller prices;

We do this in a setting with closed-form solutions. This allows us to directly compare the equilibrium fees to the total welfare maximizing fees, as well as to the fees maximizing other possible objective functions. Since we want to explore how pass-through affects the welfare results, we focus on an example where platform i just charges a per-transaction fee f_i to sellers and a lump-sum membership fee P_i^B to buyers. We also characterize the outcome if the platforms cannot charge a lump-sum membership fee P_i^B but rather rely on alternative monetization involving A_i as in Section 5.1.

There is a continuum of product categories with mass 1 indexed by the buyers' interaction benefit parameter v, where $v \in [0, v_{\max}]$ is drawn from some distribution G on $[0, v_{\max}]$. There are $n \geq 1$ potential competing sellers in each product category. A representative buyer's gross utility function for purchasing q_l units from each seller $l = 1, .., n$ in a particular product category is

$$u(q_1, ..., q_n) = v\sum_{l=1}^{n} q_l - \frac{n}{2}\left((1-\theta)\sum_{l=1}^{n} q_l^2 + \frac{\theta}{n}\left(\sum_{l=1}^{n} q_l\right)^2\right),$$

and $\theta \in [0,1]$ is a measure of seller differentiation within the category. This is the model by Shubik and Leviatan (1980).[3] Then, buyer demand for seller l in category v is

$$q_v = \frac{1}{n}\left(v - \frac{p_l}{1-\theta} + \frac{\theta}{1-\theta}\sum_{l=1}^{n}\frac{p_l}{n}\right).$$

[3] Shubik, M., and Levitan, R. (1980). Market structure and behavior. Harvard University Press.

We normalize sellers' marginal costs to zero, and for simplicity, assume sellers face no fixed costs of participating on a platform.

Solving for the symmetric equilibrium between sellers yields the equilibrium price on platform i

$$p_v^*\left(f_i\right)=f_i+\frac{(1-\theta)\, n}{(2-\theta)\, n-\theta}(v-f_i),$$

which implies a pass-through rate $\rho \equiv \partial p_v^*\left(f_i\right)/\partial f_i=\frac{n-\theta}{(2-\theta) n-\theta}$, and $\rho \in\left[\frac{1}{2}, 1\right]$. The demand and profit an individual seller gets in product category v from a representative buyer is $q_v^*\left(f_i\right)=\rho\left(\frac{v-f_i}{n}\right)$, $\pi_v^*\left(f_i\right)=\frac{(1-\theta) n^2}{n-\theta} q_v^*\left(f_i\right)^2$, and per-buyer utility in product category v is $u_v^*(f_i)=\frac{n^2}{2} q_v^*\left(f_i\right)^2$. Once joined platform i, each participating seller in product category v will set the price $p_v^*\left(f_i\right)$ on platform i and transact with each buyer on that platform once, with the representative buyer consuming $q_v^*\left(f_i\right)$ units from such a seller. Notice each seller's profit $\pi_v^*\left(f_i\right)$ is positive if and only if $f_i \leq v$. Therefore, in the absence of any seller fixed costs of participation, if $f_i \leq v$, all n sellers in category v participate on platform i; if $f_i>v$, none of them participate on platform i. The measure of product categories where sellers participate on platform i is $1-G\left(f_i\right)$.

We are now ready to define the key functions U_i and R_i in (2) and (4). We have

$$U_i=\int_{f_i}^{v_{\max}} u_v^*\left(f_i\right) dG\left(v\right).$$

Here f_i affects buyer utility $u_v^*\left(f_i\right)$ through the positive pass-through in sellers' pricing, while f_i also affects how many product categories will be active, and so buyers' utility via cross-side network effects. And the platform's revenue from transaction fees is

$$R_i=f_i \int_{f_i}^{v_{\max}} n q_v^*\left(f_i\right) dG(v) s_i.$$

Note both U_i and R_i are independent of f_j (when holding s_i fixed), thus satisfying the no spillover condition, meaning the equilibrium fee characterized below corresponds to the seller-excluded outcome. As noted in Section A, this result remains true even if we allow for incomplete coverage on the buyer side given that U_i only depends on f_i and R_i is proportional to s_i, provided $m=2$ and the assumptions on (ϵ_1, ϵ_2) noted there hold.

As in the general framework, consider a deviation platform i setting $P_i^B \neq P^{B*}$ and $f_i \neq f^*$. Then, the one-to-one relation between P_i^B and s_i (for given P^{B*} and f^* by platforms $j \neq i$) means we can reframe platform i's problem as choosing s_i and f_i to maximize

$$\begin{aligned}
\Pi_i & = \left(P_i^B-c\right) s_i+R_i \\
& = (U_i-U_j-\Phi^{-1}\left(s_i\right)+P_j^B-c) s_i+R_i .
\end{aligned}$$

Each platform's optimal fee is therefore determined by

$$\begin{aligned}
f^* & = \arg \max_{f_i}\left\{U_i s_i+f_i \int_{f_i}^{v_{\max}} n q_v^*\left(f_i\right) dG(v) s_i\right\} \\
& = \arg \max_{f_i}\left\{\int_{f_i}^{v_{\max}}\left(u_v^*\left(f_i\right)+n f_i q_v^*\left(f_i\right)\right) s_i dG\left(v\right)\right\} .
\end{aligned}$$

Assuming $G\left(v\right)$ is linear on $[0, v_{\max}]$, the transaction fee in the equilibrium outcome (and seller-excluded outcome) is

$$f^*=\left(\frac{1-\rho}{3-\rho}\right) v_{\max}=\frac{(1-\theta)\, n}{3\,(1-\theta)\, n+2\,(n-\theta)} v_{\max}.$$

Note second-order conditions hold here: the first derivative of the objective above (using $q_v^*(f_i) = 0$ when $v = f_i$ and $\frac{dq_v^*(f_i)}{df_i} = -\frac{\rho}{n}$) is

$$\begin{aligned} & \int_{f_i}^{v_{\max}} n\left(nq_v^*(f_i)\frac{dq_v^*(f_i)}{df_i} + q_v^*(f_i) + f_i\frac{dq_v^*(f_i)}{df_i}\right)dG(v) \\ = & \int_{f_i}^{v_{\max}} n\left((1-\rho)\rho\left(\frac{v-f_i}{n}\right) - f_i\frac{\rho}{n}\right)dG(v), \end{aligned}$$

which is point-wise decreasing in f_i and hence the objective function is concave.

Ultimately the equilibrium fee is determined solely by the pass-through rate, with the fee decreasing as the rate of pass-through increases. Since the pass-through rate is increasing in the degree of substitution between sellers θ within each product category and the number of sellers n that compete in each product category, an increase in either of these also decreases the equilibrium fee. This also highlights it is the pass-through rate and not seller profits that drive the result. As we increase n, the equilibrium fee decreases despite the fact total seller profit in each product category increases in n.

If $\theta = 0$ and/or $n = 1$, so each seller is independent, then pass-through is at its lowest possible level ($\rho = 1/2$) and the equilibrium fee is at its highest ($f^* = \frac{1}{5}v_{\max}$). As $\theta \to 1$ for a fixed n, this converges to the case with homogenous sellers, and pass-through $\rho \to 1$, and as a result $f^* \to 0$. With the per-transaction fee fully passed through to buyers, the platforms do not benefit from inflating the fee above cost given in the end they are just competing for buyers. Finally, even as $n \to \infty$, so each individual seller's profit goes to zero, we find $f^* \to \frac{1-\theta}{5-3\theta}v_{\max}$, which remains positive for $\theta < 1$, since pass-through remains strictly less than one in this case and total seller profit in each product category does not go to zero.

We can compare the equilibrium fee to various welfare benchmarks, and calculate the associated welfare loss. Ignoring terms that don't depend on the per-transaction fee f_i, total welfare created from transactions on platform i is

$$W^T(f_i) = \int_{f_i}^{v_{\max}} \left(u_v^*(f_i) + n\pi_v^*(f_i) + nf_iq_v^*(f_i)\right)s_i dG(v).$$

Note that platform i's buyer price P_i^B cancels out as it represents a pure transfer between buyers and the platform. The integrand term $\left(u_v^*(f_i) + n\pi_v^*(f_i) + nf_iq_v^*(f_i)\right)s_i$ is just the gross surplus the representative buyer gets from transactions in product category v on platform i. This is clearly non-negative and strictly decreasing in f_i for all $v \geq f_i$. It follows that the fee that maximizes W^T must involve $f^W \leq 0$. Indeed, without any constraint on negative fees, the fee maximizing total welfare is

$$f^W = -\frac{(1-\theta)n}{(3-\theta)n - 2\theta}v_{\max} \leq 0.$$

Given our requirement that $f_i \geq 0$, the constrained efficient fee is then $f^W = 0$.[4]

There are two types of inefficiency caused by the seller-excluded outcome. First, fewer sellers join in the seller-excluded outcome, so there is efficiency loss from lost transactions from the missing sellers. Second, $f^* > f^W = 0$ results in sellers that do join setting their prices inefficiently high, decreasing the quantity demanded. The fraction of total transaction welfare lost in the equilibrium when compared to total transaction welfare obtainable at $f^W = 0$ is given by

$$\frac{W^T(f^W) - W^T(f^*)}{W^T(f^W)} = \frac{(1-\rho)^2(26 - 9\rho + \rho^2)}{(2-\rho)(3-\rho)^3}.$$

As can be seen, this welfare loss measure only depends on the pass-through rate ρ, and indeed, it is decreasing in that rate. The relative loss varies from no loss up to a loss of $\frac{29}{125}$ of the relevant welfare as the pass-through

[4]One reason negative fees may not be viable is they could induce sellers to fabricate fake transactions to generate payments from the platform.

rate ρ varies from 1 down to $\frac{1}{2}$.

Let's now consider the fee that maximizes other objectives.

1. Total user surplus: An alternative welfare benchmark that has been used in platform contexts (Rochet and Tirole, 2011) is total user surplus (total buyer and seller surpluses, ignoring the profit of the platform). Focusing only on terms that depend on f_i, this is the same as $W^T(f_i)$ above. This reflects the equilibrium buyer membership fee for platform i is one-for-one decreasing in its seller fee revenue per buyer attracted; i.e.,

$$P_i^B = c + t - f_i \int_{f_i}^{v_{\max}} n q_v^*(f_i) dG(v).$$

Thus,

$$\begin{aligned} W^{TUS}(f_i) &= \int_{f_i}^{v_{\max}} \left(u_v^*(f_i) + n\pi_v^*(f_i)\right) dG(v)\, s_i + P_i^B s_i \\ &= \int_{f_i}^{v_{\max}} \left(u_v^*(f_i) + n\pi_v^*(f_i) + n f_i q_v^*(f_i)\right) dG(v)\, s_i - (c+t)\, s_i, \end{aligned}$$

where we have ignored buyers' transport costs which do not depend on the level of f_i given the platforms are symmetric.

2. Buyer surplus. Focusing only on terms that depend on f_i, this is the same as W^{TUS} without the term for sellers' profit $n\pi_v^*(f_i)$, and so equals

$$W^B(f_i) = \int_{f_i}^{v_{\max}} \left(u_v^*(f_i) + n f_i q_v^*(f_i)\right) dG(v).$$

This is the same objective function that each platform maximizes. Thus, f^* also maximizes buyer surplus. However, in this two-sided setting, sellers are customers of the platforms too, so there is no reason not to consider their interests. Moreover, this ignores any of the sources of spillovers discussed in Section 4.1, as well as myopic buyers and different buyer monetization methods discussed in Section 5, which can distort the equilibrium fee from the level maximizing buyer surplus.

3. Platform transaction fee revenue. If each platform just maximizes transaction fee revenue, it will set f_i to maximize

$$\int_{f_i}^{v_{\max}} n f_i q_v^*(f_i)\, dG(v)$$

which implies

$$f^R = \frac{v_{\max}}{3} > f^*$$

and

$$\frac{W^T\left(f^W\right) - W^T\left(f^R\right)}{W^T\left(f^W\right)} = \frac{26 - 19\rho}{27(2-\rho)},$$

so the loss varies from $\frac{7}{27}$ to $\frac{11}{27}$ of the relevant welfare as the pass-through rate ρ varies from 1 down to $\frac{1}{2}$.

Finally, we consider the two alternative sources of deviations from the seller-excluded outcome studied in Section 5.

1. In case platforms extract revenue on the buyer side with the alternative instrument A_i that can be more efficient than lump-sum membership fees ($P' < 0$) or less efficient ($P' > 0$) as in Section 5.1, we calculate f^* using (17) and calculate f^{SE} using (18) and $\frac{dA_i}{df_i}$ from Proposition 4. This implies

$$f^* = f^{SE} = \max\left\{\frac{(P' - \rho)\, v_{\max}}{3P' - \rho}, 0\right\},$$

where f^* is increasing in P'.

2. In case buyers discount their surplus from transactions by $0 < \delta < 1$ when making their decision over which platform to join, as in Section 5.2, then using (20) we find

$$f^* = \frac{(1-\delta\rho)\, v_{\max}}{3-\delta\rho} > \frac{(1-\rho)\, v_{\max}}{3-\rho} = f^{SE},$$

so the equilibrium fee is inflated above the seller-excluded benchmark. The extent of this "inflation" increases in the degree to which buyers discount their surplus from transactions (i.e. the lower is δ).

C Details for Sections 2.2 and 3.3

□ **Application 1 (Two-part tariffs).** As stated in the main text, $\hat{W}$ is clearly decreasing in platform fees $\left(f_i, P_i^S\right)$.

□ **Application 2 (Platform investment).** Imposing symmetry and dropping constant terms, the total welfare objective function that is relevant for determining a_i^W is

$$\hat{W} = I_i(u^* + \pi^*)G\left(\bar{k}_i\right) - m\int_{k_{\min}}^{\bar{k}_i} kdG\left(k\right) - C(I_i),$$

where $\bar{k}_i \equiv (1-r_i)\frac{I_i\pi^*}{m}$. Then

$$\frac{d\hat{W}}{dr_i} = -I_i(u^* + r_i\pi^*)g\left(\bar{k}_i\right)\frac{I_i\pi^*}{m} < 0,$$

Thus, dW_i/dr_i is single-crossing in I_i. Meanwhile, $\hat{W}$ is non-monotonic in I_i, and so to establish single-crossing, we look at the cross-derivative:

$$\begin{aligned}\frac{d^2\hat{W}}{dI_i dr_i} &= -2\left(u^* + r_i\pi^*\right)g\left(\bar{k}_i\right)\frac{I_i\pi^*}{m} - I_i(u^* + r_i\pi^*)\frac{(1-r_i)\,\pi^*}{m}g'\left(\bar{k}_i\right)\frac{I_i\pi^*}{m} \\ &= -\left(2 + \bar{k}_i\frac{g'\left(\bar{k}_i\right)}{g\left(\bar{k}_i\right)}\right)\left(u^* + r_i\pi^*\right)\frac{I_i\pi^*}{m}g\left(\bar{k}_i\right),\end{aligned}$$

which is negative if elasticity of g is greater than -2, a sufficient condition for which is that G is weakly convex. Thus, dW_i/dI_i is single-crossing in r_i, and we conclude $\hat{W}$ satisfies quasi-supermodularity in $(r_i, -I_i)$.

□ **Application 3 (First-party entry and self-preferencing).** Imposing symmetry and dropping constant terms, the total welfare objective function that is relevant for determining a_i^W is

$$\hat{W} = \left(u^* + \pi^* + \alpha e_i\left(l_i\Delta^{sp} + (1-l_i)\,\Delta^{fp}\right)\right)G\left(\bar{k}_i\right) - m\int_{k_{\min}}^{\bar{k}_i} kdG\left(k\right),$$

where $\bar{k}_i = (1-r_i)(\pi^* - \alpha e_i(\pi^* - (1-l_i)\,\pi^d))\frac{1}{m}$. Observe that $\bar{k}_i$ is decreasing in r_i, e_i, and l_i.

Define $\Delta^{sp} = \pi^{sp} + u^{sp} - \pi^* - u^*$ and $\Delta^{fp} = \pi^{fp} + \pi^d + u^d - \pi^* - u^*$ as the ex-post efficiency gain from first-party entry with and without self-preferencing. Recall that we assume $\Delta^{fp} > \Delta^{sp}$. Then,

$$\frac{d\hat{W}}{dr_i} = (\underbrace{u^* + \pi^* + \alpha e_i\left(l_i\Delta^{sp} + (1-l_i)\,\Delta^{fp}\right) - m\bar{k}_i}_{>0 \text{ because } m\bar{k}_i<(1-r_i)\pi^*})g\left(\bar{k}_i\right)\frac{d\bar{k}_i}{dr_i} < 0;$$

while

$$\frac{d\hat{W}}{dl_i} = \left(u^* + \pi^* + \alpha e_i \left(l_i \Delta^{sp} + (1-l_i)\Delta^{fp}\right) - m\bar{k}_i\right) g\left(\bar{k}_i\right) \frac{d\bar{k}_i}{dl_i} + \alpha e_i \left(\Delta^{sp} - \Delta^{fp}\right) G\left(\bar{k}_i\right) < 0$$

because $\Delta^{fp} > \Delta^{sp}$; and

$$\frac{d\hat{W}}{de_i} = \left(u^* + \pi^* + \alpha e_i \left(l_i \Delta^{sp} + (1-l_i)\Delta^{fp}\right) - m\bar{k}_i\right) g\left(\bar{k}_i\right) \frac{d\bar{k}_i}{de_i} + \alpha \left(l_i \Delta^{sp} + (1-l_i)\Delta^{fp}\right) G\left(\bar{k}_i\right) < 0$$

because $\Delta^{fp} > \Delta^{sp}$ is not too large.

□ **Application 4 (Leakage prevention).** As stated in the main text, $\hat{W}$ is clearly decreasing in platform fees $(r_i\ , \lambda_i)$.

□ **Application 5 (App tracking).** As noted in the text, a typical seller on platform i chooses p_i to maximize

$$\sum_{i \in \phi} \left((1-r_i)\, p_i q(p_i)(1-H(p_i)) + \pi_a (1-\tau_i) \int_0^{p_i} q(z) dH(z) \right) s_i.$$

Assuming the seller objective function is strictly quasiconcave, then by additive separability, the optimal price p_i^* satisfies FOC

$$p_i^* = \frac{\pi_a (1-\tau_i)}{1-r_i} + \left(1 + p_i^* \frac{q'(p_i^*)}{q(p_i^*)}\right) \frac{1-H(p_i^*)}{h(p_i^*)}.$$

Observe that p_i^* is an increasing function of $\frac{1-\tau_i}{1-r_i}$ as claimed in the text. That is, sellers set a higher price for their apps (to divert buyers to watch ads) when ads becomes more profitable relative to their share of transacton revenue $1-r_i$. To check strict quasiconcavity of the seller objective function, notice $d\pi/dp_i$ has the same sign as

$$-p_i + \frac{\pi_a (1-\tau_i)}{1-r_i} + (1+e_q) \frac{1-H(p_i)}{h(p_i)}, \tag{32}$$

where $e_q \equiv p_i \frac{q'(p_i)}{q(p_i)} < 0$ is elasticity of $q(.)$. By standard results, e_q is weakly decreasing in p_i if $q(.)$ is weakly log-concave or admits constant-elasticity. Therefore, as long as $(1+e_q) > 0$ then we know $(1+e_q)\frac{1-H(p_i)}{h(p_i)}$ is decreasing in p_i by log-concavity of $1-H$, and so (32) is always decreasing in p_i, which establishes strict-quasiconcavity.

Imposing symmetry and dropping constant terms, the total welfare objective function that is relevant for determining a_i^W is

$$\hat{W} = U_0(p_i^*) G\left(\bar{k}_i\right) + r_i R_0(p_i^*) G\left(\bar{k}_i\right) + m \int_0^{\bar{k}_i} (\bar{k}_i - k_i) dG,$$

where

$$\begin{aligned} U_0(p_i^*) &= \int_0^{p_i^*} u(q(z)) - z q(z) dH(z) + \int_{p_i^*}^{\infty} u(q(p_i^*)) - p_i^* q(p_i^*) dH(z) \\ R_0(p_i^*) &= p_i^* q(p_i^*)(1-H(p_i^*)) \\ \bar{k}_i &= \frac{(1-r_i)}{m} p_i^* q(p_i^*)(1-H(p_i^*)) + \frac{\pi_a (1-\tau_i)}{m} \int_0^{p_i^*} q(z) dH(z). \end{aligned}$$

To establish quasi-supermodularity, we reframe the platform's problem as choosing $a_i = (r_i, -p_i^*)$, where

$$\tau_i = \tau(r_i, p_i^*) = 1 + \psi(p_i^*) \left(\frac{1-r_i}{\pi_a}\right)$$

and

$$\psi(p_i^*) \equiv (1+e_q)\frac{1-H(p_i^*)}{h(p_i^*)} - p_i^* < 0$$

is strictly decreasing in p_i^* by the properties on (32) as established above. Then

$$\frac{1}{G(\bar{k}_i)}\frac{d\hat{W}}{dr_i} = (U_0(p_i^*) + r_i R_0(p_i^*))\frac{g(\bar{k}_i)}{G(\bar{k}_i)}\frac{d\bar{k}_i}{dr_i} + m\frac{d\bar{k}_i}{dr_i} < 0$$

for all p_i^* because $\frac{d\bar{k}_i}{dr_i} = -\frac{1}{1-r_i}\bar{k}_i < 0$. Thus, dW_i/dr_i^* is single-crossing in p_i^*, as required. Likewise,

$$\frac{1}{G(\bar{k}_i)}\frac{d\hat{W}}{dp_i^*} = \left(\frac{dU_0}{dp_i^*} + \frac{dR_0}{dp_i^*}r_i\right) + (U_0(p_i^*) + r_i R_0(p_i^*))\,\varphi(\bar{k}_i)\frac{d\bar{k}_i/dp_i^*}{\bar{k}_i} + m\frac{d\bar{k}_i}{dp_i^*}$$

where $\varphi(x) \equiv \frac{xg(x)}{G(x)}$ is the elasticity of G with respect to its argument. If we impose constant-elasticity $G(k) = \left(\frac{k}{k_{\max}}\right)^{\varphi}$ on $[0, k_{\max}]$, and let $\varphi \to 0$, then

$$\frac{1}{G(\bar{k}_i)}\frac{d^2\hat{W}}{dp_i^* dr_i} \to \frac{dR_0}{dp_i^*} + m\frac{d^2\bar{k}_i}{dp_i^* dr_i} < 0$$

because $\frac{dR_0}{dp_i^*} < 0$ by (32), and

$$\frac{d^2\bar{k}_i}{dp_i^* dr_i} = -\frac{1}{1-r_i}\frac{d\bar{k}_i}{dp_i^*} = \frac{1}{m}\psi'(p_i^*)\int_0^{p_i^*} q(z)dH(z) < 0.$$

Thus, dW_i/dp_i^* is single-crossing in r_i, as required.

D Details for Section 4.1

We provide the additional details referred to for each of the sources of spillovers in Section 4.1.

D.1 Within-seller economies of scale

□ **Quasi-supermodularity.** Given symmetry and after dropping constant terms, we have $\bar{k}_i = (1-r_i)\, I_i \pi^*$ and

$$\begin{aligned}
\hat{W}^{SE} &= I_i\,(u^* + r_i\pi^*)\,G(\bar{k}_i) - C(I_i) \\
\frac{d\hat{W}^{SE}}{dr_i} &= I_i\pi^* G(\bar{k}_i) - I_i\,(u^* + r_i\pi^*)\,I_i\pi^* g(\bar{k}_i)\,. \\
\frac{d\hat{W}^{SE}}{dI_i} &= (u^* + r_i\pi^*)\,G(\bar{k}_i) + I_i\,(u^* + r_i\pi^*)\,(1-r_i)\,\pi^* g(\bar{k}_i) - C'(I_i)\,.
\end{aligned}$$

To establish quasi-supermodularity, we will establish pairwise single crossing in $a_i = (r_i, -I_i)$ for all $a_i \geq \min\{a^{SE}, a^*\}$. We impose constant-elasticity G:

$$G(k) = \left(\frac{k}{k_{\max}}\right)^{\varphi} \text{ on } [0, k_{\max}]\,,$$

so that $g(k) = \frac{\varphi}{k_{\max}^{\varphi}}k^{\varphi-1}$, and assume $\varphi \geq 1$. Note $\varphi = 1$ corresponds to linearity.

We first show $\frac{d\hat{W}^{SE}}{dr_i}$ is single-crossing-from-above in I_i for all I_i. Dropping the common factor $I_i^2 g(\bar{k}_i)$, it suffices to show the following is weakly decreasing in I_i:

$$\begin{aligned} & \frac{1}{I_i}\frac{G\left(\bar{k}_i\right)}{g\left(\bar{k}_i\right)} - \left(u^* + r_i\pi^*\right) \\ = \quad & \left(1-2r_i\right)\pi^* - u^*, \end{aligned}$$

which is independent (hence weakly decreasing) in I_i. Notice the analysis also means

$$r^{SE} = \max\left\{\frac{1}{2} - \frac{u^*}{2\pi^*}, 0\right\},$$

which is independent of I_i. Therefore, the negative spillover logic for the scalar case immediately implies $r^{SE} \leq r^*$. We then show $\frac{d\hat{W}^{SE}}{dI_i}$ is decreasing (hence single-crossing-from-above) in r_i for all $r_i \geq \min\{r^{SE}, r^*\}$. Using the functional form of G and simplifying,

$$\frac{d\hat{W}^{SE}}{dI_i} = \frac{2\varphi}{k_{\max}^{\varphi}} \underbrace{\left(\left(1-r_i\right)I_i\pi^*\right)^{\varphi-1}}_{\text{decreasing in } r_i \text{ given } \varphi\geq 1} \times \underbrace{\left(u^* + r_i\pi^*\right)\left(1-r_i\right)}_{\text{decreasing in } r_i \text{ for } r_i \geq r^{SE}} I_i\pi^* - C'\left(I_i\right),$$

which is decreasing for all $r_i \geq r^{SE} = \min\{r^{SE}, r^*\}$. Thus, we conclude $\hat{W}^{SE}(a_i)$ obeys quasi-supermodularity in $a_i = \{r_i, -I_i\}$ for all $a_i \geq \min\{a^{SE}, a^*\} = (\min\{r^{SE}, r^*\}, \min\{-I^{SE}, -I^*\})$.

□ **Proxied-spillover approach.** It remains to check the welfare comparison. We note that, given symmetry and after dropping constant terms, we have

$$\begin{aligned} \hat{W}^{SE}(a_i) &= I_i\left(u^* + r_i\pi^*\right)G\left(\bar{k}_i\right) - C\left(I_i\right) \\ \hat{W}(a_i) &= \hat{W}^{SE}(a_i) + \int_{k_{\min}}^{\bar{k}_i}(\bar{k}_i - k)dG\left(k\right), \end{aligned}$$

which have the same expressions as Application 2 without spillovers. Observe that seller surplus is fully summarized by $\bar{k}_i = \left(1-r_i\right)I_i\pi^*$, and so the logic of Proposition 2 implies $\left(1-r_{SE}\right)I_{SE} \geq \left(1-r_W\right)I_W$ and $\bar{k}^{SE} \leq \bar{k}^W$.

□ **Alternative specification: seller invesment.** Consider the model of Application 2 where platform i's investment I_i scales up the buyer's gross utility obtained from transacting with any seller. As an alternative source of within-seller economies of scale, suppose sellers can now choose how much to invest to raise their product quality. Assume, as seems most natural, a seller's investment in its product is a complement to each platform's investment in helping buyers transact with sellers. Thus, the gross utility of buyers is $u\left(q_i\right)I_iI_s$, where I_s is a seller's investment with the corresponding cost function $K(I_s)$. As is standard, we assume K is increasing and strictly convex, with boundary conditions $\lim_{I_s\to\infty}K'(I_s) = \infty$ and $K'(0) = 0$ so that each seller's optimal investment is unique, strictly positive, and finite. In order to show spillovers can arise absent any fixed participation cost or source of seller heterogeneity, assume there are no fixed costs for sellers to participate and all sellers (measure one in total) will therefore participate in equilibrium.

Defining the seller's quality-adjusted price $\hat{p}_i = \frac{p_i}{I_iI_s}$, each seller sets $\hat{p}_i$ to maximize $\left(1-r_i\right)I_iI_s\hat{p}_iq_i\left(\hat{p}_i\right)$. Let the resulting profit maximizing price be denoted $\hat{p}^*$, which note doesn't depend on r_i, I_i or I_s. Therefore, we know each seller's optimal investment maximizes

$$\pi = \sum\nolimits_{i=1}^{m}\left(1-r_i\right)I_is_i\pi^*I_s - K(I_s),$$

where $\pi^* = \hat{p}^* q\left(\hat{p}^*\right)$. A seller's optimal investment is I_s^* satisfying the first-order condition

$$\sum\nolimits_{i=1}^{m} \left(1 - r_i\right) I_i s_i \pi^* = K'(I_s^*)$$

and is increasing in $\left(1 - r_i\right) I_i$ on each platform i.

We have

$$\begin{aligned} U_i &= I_i I_s^* u^* \\ R_i &= r_i I_i I_s^* \pi^* s_i - C\left(I_i\right). \end{aligned}$$

Again, U_i and R_i are increasing in I_j and decreasing in r_j because both a higher platform investment and a lower commisson result in a higher I_s chosen by the seller. Moreover, since K' is assumed strictly increasing, we can take the inverse of it, which is an increasing function, that plays the same role of G in Application 2. If this inverse function is denoted $(K')^{-1}$, then

$$I_s^* = (K')^{-1}\left(\sum\nolimits_{i=1}^{m} \left(1 - r_i\right) I_i s_i \pi^*\right),$$

and replacing $G\left(\overline{k}\right)$ with $(K')^{-1}\left(\overline{k}\right)$ in the existing Application 2, yields essentially the same specification here. Thus, from Proposition 3, we still have that $r^* > r^{SE}$ and $I^* \leq I^{SE}$ in case we consider each choice holding the other fixed, and in the multidimensional case, we have $I_s^* \leq I_s^{SE}$.

D.2 Price coherence

☐ **Quasi-supermodularity.** The condition trivially holds because $a_i = f_i$ is one-dimensional.

☐ **Verify equilibrium construction.** We verify that all sellers will multihome on all platforms as long as the fee difference $\max_{j \neq i} |f_i - f_j|$ is not too large, and that the platforms have no incentive to deviate and induce large fee differences if β is small enough. Given the symmetry assumption, it suffices to focus on the case where platform i sets $f_i \neq f^*$ while all other platforms $j \neq i$ set $f_j = f^*$.

We can analyze an individual seller's decision on whether to multihome. Consider first $f_i \leq f^*$. Clearly, all sellers who are not subjected to price coherence would prefer to multihome. For the sellers subjected to price coherence, multihoming is better than singlehoming on the higher fee platforms (platform $j \neq i$) because $\pi^*(f^{avg}) > \pi^*(f^*)(1 - s_i)$ and $\pi^*(.)$ is a decreasing function. Meanwhile, multihoming is better than singlehoming on the lower fee platform (platform i) if and only if

$$\pi^*(f^{avg}) > \pi^*(f_i)s_i,$$

which holds if and only if the fee difference $f^* - f_i$ is small enough. We verify ex-post that platforms have no incentive to set such fees when β is sufficiently small, and so all sellers multihome in the equilibrium, with a fraction β of them subjected to price coherence.

We first pin down the equilibrium transaction fee f^* which, assuming all sellers multihoming on all platforms, satisfies the FOC:

$$\begin{aligned} &\left(\frac{\partial U_i}{\partial f_i} - \frac{\partial U_{-i}}{\partial f_i}\right)\frac{1}{m} + \frac{\partial R_i}{\partial f_i} = 0 \\ \iff \quad &\frac{(1-\beta)u^*(f^*)'}{m} + q^*(f^*) + f^*\left(1 - \frac{\beta}{m}\right)\frac{dq^*(f^*)}{df} = 0, \end{aligned}$$

which is increasing in β.

Suppose platform i wants to deviate by choosing $(f_i, P_i^B) \neq (f^*, P^{B*})$ to induce some sellers to singlehome. Recall this necessarily requires $f_i < f^*$. Note this is applicable only to the mass β of sellers that are subjected to price coherence. A successful deviation requires

$$\pi^*(s_i f_i + (1-s_i)f^*) < \pi^*(f_i)s_i.$$

Let us denote the maximum deviation fee as f^{dev}, which we know is *strictly* below f^* as long as $s_i < 1$ (i.e., buyer-side heterogeneity is not too small), for all $\beta \geq 0$.

With this undercutting strategy, buyers expect utility difference

$$U_i - U_{-i} = u^*(f^{dev}) - (1-\beta)u^*(f^*) + P_i^B - P^{B*}$$

and the deviation platform profit is

$$\Pi^{dev} = \max_{P_i^B; f_i \leq f^{dev}} (P_i^B - c + f^{dev} q^*(f^{dev}))\Phi(u^*(f^{dev}) - (1-\beta)u^*(f^*) + P_i^B - P^{B*}).$$

Observe that the equilibrium platform profit can be expressed as

$$\begin{aligned} \Pi^* &= (P^{B*} - c + f^* q^*(f^*))\frac{1}{m} \\ &= \max_{P_i^B; f_i} (P_i^B - c + f_i (\beta q^*(f^{avg}) + (1-\beta)q^*(f_i)))\Phi((1-\beta)(u^*(f_i) - u^*(f^*)) + P_i^B - P^{B*}). \quad (33) \end{aligned}$$

Therefore, if $\beta \to 0$, then the two objective functions coincide. Assuming that the objective function in (33) is strictly quasiconcave, the constraint of $f^{dev} < f^*$ implies $\Pi^{dev} < \Pi^*$.

D.3 Building a direct channel

□ **Quasi-supermodularity.** Given symmetry and after dropping constant terms, we have

$$\hat{W}^{SE} = r_i (1 - (1-\lambda_i)H(\bar{\kappa}))\pi^*,$$

where $\bar{\kappa} = (1-\lambda_i)r_i\pi^*$. Then

$$\begin{aligned} \frac{d\hat{W}^{SE}}{dr_i} &= (1 - (1-\lambda_i)H(\bar{\kappa}))\pi^* - r_i ((1-\lambda_i)\pi^*)^2 h(\bar{\kappa}) \\ \frac{d\hat{W}^{SE}}{d\lambda_i} &= r_i H(\bar{\kappa})\pi^* + (r_i\pi^*)^2 (1-\lambda_i)h(\bar{\kappa}) \geq 0. \end{aligned}$$

To establish quasi-supermodularity, we will establish pairwise single crossing in $a_i = (r_i, \lambda_i)$.

We first show $\frac{d\hat{W}^{SE}}{dr_i}$ is single-crossing-from-below in λ_i for all λ_i. Dropping the common factor $\pi^* h(\bar{\kappa})$, it suffices to show the following is weakly increasing in λ_i:

$$\begin{aligned} & \frac{1 - (1-\lambda_i)H(\bar{\kappa})}{\pi^* h(\bar{\kappa})} - r_i(1-\lambda_i)^2 \\ = & \frac{\kappa_{\max}}{\varphi\pi^*\bar{\kappa}} - \left(\frac{1-\lambda_i}{\pi^*}\right)\bar{\kappa} - r_i(1-\lambda_i)^2, \end{aligned}$$

which is indeed increasing in λ_i if h is weakly increasing (i.e., weakly convex H). Thus, we conclude $\hat{W}^{SE}(a_i)$ obeys quasi-supermodularity in $a_i = \{r_i, \lambda_i\}$.

□ **Proxied-spillover approach.** It remains to show $\bar{\kappa}^{SE} \leq \bar{\kappa}^W$. Recall that

$$\hat{SS}(r_i, \lambda_i) = \int_0^{\kappa_{\max}} \max\left\{\left(1 - \sum_{i=1}^m \lambda_i r_i s_i\right)\pi^* - \bar{\kappa}, \left(1 - \sum_{i=1}^m r_i s_i\right)\pi^*\right\} dH(\kappa),$$

which can be written as

$$\hat{SS}(r_i, \bar{\kappa}) = \int_0^{\kappa_{\max}} (1 - mr_i s_i)\,\pi^* + \max\{\bar{\kappa} - \kappa, 0\}\, dH(\kappa),$$

which is decreasing in r_i and increasing in $\bar{\kappa}$, and so decreasing in vector $(r_i, -\bar{\kappa})$. By reframing $a_i = (r_i, -\bar{\kappa})$ in the seller-excluded welfare and total welfare maximizations, Proposition 2 implies $a^{SE} \geq a^W$, and so $\bar{\kappa}^{SE} \leq \bar{\kappa}^W$.

E Details for Section 5.1

We want to evaluate (19) for each of our applications. We first prove the following technical claims that repeatedly used in most of the applications below.

Lemma 1 *Suppose*

$$\frac{\partial^2 U_i}{\partial s_i \partial a_i} = \frac{1}{s_i}\frac{\partial U_i}{\partial a_i} \quad \text{and} \quad \frac{\partial^2 R_i}{\partial s_i \partial a_i} = \frac{2}{s_i}\frac{\partial R_i}{\partial a_i}$$

for any a_i. Then, $dA_i^*/da_i = 0$ when evaluated at the symmetric equilibrium $a_i = a^*$. Therefore, (19) holds (that is, $a^* \geq a^{SE}$) if and only if

$$(1 - P')\frac{\partial U_i(a^*; \mathbf{1}/m)}{\partial a_i} \geq 0.$$

Proof. (Lemma 1). For given a_i, recall from the definition in (26) that

$$\frac{dA_i^*}{da_i} = -\frac{1}{P'}\frac{\partial^2 U_i}{\partial s_i \partial a_i} - \frac{\partial^2 R_i}{\partial s_i \partial a_i}.$$

By the supposition,

$$\frac{dA_i^*}{da_i} = -\frac{2}{s_i}\left(\frac{1}{2P'}\frac{\partial U_i}{\partial a_i} + \frac{\partial R_i}{\partial a_i}\right)$$

which equals to zero when evaluated at $a_i = a^*$ and $\boldsymbol{s} = \mathbf{1}/m$ due to the FOC definition of a^*. Then, applying Proposition 4, (19) holds if and only if

$$(1 - P')\frac{\partial U_i(a^*; \mathbf{1}/m)}{\partial a_i} \geq 0.$$

■

Lemma 2 *Suppose*

$$\frac{\partial^2 U_i}{\partial s_i \partial a_i} \to 0 \quad \text{and} \quad \frac{\partial^2 R_i}{\partial s_i \partial a_i} \to \frac{1}{s_i}\frac{\partial R_i}{\partial a_i}$$

for any a_i. Then (19) holds in equality in the limit (that is, $a^* \to a^{SE}$).

Proof. (Lemma 2). By the supposition,

$$\frac{dA_i^*}{da_i} \equiv -\frac{1}{P'}\frac{\partial^2 U_i}{\partial s_i \partial a_i} - \frac{\partial^2 R_i}{\partial s_i \partial a_i} \to -\frac{1}{s_i}\frac{\partial R_i}{\partial a_i}$$

which equals to $\frac{1}{P'}\frac{\partial U_i}{\partial a_i}$ when evaluated at $a_i = a^*$ and $\boldsymbol{s} = \mathbf{1}/m$ due to the FOC definition of a^*. Then, (19) becomes

$$(1 - P')\left[\frac{dA_i^*}{da_i} - \frac{1}{P'}\frac{\partial U_i(a^*; \mathbf{1}/m)}{\partial a_i}\right] \to 0.$$

■

In what follows, we assume throughout

$$G(k) = \left(\frac{k}{k_{\max}}\right)^{\varphi} \quad \text{on } [0, k_{\max}], \tag{34}$$

where $\varphi > 0$ is constant elasticity of distribution G with respect to its argument, i.e., $\varphi = \frac{kg(k)}{G(k)}$. We normalize $k_{\max} = 1$ to avoid carrying additional notation which does not change any results. For each application below, we will focus on $\varphi = 1$ (linear G) and $\varphi \to 0$ (sufficiently inelastic G).

□ **Application 1.** Consider $\varphi = 1$, so

$$\begin{aligned} U_i &= u^*(f_i)\left(\pi^*(f_i)s_i - P_i^S\right) \\ R_i &= \left(f_i q^*(f_i)s_i + P_i^S\right)\left(\pi^*(f_i)s_i - P_i^S\right). \end{aligned}$$

For instrument $a_i = P_i^S$ we have $\frac{\partial^2 U_i}{\partial s_i \partial P_i^S} = 0$, and so $\frac{dA_i^*}{dP_i^S} = -\frac{\partial^2 R_i}{\partial s_i \partial P_i^S}$. Then,

$$\begin{aligned} \left[\frac{dA_i^*}{dP_i^S} - \frac{1}{P'}\frac{\partial U_i}{\partial P_i^S}\right]_{P_i^s = P_i^{s*}, s_i = 1/m} &= -\frac{\partial^2 R_i}{\partial s_i \partial P_i^S} + 2\frac{\partial R_i}{\partial P_i^s} \\ &= -4P_i^S \leq 0, \end{aligned}$$

where the first equality uses the FOC definition of P_i^* ($\frac{1}{m}\frac{\partial U_i}{\partial P_i^s} + \frac{\partial R_i}{\partial P_i^s} = 0$) and the second equality uses symmetry and

$$\frac{\partial R_i}{\partial P_i^S} = \pi^*(f_i)s_i - f_i q^*(f_i)s_i - 2P_i^S \quad \text{and} \quad \frac{\partial^2 R_i}{\partial s_i \partial P_i^S} = \pi^*(f_i) - f_i q^*(f_i).$$

Therefore, (19) holds if and only if

$$0 \geq (1 - P')\left[\frac{dA_i^*}{dP_i^S} - \frac{1}{P'}\frac{\partial U_i}{\partial P_i^S}\right] \iff P' < 1.$$

This implies $P_i^{S*} \geq \left(P_i^S\right)^{SE}$ if $P' < 1$ and $P_i^{S*} \leq \left(P_i^S\right)^{SE}$ if $P' > 1$. Notice this is opposite to most other cases considered below— whereby (19) holds if and only if $P' > 1$.

For instrument $a_i = f_i$, we focus on the case of $P_i^S = 0$ since otherwise (19) remains ambiguous even if $P' > 1$ or $P' < 1$. Then $U_i = u^*(f_i)\pi^*(f_i)s_i$ and $R_i = f_i q^*(f_i)\pi^*(f_i)s_i^2$. The condition for Lemma 1 clearly holds, and so the lemma implies (19) holds for $a_i = f_i$ if and only if $P' > 1$ given $\partial U_i/\partial f_i < 0$. This implies $f_i^* \geq f_i^{SE}$ if $P' > 1$ and $f_i^* \leq f_i^{SE}$ if $P' < 1$.

Consider $\varphi > 0$. For instrument $a_i = P_i^S$, we have

$$\begin{aligned} \frac{\partial U_i}{\partial P_i^S} &= \varphi u^*(f_i)\frac{\partial \bar{k}_i/\partial P_i^S}{\bar{k}_i}G(\bar{k}_i) \\ \frac{\partial^2 U_i}{\partial s_i \partial P_i^S} &= \varphi \frac{\partial}{\partial s_i}\left[u^*(f_i)\frac{\partial \bar{k}_i/\partial P_i^S}{\bar{k}_i}G(\bar{k}_i)\right] \\ \frac{\partial^2 R_i}{\partial s_i \partial P_i^S} &= \varphi \frac{\partial}{\partial P_i^S}\left[\left(f_i q^*(f_i)s_i + P_i^S\right)\frac{\partial \bar{k}_i/\partial s_i}{\bar{k}_i}G(\bar{k}_i)\right] - \varphi f_i q^*(f_i)\frac{\partial \bar{k}_i/\partial P_i^S}{\bar{k}_i}G(\bar{k}_i). \end{aligned}$$

If $\varphi \to 0$, then

$$-\frac{1}{P'}\frac{\partial^2 U_i}{\partial s_i \partial P_i^S} - \frac{\partial^2 R_i}{\partial s_i \partial P_i^S} - \frac{1}{P'}\frac{\partial U_i}{\partial P_i^S} \to 0,$$

and so (19) holds in the limit, implying $P_i^{S*} \to \left(P_i^S\right)^{SE}$. For instrument $a_i = f_i$ (and again assuming

$P_i^S = 0$), we have

$$\begin{aligned}\frac{\partial^2 U_i}{\partial s_i \partial f_i} &= \varphi \frac{\partial}{\partial f_i}\left[u^*(f_i)\frac{\partial \bar{k}_i/\partial s_i}{\bar{k}_i}G(\bar{k}_i)\right] \\ \frac{\partial^2 R_i}{\partial s_i \partial f_i} &= \varphi \frac{\partial}{\partial f_i}\left[f_i q^*(f_i)\frac{\partial \bar{k}_i/\partial s_i}{\bar{k}_i}G(\bar{k}_i)\right] + \frac{1}{s_i}\frac{\partial R_i}{\partial f_i}.\end{aligned}$$

If $\varphi \to 0$, the condition for Lemma 2 holds, implying $f^* \to f^{SE}$.

□ **Application 2.** Consider $\varphi = 1$, so

$$\begin{aligned}U_i &= (1-r_i) I_i^2 u^* \pi^* s_i \\ R_i &= r_i(1-r_i)(I_i \pi^* s_i)^2 - C(I_i).\end{aligned}$$

For instrument $a_i = r_i$, the condition for Lemma 1 clearly holds, and so the lemma implies (19) holds for $a_i = r_i$ if and only if $P' > 1$ given $\partial U_i/\partial r_i < 0$. This implies $r_i^* \geq r_i^{SE}$ if $P' > 1$ and $r_i^* \leq r_i^{SE}$ if $P' < 1$.

Next, consider the choice of $a_i = -I_i$ (we proceed in terms of I_i, but note the sign of the result will take the opposite interpretation of in Proposition 4). Then,

$$\begin{aligned}\frac{\partial^2 U_i}{\partial s_i \partial I_i} &= \frac{1}{s_i}\frac{\partial U_i}{\partial I_i} \\ \frac{\partial^2 R_i}{\partial s_i \partial I_i} &= \frac{2}{s_i}\left(\frac{\partial R_i}{\partial I_i} + C'(I_i)\right) > \frac{2}{s_i}\frac{\partial R_i}{\partial I_i}\end{aligned}$$

meaning

$$\begin{aligned}\frac{dA_i^*}{dI_i}|_{I_i = I_i^*} &= -\frac{1}{P'}\frac{\partial^2 U_i}{\partial s_i \partial I_i} - \frac{\partial^2 R_i}{\partial s_i \partial I_i} \\ &< -\frac{1}{P'}\frac{1}{s_i}\frac{\partial U_i}{\partial I_i} - \frac{2}{s_i}\frac{\partial R_i}{\partial I_i} = 0,\end{aligned}$$

where the last equality uses the FOC definition of the equilibrium instrument I_i^*. Given $\frac{\partial U_i}{\partial I_i} > 0$, we conclude

$$\left[\frac{dA_i^*}{dI_i} - \frac{1}{P'}\frac{\partial U_i}{\partial I_i}\right]_{I_i = I^*, s_i = 1/m} < 0,$$

meaning (19) holds for $a_i^* = I_i$ if and only if $P' < 1$. That is, $I_i^* \geq I_i^{SE}$ if $P' < 1$ and $I_i^* \leq I_i^{SE}$ if $P' > 1$.

Consider $\varphi > 0$. For instrument $a_i = I_i$, we have

$$\begin{aligned}\frac{\partial^2 U_i}{\partial s_i \partial I_i} &= \varphi \frac{\partial}{\partial I_i}\left[I_i u^* \frac{\partial \bar{k}_i/\partial s_i}{\bar{k}_i}G(\bar{k}_i)\right] \\ \frac{\partial^2 R_i}{\partial s_i \partial I_i} &= \varphi \frac{\partial}{\partial I_i}\left[r_i I_i \pi^* \frac{\partial \bar{k}_i/\partial s_i}{\bar{k}_i}G(\bar{k}_i)s_i\right] + \underbrace{\frac{\partial}{\partial I_i}\left(r_i I_i \pi^* G\left(\bar{k}_i\right)\right)}_{>0}.\end{aligned}$$

If $\varphi \to 0$, then

$$-\frac{1}{P'}\frac{\partial^2 U_i}{\partial s_i \partial I_i} - \frac{\partial^2 R_i}{\partial s_i \partial I_i} - \frac{1}{P'}\frac{\partial U_i}{\partial I_i} \to -\frac{\partial}{\partial I_i}\left(r_i I_i \pi^* G\left(\bar{k}_i\right)\right) - \frac{1}{P'}\frac{\partial U_i}{\partial I_i} < 0,$$

meaning (19) holds for this instrument in the limit if and only if $P' < 1$, which is the same condition as the case with $\varphi = 1$.

For instrument $a_i = r_i$, we have

$$\begin{aligned}
\frac{\partial^2 U_i}{\partial s_i \partial r_i} &= \varphi \frac{\partial}{\partial r_i}\left[I_i u^* \frac{\partial \bar{k}_i / \partial s_i}{\bar{k}_i} \right] \\
\frac{\partial^2 R_i}{\partial s_i \partial r_i} &= \varphi \frac{\partial}{\partial r_i}\left[r_i I_i \pi^* \frac{\partial \bar{k}_i / \partial s_i}{\bar{k}_i} s_i \right] + \frac{1}{s_i}\frac{\partial R_i}{\partial r_i}.
\end{aligned}$$

If $\varphi \to 0$, the condition for Lemma 2 holds in the limit, implying $r^* \to r^{SE}$.

□ **Application 3.** In what follows, we treat the platform first-party entry variable as if it is continuous. Consider $\varphi = 1$, so

$$\begin{aligned}
U_i &= (u^* + \alpha e_i (l_i u^{sp} + (1 - l_i) u^d - u^*))\bar{k}_i \\
R_i &= (r_i \pi^* + \alpha e_i (l_i \pi^{sp} + (1 - l_i) \left(r_i \pi^d + \pi^{fp} \right) - r_i \pi^*))\bar{k}_i s_i,
\end{aligned}$$

where $\bar{k}_i \equiv (1 - r_i)(\pi^* - \alpha e_i (\pi^* - (1 - l_i) \pi^d)) s_i$. For each instrument $a_i \in \{r_i, e_i, l_i\}$, the condition for Lemma 1 clearly holds, and so the lemma applies. Moreover, clearly $\partial U_i / \partial r_i < 0$ and $\partial U_i / \partial l_i < 0$. Meanwhile $\frac{\partial U_i}{\partial e_i} \leq 0$ if and only if

$$\begin{aligned}
& (l_i u^{sp} + (1 - l_i) u^d - u^*))(1 - r_i)(\pi^* - \alpha e_i (\pi^* - (1 - l_i) \pi^d)) s_i \\
\leq \; & (u^* + \alpha e_i (l_i u^{sp} + (1 - l_i) u^d - u^*))(1 - r_i)(\pi^* - (1 - l_i) \pi^d) s_i.
\end{aligned}$$

If $l_i u^{sp} + (1 - l_i) u^d \leq u^*$, then this is clearly true. So suppose $l_i u^{sp} + (1 - l_i) u^d > u^*$. A sufficient condition for this to hold is that it holds when $l_i = 0$, and so this holds for all $e_i \in [0, 1]$ if

$$\frac{u^d - u^*}{u^*} \leq \frac{\pi^* - \pi^d}{\pi^*}, \tag{35}$$

i.e., ignoring any self-preferencing, the percentage per-buyer gain in utility from entry is weakly lower than the percentage loss in seller profit. Then (19) holds for $a_i \in \{r_i, l_i\}$ if and only if $P' > 1$, and provided (35) also holds, likewise for $a_i = e_i$. Thus, with this extra condition, each of $a_i \in \{r_i, e_i, l_i\}$ satisfies $a_i^* \geq a_i^{SE}$ if $P' > 1$ and $a_i^* \leq a_i^{SE}$ if $P' < 1$.

Consider $\varphi > 0$. For each instrument $a_i \in \{r_i, e_i, l_i\}$, we have

$$\begin{aligned}
\frac{\partial^2 U_i}{\partial s_i \partial a_i} &= \varphi \frac{\partial}{\partial a_i}\left[(u^* + \alpha e_i (l_i u^{sp} + (1 - l_i) u^d - u^*)) \frac{\partial \bar{k}_i / \partial s_i}{\bar{k}_i} G(\bar{k}_i) \right] \\
\frac{\partial^2 R_i}{\partial s_i \partial a_i} &= \varphi \frac{\partial}{\partial a_i}\left[(r_i \pi^* + \alpha e_i (l_i \pi^{sp} + (1 - l_i) \left(r_i \pi^d + \pi^{fp} \right) - r_i \pi^*)) \frac{\partial \bar{k}_i / \partial s_i}{\bar{k}_i} G(\bar{k}_i) s_i \right] + \frac{1}{s_i}\frac{\partial R_i}{\partial a_i}.
\end{aligned}$$

If $\varphi \to 0$, the condition for Lemma 2 holds in the limit, implying $r^* \to r^{SE}$, $e^* \to e^{SE}$, and $l^* \to l^{SE}$.

□ **Application 4.** Consider $\varphi = 1$, so

$$\begin{aligned}
U_i &= (\beta (1 - \lambda_i r_i) + (1 - \beta)(1 - r_i)) u^* \pi^* s_i \\
R_i &= r_i (\beta \lambda_i (1 - \lambda_i r_i) + (1 - \beta)(1 - r_i)) (\pi^* s_i)^2.
\end{aligned}$$

For each instrument $a_i \in \{r_i, \lambda_i\}$, the condition for Lemma 1 clearly holds, and so the lemma applies. Given $\partial U_i / \partial r_i < 0$ and $\partial U_i / \partial \lambda_i < 0$, (19) holds for $a_i \in \{r_i, \lambda_i\}$ if and only if $P' > 1$, thus implying $a_i^* \geq a_i^{SE}$ if $P' > 1$ and $a_i^* \leq a_i^{SE}$ if $P' < 1$.

Consider $\varphi > 0$. For each instrument $a_i \in \{r_i, \lambda_i\}$, we have

$$\begin{aligned} \frac{\partial^2 U_i}{\partial s_i \partial a_i} &= \varphi \frac{\partial}{\partial a_i} \left[\beta u^* \frac{\partial \bar{k}_i / \partial s_i}{\bar{k}_i} G(\bar{k}_i) + (1-\beta) u^* \frac{\partial \bar{k}_i^n / \partial s_i}{\bar{k}_i} G(\bar{k}_i^n) \right] \\ \frac{\partial^2 R_i}{\partial s_i \partial a_i} &= \varphi \frac{\partial}{\partial a_i} \left[\beta r_i \lambda_i \pi^* \frac{\partial \bar{k}_i / \partial s_i}{\bar{k}_i} G(\bar{k}_i) s_i + (1-\beta) \pi^* \frac{\partial \bar{k}_i^n / \partial s_i}{\bar{k}_i} G(\bar{k}_i^n) s_i \right] + \frac{1}{s_i} \frac{\partial R_i}{\partial a_i}. \end{aligned}$$

If $\varphi \to 0$, the condition for Lemma 2 holds in the limit, implying $r^* \to r^{SE}$ and $\lambda^* \to \lambda^{SE}$.

□ **Application 5.** Consider $\varphi = 1$,

$$\begin{aligned} U_i &= \left(\int_0^\infty u(q(\min(p_i^*, z)) - \min(p_i^*, z) q(\min(p_i^*, z)) dH(z) \right) \bar{k}_i \\ R_i &= r_i p_i^* q(p_i^*)(1 - H(p_i^*)) \bar{k}_i s_i \end{aligned}$$

where

$$\bar{k}_i = \left((1 - r_i)\, p_i^* q(p_i^*)(1 - H(p_i^*)) + \pi_a \,(1 - \tau_i) \int_0^{p_i^*} q(z) dH(z) \right) s_i.$$

For each instrument $a_i \in \{r_i, \tau_i\}$, the condition for Lemma 1 clearly holds, and so the lemma applies. Then, $\partial U_i / \partial r_i < 0$ is obvious because

$$p_i^* = \frac{1 - \tau_i}{1 - r_i} + \frac{1 - H(p_i^*)}{h(p_i^*)} + p_i^* \frac{q'(p_i^*)}{q(p_i^*)} \frac{1 - H(p_i^*)}{h(p_i^*)}$$

is increasing in r_i while $\bar{k}_i$ is decreasing in r_i. Therefore, (19) holds for $a_i = r_i$ if and only if $P' > 1$, implying $r_i^* \geq r_i^{SE}$ if $P' > 1$ and $r_i^* \leq r_i^{SE}$ if $P' < 1$. Meanwhile, the sign of $\partial U_i / \partial \tau_i$ is not obvious because both p_i^* and k_i are decreasing in τ_i. That is, a more stringent tracking policies increases per-seller surplus of buyers but reduces seller participation.

Consider $\varphi > 0$. For each instrument $a_i \in \{r_i, \tau_i\}$, we have

$$\begin{aligned} \frac{\partial^2 U_i}{\partial s_i \partial a_i} &= \varphi \frac{\partial}{\partial a_i} \left[\left(\int_0^\infty u(q(\min(p_i^*, z)) - \min(p_i^*, z) q(\min(p_i^*, z)) dH(z) \right) \frac{\partial \bar{k}_i / \partial s_i}{\bar{k}_i} G(\bar{k}_i) \right] \\ \frac{\partial^2 R_i}{\partial s_i \partial a_i} &= \varphi \frac{\partial}{\partial a_i} \left[r_i p_i^* q(p_i^*)(1 - H(p_i^*)) \frac{\partial \bar{k}_i / \partial s_i}{\bar{k}_i} G(\bar{k}_i) s_i \right] + \frac{1}{s_i} \frac{\partial R_i}{\partial a_i} \end{aligned}$$

If $\varphi \to 0$, the condition for Lemma 2 holds in the limit, implying $r^* \to r^{SE}$ and $\tau^* \to \tau^{SE}$.

F Total user surplus

We want to compare a^* with the level of a that maximizes total user surplus (TUS) for each of our applications, where platforms remain free to set their profit-maximizing buyer-side price P_i^B. For these applications, the conditions of Proposition 2 apply, as detailed in Sections 2.2 and 3.3. Similar to Section 5.1, we focus on the case where platform instrument a_i is a scalar. Focusing on the choice of any particular scalar instrument we have that $a^* \equiv a^{SE} \geq a^W$. Note that in the symmetric outcome

$$\hat{W}^{TUS}(a_i) = \underbrace{U_i((a_i, ..., a_i); \mathbf{1}/m) - P^{B*}}_{\text{buyer surplus}} + \underbrace{SS((a_i, ..., a_i))}_{\text{seller surplus}}. \tag{36}$$

We want to show the conditions under which

$$\frac{d\hat{W}^{TUS}(a_i)}{da_i}|_{a_i = a^*} \leq 0, \tag{37}$$

which implies $a^* \geq a^{TUS}$ provided $\hat{W}^{TUS}$ is single-peaked.

Recall we have defined a_i in a way such that $SS((a_i, ..., a_i))$ is weakly decreasing in a_i, so from (36) it remains to show that buyer surplus is also weakly decreasing in a. For this, we can make use of the analysis in Section 5.1 since P_i^B is the same as A_i when $P' = 1$. We divide the results into three sets of cases that we can sign:

1. Suppose we assume G is linear (i.e. $\varphi = 1$ in (34)) and the conditions in Lemma 1 hold (in Section E). From the analysis in Section 5.1, we know $dP_i^B/da_i = 0$ when evaluated at the symmetric equilibrium outcome. Thus, buyer surplus is weakly decreasing in a at the symmetric equilibrium outcome iff $\frac{\partial U_i}{\partial a_i} \leq 0$ at that outcome. Using the analysis in Section 5.1, we have the conditions in Lemma 1 hold and $\frac{\partial U_i}{\partial a_i} \leq 0$ at the symmetric equilibrium outcome, and thus $a^* \geq a^{TUS}$ for each of the following cases:

   (a) Application 1 with $a_i = f_i$ and assuming $P_i^S = 0$.

   (b) Application 2 with $a_i = r_i$.

   (c) Application 3 for each instrument $a_i \in \{r_i, l_i\}$ and likewise for $a_i = e_i$ provided (35) also holds.

   (d) Application 4 for each instrument $a_i \in \{r_i, \lambda_i\}$.

   (e) Application 5 with $a_i = r_i$.

2. Suppose we assume sufficiently inelastic seller participation (i.e. $\varphi \to 0$ in (34)) and the conditions in Lemma 2 (in Section E) hold, so

$$\frac{\partial^2 U_i}{\partial s_i \partial a_i} \to 0 \quad \text{and} \quad \frac{\partial^2 R_i}{\partial s_i \partial a_i} \to \frac{1}{s_i} \frac{\partial R_i}{\partial a_i}$$

   for any a_i. Then from Lemma 2, we get that

$$\frac{dP_i^B}{da_i} \to -\frac{1}{s_i} \frac{\partial R_i}{\partial a_i}. \tag{38}$$

   Now $\hat{W} = \hat{W}^{TUS} + \Pi$, where Π is the platforms' joint profit in the symmetric outcome with every platform setting instrument a_i. Ignoring constant terms, this equals

$$\Pi = P^{B*} + mR_i.$$

   Evaluated at the symmetric equilibrium outcome, this implies

$$\frac{d\Pi(a_i)}{da_i} = \frac{\partial P_i^B}{\partial a_i} + m \frac{\partial R_i}{\partial a_i} \to 0$$

   given (38). This means $\frac{d\hat{W}^{TUS}}{da_i} \to \frac{d\hat{W}}{da_i}$. Assuming the objective functions are singled peaked over the relevant range, from the analysis in Section 5.1 it follows that $a^{TUS} \to a^W \leq a^*$ in the following cases:

   (a) Application 1 with $a_i = f_i$ and assuming $P_i^S = 0$.

   (b) Application 2 with $a_i = r_i$.

   (c) Application 3 for each instrument $a_i \in \{r_i, l_i, e_i\}$.

   (d) Application 4 for each instrument $a_i \in \{r_i, \lambda_i\}$.

   (e) Application 5 for each instrument $a_i \in \{r_i, \tau_i\}$.

3. Consider Application 2 with $a_i = -I_i$ and linear G (i.e. $\varphi = 1$). For this case, we need to calculate things directly. Specifically, we have

$$\begin{aligned}\hat{W}^{TUS}(a_i) &= \underbrace{\left(I_i u^* G\left(\bar{k}_i\right) - P_i^B\right)}_{\text{buyer surplus}} + \underbrace{\left(\left(1-r_i\right) I_i \pi^*\right) G\left(\bar{k}_i\right) - m\int_0^{\bar{k}_i} kdG\left(k\right)}_{\text{seller surplus}} \\ &= \frac{m\left(1-r_i\right) I_i^2 \pi^*\left(\pi^* + 6u^* + 3\pi^* r_i\right)}{4k_{\max}},\end{aligned}$$

where we have used that

$$P_i^B = c + \frac{1}{m\Phi'\left(0\right)} - \frac{\left(1-r_i\right) I_i^2 \pi^*\left(u^* + r_i \pi^*\right)}{k_{\max}}$$

from (25) with $A_i = P_i^B$ and $P' = 1$. Clearly, TUS is maximized for infinitely high I_i, so that $I^{TUS} > I^*$, while for completeness we note in this case: $r^{TUS} = \frac{1}{3} - \frac{u^*}{\pi^*} \leq \frac{1}{2} - \frac{u^*}{2\pi^*} = r^*$. That $\hat{W}^{TUS}$ is strictly increasing in I_i reflects that investment directly increase buyer-side utility and seller-side profit but users do not incur the associated fixed costs of the platforms' investments.

G Effect of number of platforms on commissions

Consider Application 2. We wish to explore how $r_i^* - r_i^W$ changes with the number of platforms m, both in the case without spillovers, and when we add within-seller economies of scale spillovers as in Section 4.1.

G.1 Case without spillovers

Recall from Application 2 we have $\bar{k}_i \equiv \left(1-r_i\right) I_i \pi^* s_i$,

$$\begin{aligned}U_i &= I_i u^* G\left(\bar{k}_i\right) \\ R_i &= r_i I_i \pi^* s_i G\left(\bar{k}_i\right) - C\left(I_i\right)\end{aligned}$$

and the total welfare at the symmetric point is:

$$\hat{W} = -c + I_i(u^* + \pi^*) G\left(\bar{k}_i\right) - m\int_{k_{\min}}^{\bar{k}_i} kdG\left(k\right) - mC(I_i).$$

The first thing to note is that

$$\frac{\partial \hat{W}}{\partial r_i} = -(u^* + r_i \pi^*)\frac{1}{m} I_i^2 \pi^* g\left(\bar{k}_i\right) < 0$$

so that given the constraint that $r_i \geq 0$, we always get $r_i^W = 0$.

We compare this to r_i^* which solves the SE maximization problem: $\max_{r_i}\left\{\frac{1}{m}U_i + R_i\right\}$. The corresponding first-order condition can be written as

$$\left(u^* + r_i \pi^*\right)\frac{1}{m} I_i = \Omega\left(\bar{k}_i\right),$$

where we define the reciprocal of the reverse hazard rate of G as

$$\Omega\left(x\right) = \frac{G\left(x\right)}{g\left(x\right)}.$$

Assuming $r_i^* > 0$,[5] so r_i^* is determined by the first-order condition rather than the non-negativity constraint $r_i \geq 0$, we can totally differentiate the first-order condition, and after substituting back in the first-order condition, we get

$$\frac{dr_i^*}{ds_i} = \frac{(1 - r_i^*)\,\pi^* - \frac{\Omega(\bar{k}_i)}{s_i I_i \Omega'(\bar{k}_i)}}{\frac{\pi^*}{m}\left(1 + \frac{1}{\Omega'(\bar{k}_i)}\right)},$$

where weak log-concavity of G ensures $\Omega' \geq 0$.

Suppose Ω takes the form $\Omega(x) = \upsilon x^{\omega}$ with $\upsilon > 0$ and constant elasticity $\omega > 0$. The derivative above then becomes

$$\frac{dr_i^*}{ds_i} = \frac{(1 - r_i^*)(\omega - 1)}{\frac{\omega}{m}\left(1 + \frac{1}{\Omega'(\bar{k}_i)}\right)},$$

where $\Omega'(\bar{k}_i) > 0$. Here $\omega = 1$ corresponds to constant-elasticity G (which includes linear G), while $\omega < 1$ corresponds to the elasticity of G being increasing in its argument and $\omega > 1$ corresponds to the elasticity of G being decreasing in its argument. Thus, provided we restrict to Ω taking this functional form with $\omega > 0$, if the elasticity of G is constant, then $\frac{dr_i^*}{ds_i} = 0$ and r_i^* does not depend on m; if the elasticity of G is increasing, then $\frac{dr_i^*}{ds_i} < 0$ and r_i^* is increasing in m; and if the elasticity of G is decreasing, then $\frac{dr_i^*}{ds_i} > 0$ and r_i^* is decreasing in m. Given $r_i^W = 0$ is fixed, this shows the divergence $r_i^* - r_i^W$ depends on the shape of G (specifically, whether its elasticity is constant, increasing or decreasing), and the divergence can increase or decrease in m in general.

G.2 Case with spillovers

Next consider what happens when we add spillovers to the above application using the framework of within-seller economies of scale from Section 4.1. Recall, a type-k seller joins all platforms if

$$k \leq \sum_{i=1}^{m} (1 - r_i)\, I_i s_i \pi^* \equiv \bar{k},$$

and otherwise does not join any platform. The functions U_i and R_i are otherwise the same, but note in total welfare across all m platforms, the sellers' participation costs are only incurred once.

If the planner chooses a common r, it does so to maximize

$$\hat{W} = -c + I_i(u^* + \pi^*)G(\bar{k}) - m\int_{k_{\min}}^{\bar{k}} k\,dG(k) - mC(I_i),$$

so

$$\frac{\partial \hat{W}}{\partial r_i} = -(u^* + r\pi^*)\pi^*\left(\sum_{i=1}^{m} s_i I_i\right)^2 g(\bar{k}) < 0,$$

and as a result $r^W = 0$.

We compare this to r_i^* which solves the SE maximization problem: $\max_{r_i}\left\{\frac{1}{m}U_i + R_i\right\}$. This involves the same first-order condition as without spillovers, and the same resulting derivative except that Ω is now a function of $\bar{k}$ rather than k_i. Given $\Omega(x) = \upsilon x^{\omega}$ and provided the equilibrium $r^* > 0$, this implies (after

[5] For example, if G has the constant elasticity form $G(x) = x^{\varphi}$ then regardless of m, we get $r_i^* = \max\left\{\frac{1}{2} - \frac{u^*}{2\pi^*}, 0\right\}$, so $u^* < \pi^*$ ensures $r_i^* > 0$.

imposing symmetry on the solution)

$$\frac{dr_i^*}{ds_i} = \frac{(1-r^*)(\omega - m)}{\omega s_i \left(1 + \frac{1}{\Omega'(\bar{k}_i)}\right)}.$$

Provided $m > \omega$, we have that $\frac{dr_i^*}{ds_i} < 0$ and r_i^* is increasing in m, implying the divergence $r^* - r^W$ increases in the number of platforms. For instance, with constant-elasticity G, since $\omega = 1$, this is always true for any number of platforms $m \geq 2$. Indeed, we can solve for the equilibrium commission rate explicitly in this case, which equals

$$r^* = \max\left\{\frac{m}{m+1} - \frac{u^*}{(m+1)\pi^*}, 0\right\},$$

which is an increasing function of m provided $m\pi^* \geq u^*$.

Intuitively, with spillovers, the more platforms there are, the less effect an individual platform's increase in commission has on decreasing seller participation given that depends on the weighted average commission across all platforms. This results in each platform preferring a higher commission level, resulting in commissions being even more inflated above the efficient level.