Loss-leader pricing and upgrades

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Abstract

A new theory of loss-leader pricing is provided in which firms advertise low (below cost) prices for certain goods to signal that their other unadvertised (substitute) goods are not priced too high. The theory is applied to the pricing of upgrades. The results contrast with most existing loss-leader theories in that firms make a loss on some consumers (who buy the basic version of the good) and a profit on others (who buy the upgrade).

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1 Introduction

Existing theories of loss-leader pricing (*e.g.* Hess and Gerstner, 1987, Lal and Matutes, 1994, Simester, 1997, Ellison, 2005, DeGraba, 2006, and Chen and Rey, 2012a, 2012b) are based on the idea that customers buy multiple goods, and certain goods are priced at low levels (often below cost) to attract consumers who will (sometimes) buy other more expensive goods from the same store. However, in the case of big-ticket items, consumers typically only intend to buy a single good (*i.e.* a single printer, TV, or computer) when they go shopping, so existing theories which rely on consumers buying multiple goods would not explain loss-leader pricing in such cases.

In this paper we provide a new theory of loss-leader pricing, based on the idea that firms offer low advertised prices for certain goods to indicate their other (substitute) goods are also not priced too high. This theory is illustrated in a setting where firms offer different versions of the same good (*e.g.* a basic version of a good and an upgrade). Consumers only want to purchase one version of the good and firms are assumed to only advertise the price of the basic version. We show they will set a price below cost for the basic version of the good, so as to signal that the price of the upgraded version, which is not advertised, is not too high.

The theory we offer is closest to Simester (1995) who also argues advertised prices may signal unadvertised prices. However, Simester relies on prices signaling a firm's marginal costs (which are assumed perfectly correlated across independent goods offered by the retailer).[1] The current theory shows that observed prices can directly signal to consumers a firm's choice of unobserved prices, provided goods are substitutes, as is the case of different versions of the same good. The results contrast with most existing loss-leader theories in two ways: (i) here firms price the unadvertised good below the monopoly level; and (ii) firms make a loss on some consumers (who buy the basic version of the good) and a profit on others (who buy the upgrade).

[1] A related signaling theory of loss-leader pricing is offered by Bagwell and Ramey (1994) in which some stores enjoy economies of scale. Loss-leader pricing is a way for such stores to credibly signal to consumers they have economies of scale and so acts as a coordination device.

Rosato (2013) provides a recent theory of loss-leader pricing in which a firm sells substitute goods, such that the firm makes losses on some consumers and profits on others. In contrast to our setting, his model relies on loss-averse consumers and a bait-and-switch strategy of the seller. Consumers go to the store enticed by the possibility of the bargain, but if it is no longer available, they buy a substitute good as a means of reducing their disappointment.

2 Model

Suppose there are two firms 1 and 2, each of which sells a basic version of a good (which is valued at v_b) and an upgrade (which is valued at v_u) to a measure one of consumers with v_u distributed on $[v_b, V]$ from some smooth distribution function F which has a strictly increasing hazard rate. Firms face costs c_b per unit for the basic version and $c_u > c_b$ per unit for the upgrade, where $v_b > c_b$ and $V - c_u > v_b - c_b$. Although the upgrade is more costly, it offers greater surplus over cost to some high-value consumers (*i.e.* those drawing high enough values of v_u). The two firms set both their prices simultaneously. Consumers observe the prices of the basic version p_b^1 and p_b^2 offered by the two firms and have to choose one firm to go to. After choosing the firm, they observe the price of the upgraded version from the chosen firm (denoted p_u^1 and p_u^2 for firms 1 and 2 respectively). Finally, they decide which version of the good to buy. We assume that once they have chosen a firm, say firm i, they cannot switch to the rival after observing the actual price p_u^i (for instance, due to high "transport" costs of doing so).

3 Analysis

Our solution concept is perfect Bayesian equilibrium (PBE). When consumers observe p_b^i different from its expected level, the PBE concept does not restrict the consumers' beliefs about the unobserved price p_u^i. The most natural restriction, given that firms set both their prices at the same time, is that consumers believe each firm sets p_u^i optimally given its choice of p_b^i and the equilibrium strategies of all other players. This restriction has been adopted frequently in the literature analyzing situations in which asymmetric information is created by strategic players (see In and Wright, 2012 for references and a formal treatment of such endogenous signaling). We are now ready to state our results.

Proposition 1 *There exists a symmetric PBE where the basic version is sold below marginal cost and the upgrade above marginal cost. The firms' equilibrium strategies are*

$$p_b^* = c_b - (1 - F(v^*))\,\phi(v^*), \tag{1}$$

$$p_u^* = c_u + F(v^*)\,\phi(v^*), \tag{2}$$

where $\phi(v^*) = \frac{1-F(v^*)}{f(v^*)}$, v^* *is the unique value satisfying*

$$v^* = v_b + c_u - c_b + \phi(v^*) \tag{3}$$

and $v_b < v^* < V$. *The consumers' equilibrium strategies are to choose the firm with the lowest* p_b^i *(randomizing between the two firms if* $p_b^1 = p_b^2$*) and buy (i) the basic version if* $v_b - p_b^i \geq v_u - p_u^i$ *and* $v_b \geq p_b^i$*; (ii) the upgrade if* $v_u - p_u^i > v_b - p_b^i$ *and* $v_u \geq p_u^i$*; or (iii) nothing otherwise. At the equilibrium, consumers believe upon observing* $(p_b^i)_{i\in\{1,2\}}$ *that each firm* i *has chosen the following price of the upgrade:*

$$p_u^*\left(p_b^i\right) = p_b^i + c_u - c_b + \phi(v^*). \tag{4}$$

The equilibrium outcome is such that consumers who draw $v_u \leq v^*$ *buy the basic version and those who draw* $v_u > v^*$ *buy the upgrade.*

Proof. Given consumers' beliefs in (4) that the firm which has chosen the lowest price of the basic version has also chosen the lowest price of the upgrade, consumers will always do best choosing the firm with the lowest p_b^i. Obviously, randomizing between the two firms is optimal if $p_b^1 = p_b^2$. The consumers' choice of the version to buy from the chosen firm, as specified in (i)-(iii), follows trivially given they have observed both prices at that stage.

Now we show that it is not profitable for firm i to choose (p_b^i, p_u^i) different from (1)-(2) given the other firm's equilibrium strategy and that of consumers, in two steps. First, we show that for any given p_b^i, it is not profitable for firm i to choose p_u^i different from the value implied by (4) given the other firm's equilibrium strategy and that of consumers. For $p_b^i > p_b^*$, any choice of p_u^i would be optimal since firm i makes zero profit given it attracts no consumers.

For $p_b^i \leq p_b^*$, firm i will want to price the upgraded version to maximize[2]

$$\pi^i = \left(p_b^i - c_b\right) F\left(v_b + p_u^i - p_b^i\right) + \left(p_u^i - c_u\right)\left(1 - F\left(v_b + p_u^i - p_b^i\right)\right), \quad (5)$$

since consumers that go to firm i will choose the basic version if $v_b - p_b^i \geq v_u - p_u^i$ and will choose the upgrade otherwise. Note $v_b > p_b^i$ from $v_b > c_b > p_b^* \geq p_b^i$, where the first inequality is by assumption, the second from (1), and the third from the range of p_b^i under consideration. This also implies $v_u > p_u^i$ if $v_u - p_u^i > v_b - p_b^i$. Therefore, consumers will always choose to buy one of the goods. The first order condition for p_u^i to maximize π^i is

$$d\pi^i/dp_u^i = \left(p_b^i - p_u^i + c_u - c_b\right) f\left(v_b + p_u^i - p_b^i\right) + \left(1 - F\left(v_b + p_u^i - p_b^i\right)\right) = 0,$$

which can be rewritten as

$$p_u^i = p_b^i + c_u - c_b + \phi\left(v_b + p_u^i - p_b^i\right). \quad (6)$$

[2] In case $p_b^i = p_b^*$, firm i's profit will be one half of the profit in (5) since consumers will randomize between the two firms.

Let $p_u^*(p_b^i)$ be the solution of (6) and $v^* = v_b + p_u^*(p_b^i) - p_b^i$. Then (6) implies (3) and (4).[3] To show $p_u^*(p_b^i)$ maximizes profit, define

$$\begin{aligned} D^i\left(p_u^i\right) &= \frac{\frac{d\pi^i}{dp_u^i}}{1 - F\left(v_b + p_u^i - p_b^i\right)} \\ &= 1 - \frac{\left(p_u^i - p_b^i + c_b - c_u\right)}{\phi\left(v_b + p_u^i - p_b^i\right)} \end{aligned}$$

for $1 - F\left(v_b + p_u^i - p_b^i\right) > 0$. Then $D^i\left(p_u^i\right) > 0$ if $p_u^i < p_u^*\left(p_b^i\right)$, $D^i\left(p_u^i\right) = 0$ if $p_u^i = p_u^*\left(p_b^i\right)$ and $D^i\left(p_u^i\right) < 0$ if $p_u^i > p_u^*\left(p_b^i\right)$. Since $1 - F\left(v_b + p_u^i - p_b^i\right) > 0$, this also implies $d\pi^i/dp_u^i > 0$ if $p_u^i < p_u^*\left(p_b^i\right)$, $d\pi^i/dp_u^i = 0$ if $p_u^i = p_u^*\left(p_b^i\right)$ and $d\pi^i/dp_u^i < 0$ if $p_u^i > p_u^*\left(p_b^i\right)$.[4]

Secondly, we show firm i's choice of p_b^i as in (1) is optimal given it will choose p_u^i according to (4). Note $p_u^*(p_b^*)$ is equal to p_u^* as defined in (2), and it makes zero profit at the proposed equilibrium. If it sets p_b^i above p_b^* it will attract no consumers, and make zero profit. If it sets p_b^i below p_b^* it will attract all consumers, but make a loss (given both of its prices will be lower than in the proposed equilibrium).

Finally, consumers' beliefs as specified in (4) are consistent with the firms' equilibrium strategies on the equilibrium path and they also reflect their beliefs that each firm sets p_u^i optimally given its choice of p_b^i and the equilibrium strategies of all other players off the equilibrium path.

In the equilibrium outcome, consumers who draw $v_u \leq v^*$ buy the basic version and those who draw $v_u > v^*$ buy the upgrade, which can be shown by substituting (1) and (2) into (i) and (ii). ■

Compared to the full information equilibrium in which both versions are priced at their respective costs (*i.e.* $p_b^i = c_b$ and $p_u^i = c_u$), in this symmetric

[3] Note that v^* is uniquely defined with $v_b < v^* < V$. To see this define the function g such that $g\left(v\right) = v_b + c_u - c_b + \phi\left(v\right)$. Then $g\left(v_b\right) = v_b + c_u - c_b + \phi\left(v_b\right) > v_b$ and $g\left(V\right) = v_b + c_u - c_b < V$ since $F\left(V\right) = 1$. Given F is smooth and has a strictly increasing hazard rate, the function ϕ is continuous and strictly decreasing in v, and so is g, which establishes the result.

[4] If p_u^i is so high that $F\left(v_b + p_u^i - p_b^i\right) = 1$, so that no consumers will want to buy the upgrade from firm i, then i's profit will be $p_b^i - c_b$, which is less than zero given $p_b^i \leq p_b^*$ and $p_b^* < c_b$ from (1).

PBE, the basic version of the good is used as a loss leader to signal that the upgrade will not be priced too much above cost. In contrast to many loss-leader models, here firms make a loss on some consumers (who buy only the basic version of the good) and a profit on others (who buy the upgraded version only). This reflects that firms cannot tell how much consumers value the upgrade, so they end up attracting the full mix of consumers. There is no way for a firm to attract only consumers with high value on the upgrade given that consumers use the basic version price as a signal of the price of the upgrade.

Proposition 2 *The price of the upgrade at the symmetric PBE is less than both (i) the price of the upgrade that would be set by a monopolist selling both the basic and upgrade versions and (ii) the price of the upgrade that would be set by a monopolist selling only the upgrade version. The price in (ii) is also less than the price in (i).*

Proof. Let p_u^0 be the price of the upgrade that would be set by a monopolist selling only the upgrade version and p_u^m be the price of the upgrade that would be set by a monopolist selling both the basic and upgrade versions. We need to show $p_u^* < p_u^0 < p_u^m$, where p_u^* is defined in (2). We first solve for p_u^0 and p_u^m. Suppose a monopolist sells only the upgrade. Then it obtains the maximum profit $\pi_u \equiv \max_{p_u} (p_u - c_u)(1 - F(p_u))$ by setting $p_u^0 \equiv \arg\max_{p_u} (p_u - c_u)(1 - F(p_u))$. Given our assumptions, the price p_u^0 must satisfy the first-order condition $p_u^0 = c_u + \frac{1-F(p_u^0)}{f(p_u^0)} = c_u + \phi(p_u^0)$.

Now we derive the price p_u^m that would be set by a monopolist selling both the basic and upgrade versions. We first show the monopolist will indeed prefer to offer both versions. Suppose on the contrary that the monopolist aims to sell only the basic version. Then it can obtain the maximum profit by setting $p_b^i = v_b$ giving a profit of $\pi_b \equiv v_b - c_b$. However, the monopolist can do at least as well by setting $p_b^i = v_b$ and $p_u^i = p_u^*(v_b)$ to obtain a profit of $\pi_b + \phi\left(v^*\right)\left(1 - F(p_u^*(v_b)\right)$. Suppose instead it aims to sell only the upgrade. Then it can obtain the maximum profit π_u characterized above. However, the monopolist can do at least as well by setting $p_b^i = v_b$ and $p_u^i = p_u^0$ to obtain a profit of $(v_b - c_b)F(p_u^0) + \pi_u$. Therefore, without loss of generality, we only need to maximize the profit function in (5) subject to $p_b^i \leq v_b$. The optimal choice of p_u^i for any given p_b^i is still given by (4). Substituting (4) into (5) and maximizing with respect to p_b^i subject to the constraint yields the monopoly prices $(p_b^m, p_u^m) = (v_b, v_b + c_u - c_b + \phi(v^*))$.

Now we show $p_u^0 < p_u^m$. Suppose on the contrary $p_u^0 \geq p_u^m$. Then $p_u^0 \geq v_b + c_u - c_b + \phi(v^*) \geq v_b + c_u - c_b + \phi(p_u^0) > c_u + \phi(p_u^0)$, where the second inequality is from the fact that ϕ is strictly decreasing. The inequality $p_u^0 > c_u + \phi(p_u^0)$ contradicts the definition of p_u^0. Therefore, $p_u^0 < p_u^m$.

Finally we show $p_u^* < p_u^0$. Clearly, $p_u^* \equiv c_u + F(v^*)\,\phi(v^*) < c_u + \phi(v^*)$. Since ϕ is strictly decreasing, $p_u^0 < p_u^m$ implies $c_u + \phi(p_u^m) < c_u + \phi(p_u^0) \equiv p_u^0$. Combining the two inequalities noting $v^* = p_u^m$, we obtain $p_u^* < p_u^0$. ■

The result contrasts with existing loss-leader models, and implies that competition does still act to lower the price of the good not used as a loss-leader. The intuition behind this result rests on the substitutability between the basic version of the good and the upgrade. The pricing of the basic version involves Bertrand-type competition between the firms for the profit from selling the upgrade, which results in the basic version being priced below cost. Once the firm has attracted some consumers to its store, it faces consumers with different values of the upgrade, which are considering which of the two versions to purchase. The pricing of the upgrade in equilibrium is then like a standard monopoly pricing problem with downward sloping demand except for two differences. First, the consumers' outside option if they don't purchase the upgrade involves their receiving a positive surplus from buying the basic version rather than zero utility in the standard case (*i.e.* from not purchasing in case the monopolist only sells the upgrade or from purchasing the basic version at the monopoly price in case the monopolist sells both versions). This lowers the demand for the upgrade compared to the standard monopoly case. Second, the firm's outside option in case the consumers do not purchase the upgrade now involves the firm making a loss from the sale of the basic version as opposed to zero profit in case the monopolist only sells the upgrade or a positive profit in case the monopolist sells both versions.

Both factors lead the firm to optimally price the upgrade below the price set by a monopolist selling just the upgrade and below the price set by a monopolist selling both versions.

A similar logic also explains why the price of the basic version is a signal for the price of the upgrade. The higher the price of the basic version, the worse is this outside option for consumers and the higher is the seller's optimal price of the upgrade. Moreover, the higher the price of the basic version, the lower is the loss to the seller from the consumers not buying the upgrade. Both of these factors explain why the price of the upgrade version is strictly increasing in the price of the basic version, and therefore why the price of the basic version signals the price of the upgrade.

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