Competitive bottlenecks and platform spillovers*

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Abstract

We provide a general framework to analyze competition between oligopolistic platforms in competitive bottleneck settings, allowing for various pricing and non-price design choices by platforms, and a range of buyer-seller microfoundations. We show that the equilibrium choices by platforms are distorted against sellers' interests in a way that is harmful to welfare (e.g., setting excessive commission fees), and that platform entry can exacerbate this distortion. We also characterize how buyer-side heterogeneity in interaction benefits, partial market coverage, platform asymmetry, and cross-platform spillovers further amplify or mitigate welfare distortions. Based on our findings, we discuss policy implications for mobile app platforms.

Keywords: platforms, two-sided markets, welfare distortions, regulation

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1 Introduction

A useful framework for analyzing how competition may (or may not) work between rival platforms across a wide range of sectors (e.g., smartphone ecosystems, video game consoles, shopping malls, subscription-based media such as newspapers, and internet service providers) assumes that agents on one side singlehome (they can join at most one platform), while agents on the other side can multihome (they are free to join any number of platforms). Armstrong (2006) was the first to formalize this setup, coining the term competitive bottlenecks to describe it.

In Armstrong's framework, platforms compete for the singlehoming side (e.g., buyers), but act as gatekeepers for the other side (e.g., sellers). Because each platform grants sellers access to a distinct set of buyers, a seller's decision to join each platform is made independently of its decisions to join other platforms. As shown by Armstrong, the result is an equilibrium where each platform acts to maximize the surplus of its singlehoming buyers and its own profit, while ignoring the surplus of the other side (sellers). This insight has informed various policy documents and cases, as detailed in Section 1.1.

This paper develops a more general framework to study platform competition when agents on one side singlehome while agents on the other side are free to multihome, and provides broad welfare results for such a setting. In contrast to the existing literature, our baseline model allows for any number of platforms (not just two), platforms to use a range of seller-side instruments (not only lump-sum fees), and specifications that account for sellers' post-participation decisions such as pricing, competition and investment (rather than just their participation decisions).

Despite its generality, our baseline model yields a tractable equilibrium characterization. Because the model allows for more than two rival platforms, we can analyze how platform entry affects equilibrium instrument choice. Whether the level of these instruments (e.g., commission rates) rise or fall turns out to depend on whether sellers' participation elasticity is increasing or decreasing. We also establish a general welfare result: seller-side instruments that reduce seller profit (e.g., commission rates) are set at excessive levels in equilibrium. Unlike previous welfare analysis (e.g., Armstrong's Proposition 4), which focus on showing insufficient seller participation, our analysis also captures welfare effects arising from sellers' decisions after they have joined a platform. Notably, our findings still apply even if all sellers participate, so that distortions arise solely from sellers' post-participation pricing or other decisions.

Moving beyond our baseline setting, we identify new distortions that arise when not all buyers join a platform in equilibrium or when platforms are asymmetric, showing these also drive excessive commissions in competitive bottleneck settings. Meanwhile, with heterogeneous buyers, the baseline distortion toward excessive commissions is either amplified or mitigated depending on whether the correlation between buyers' horizontal preferences (across platforms) and vertical preferences (for interaction benefits) is positive or negative.

A further source of welfare distortion stems from cross-platform spillovers, which arise whenever a platform's seller-side instruments affect (i) the utility buyers obtain on other platforms, and/or (ii) the other platforms' revenue, in both cases, holding buyer-side market shares constant. Spillovers naturally emerge when sellers face fixed entry costs, enjoy product-level network effects, make product-level investments, sell via direct channels, or set uniform prices across platforms. Such spillovers can break the independence of sellers' decisions across platforms, even when buyers singlehome and sellers are free to multihome. We show how these negative (positive) utility and revenue spillovers amplify (mitigate) the usual competitive bottleneck distortion (e.g., towards excessive commission rates).

Finally, we apply our findings to the case of smartphone ecosystems, where consumers typically choose only one platform (iOS or Android), whereas app developers often serve both. Even if there is some pass-through of commissions into consumer prices, and thus into consumers' platform choice, our results imply that platforms still fail to compete for developers' interests. Consequently, platforms set commissions (and other instruments that reduce developer profit, such as first-party entry, self-preferencing, limiting disintermediation, and restrictions on app tracking) at levels that are excessive from a welfare perspective. Moreover, the economics of these ecosystems give rise to several negative cross-platform spillovers, amplifying these welfare distortions. We conclude by discussing how policymakers might address these issues to alleviate the bottleneck facing app developers.

1.1 Related literature

Our paper is closest to Armstrong (2006)'s general framework of competitive bottlenecks and subsequent theoretical work that builds on it. Among other contributions, this line of work examines how homing configurations in competitive bottleneck settings emerge endogenously (Armstrong and Wright, 2007) and how exogenously imposed homing restrictions shape equilibrium outcomes (Belleflamme and Peitz, 2019; Bakos and Halaburda, 2020; Tremblay et al., 2023). However, these authors assume duopoly platforms,[1] lump-sum fees on both sides,[2] and platforms having no influence on sellers' post-participation decisions or seller competition.[3] As outlined in our introduction, our framework generalizes the existing literature on all three fronts and derives corresponding welfare implications.

Some other recent works also examine welfare distortions in equilibrium platform design in the context of a monopoly marketplace (Teh, 2022) or competing duopoly platforms connecting consumers and advertisers (Choi and Jeon, 2023). These studies focus on how platforms' monetization methods drive welfare distortions, but they depart from the canonical timing of two-sided market models by assuming away participation problems faced by sellers (or advertisers).

The high-level intuition of our baseline welfare analysis relates to Section 5 of Armstrong (2006). He conditions on a fixed allocation of singlehoming agents (i.e., buyers in our terminology) across platforms and shows that each platform attracts too few agents from the other side (i.e., sellers). Our approach differs in two important aspects. First, our welfare results are in terms of the seller-side instrument(s) that platforms set, rather than in terms of seller participation. As noted in the introduction, this distinction matters when sellers make post-participation decisions. Second, our welfare results do not condition on a fixed allocation of buyers across platforms. Instead, we allow platforms to reoptimize buyer-side pricing, and buyers to select their preferred platform

[1] A strand of recent studies develop frameworks for oligopoly platform competition (Anderson and Peitz (2020), Tan and Zhou (2021), Teh et al. (2023), and Peitz and Sato (forthcoming)), but they depart from Armstrong (2006)'s competitive bottleneck setting.

[2] A notable exception is Reisinger (2014), who studies platforms charging two-part tariffs in Armstrong (2006)'s model but does not study welfare implications.

[3] Hagiu (2009) incorporates seller competition but still focuses on duopoly platforms charging lump-sum fees on both sides, and does not study welfare implications.

under these new prices. This approach yields implications aligned with typical second-best welfare analysis. This distinction matters when not all buyers participate in equilibrium or the platforms are not symmetric, and as we will show, in such cases it introduces new distortions in competitive bottleneck settings.

Competitive bottlenecks are often referenced in policy discussions (Franck and Peitz, 2019; Cabral et al., 2021; Jullien and Sand-Zantman, 2021).[4] The framework's basic building block — the homing assumption where agents on one side are restricted to singlehome while agents on the other side face no such restrictions — has been used to construct specialized models that study various policy and managerial questions. These include competition in advertising markets (Anderson and Coate, 2005; Crampes et al., 2009; Anderson and Peitz, 2020), exclusivity arrangements (Hagiu and Lee, 2011; Carroni et al., 2024), termination charges in mobile telecommunications (Armstrong, 2002; Wright, 2002), commissions set by mobile app platforms (Etro, 2023; Jeon and Rey, 2024), net neutrality (Bourreau et al., 2015; Choi et al., 2015; Greenstein et al., 2016), interchange fees in payment cards (Bedre-Defolie and Calvano, 2013), business models in the creator economy (Casner and Teh, forthcoming), price parity restrictions (Edelman and Wright, 2015), and tying (Choi, 2010; Choi and Jeon, 2021). Notable empirical applications include, among others, magazines (Kaiser and Wright, 2006; Song, 2021), video game consoles (Lee, 2013), and Yellow Pages (Rysman, 2004). However, there is a potential conceptual ambiguity in what different authors mean by "competitive bottlenecks". Some authors refer to a "competitive bottleneck" whenever the homing assumption holds, while others also require the non-singlehoming side (i.e., the seller side) makes their joining decisions independently across rival platforms. This ambiguity arises partly because, in many models, the homing assumption implies the independence feature (see, the survey by Jullien et al. (2021)). Our paper shows this linkage may fail once we account for richer specifications of sellers' participation and post-participation decisions that result in cross-platform spillovers. For this reason, we use "competitive bottlenecks" to refer only to situations where both the homing assumption and the independence feature are satisfied.

In applying our framework to the context of mobile app platforms, our paper is related to recent contributions by Etro (2023) and Jeon and Rey (2024). Both of these papers consider platforms that compete for singlehoming buyers via device prices, and charge sellers (i.e., developers) ad valorem commissions. In Etro (2023), sellers are free to multihome, and their participation decisions are independent across platforms. In Jeon and Rey (2024), sellers' participation decisions across platforms are generally interdependent because sellers face potential scale economy effects when listing their apps on multiple platforms. This corresponds to one type of cross-platform spillover we look at (Section 5.1). These papers find that, in the absence of scale effects, competition through device prices redistributes commission revenue back to buyers in such a way that the equilibrium ad valorem commissions maximize buyer surplus. As Jeon and Rey (2024) show, the equilibrium commissions are above the level that maximizes buyer surplus if there are economies of scale. In contrast, we find that the equilibrium commission is generally excessive from the buyer's viewpoint even without any scale effects, a finding we explain in Section 3.1.

[4]See also statements by the European Commission in OECD (OECD, 2016) and the German Cartel Office (BKartA, 2016).

2 Baseline model

There are $m \geq 2$ two-sided platforms, a continuum of buyers, and a continuum of sellers. We adopt the canonical homing assumptions of existing competitive bottleneck models: buyers singlehome (each chooses one of m platforms to join) while sellers are free to multihome (each can join any number of platforms, including none, one, two, ..., all m platforms).

Following the platform literature (Armstrong, 2006; Armstrong and Wright, 2007; Belleflamme and Peitz, 2019, among others), each platform $i \in \{1, ..., m\}$ charges buyers a lump-sum membership (or device) price, P_i^B, but we generalize the earlier literature by allowing it to also choose some other scalar "instrument" $a_i \in \mathcal{A} \subseteq \mathbb{R}$. This seller-side instrument has a general interpretation. It could be a lump-sum fee P_i^S charged to sellers, as in the previous literature. The instrument could instead be an ad-valorem transaction fee r_i charged to sellers, or even a (possibly discrete) platform design or investment choice that affects sellers (and potentially buyers as well). To ensure each platform's maximization problem is well-defined, we assume the set $\mathcal{A}$ is compact.[5] We order a_i so that a higher a_i corresponds to a lower seller surplus, in a sense that will be made precise below.

We next lay out the assumptions for each type of player in our baseline model. Section 2.2 discusses the rationale behind these assumptions and how some of them can be relaxed.

□ **Buyers.** Each buyer is indexed by $\boldsymbol{\epsilon} = (\epsilon_1, ..., \epsilon_m)$, representing idiosyncratic match values (i.e., horizontal preferences) for the m platforms.[6] Let $s_i \in [0, 1]$ denote platform i's market share of buyers, which is endogenously determined. Without imposing any particular microfoundations, we formulate each buyer's net utility from joining each platform i as:

$$U_i(a_i; s_i) - P_i^B + \epsilon_i. \tag{1}$$

Here, U_i is the gross participation utility buyers get from interacting with sellers on platform i, which is homogenous across buyers. This utility typically depends on the mass of participating sellers on platform i and their post-participation pricing decisions, reflecting cross-group network effects. However, in competitive bottleneck settings (see our discussion in Section 2.2), an important observation is that sellers' participation and pricing decisions on platform i depend only on the fee levels and design choice of platform i (as captured by instrument a_i) and the mass of buyers on platform i, s_i, due to cross-group network effects.[7] Hence, the dependency of buyer participation utility on seller behavior can simply be subsumed by expressing U_i as a general function of (a_i, s_i). For concreteness, we illustrate this baseline formulation with a fully worked microfoundation in Section 2.1.

Let $F(\cdot)$ denote the joint cumulative distribution function (CDF) of $\boldsymbol{\epsilon}$ with support $[\underline{\epsilon}, \bar{\epsilon}]^m$,

[5] For theoretically unbounded choices like fees or investment, one can still obtain bounds by introducing a "choke price" (above which no seller participates) or a "choke investment level"(above which investment costs become prohibitive).

[6] Throughout the paper, we use the bold form to denote profiles of objects involving all m platforms.

[7] This formulation also allows us to capture any same-side platform-specific network effects on the buyer side.

where $\underline{\epsilon} \geq -\infty$ and $\bar{\epsilon} \leq \infty$. Then, the measure of buyers joining platform i is

$$s_i = \Pr\left(U_i - P_i^B + \epsilon_i \geq \max_{j \neq i}\left\{U_j - P_j^B + \epsilon_j\right\}\right), \tag{2}$$

where the probability is based on $F(\cdot)$. Following the oligopolistic demand formulation by Tan and Zhou (2021), we assume $F(\cdot)$ is continuously differentiable and symmetric across the m platforms (in the sense that the joint distribution of $(\epsilon_1, ..., \epsilon_m)$ is invariant under any permutation of the order of these m random variables). In our baseline model, we assume full market coverage: each buyer participates in one and only one platform, so $\sum_{i=1}^m s_i = 1$.[8] This formulation is general enough to encompass standard competition models such as independent and identically distributed (IID) shocks (Perloff and Salop, 1985), and alternative correlation structures such as the spokes model (Chen and Riordan, 2007) and the Hotelling model in case $m = 2$.

□ **Platforms**. Given our general approach for modeling platform i's seller-side instrument a_i, we write its profit as

$$\Pi_i = P_i^B s_i + R_i(a_i; s_i), \tag{3}$$

where we assume that all platforms have the same per-buyer marginal cost, and, without loss of generality, normalize this common marginal cost to zero. Here, R_i is platform i's "net revenue" or "residual profit", which captures platform i's revenue beyond its revenue from buyer-side prices. For example, if platform i's instrument a_i is an ad-valorem transaction fee, then $R_i = a_i \times$(Transaction revenue on i). Observe that transaction revenue on platform i depends on the mass of participating sellers on platform i and those sellers' post-participation pricing decisions which, as pointed out above, just depends on (a_i, s_i).

□ **Sellers**. So far, the seller-side payoffs and behavior have been encapsulated within the U_i and R_i functions. This reduced-form formulation accommodates a wide range of specifications on the seller side and different platform instruments. That is, many microfoundations of interest can be described by appropriately specifying the corresponding functions U_i and R_i. We provide a worked example in Section 2.1 and note there how this approach can capture a range of other instrument choices and settings.

For our subsequent welfare analysis, it is useful to denote $SS_i(a_i; s_i)$ as the total surplus sellers get from platform i (so $\sum_{i=1}^m SS_i$ is total seller surplus). To capture the ordering that a higher a_i corresponds to a lower seller surplus, we assume that $SS_i(a_i; s_i)$ is decreasing in a_i, holding s_i constant (and strictly decreasing if $s_i > 0$).

□ **Timing**. We adopt the canonical timing in the two-sided market literature: (i) the platforms set their buyer-side price P_i^B and seller-side instrument a_i simultaneously; (ii) observing these prices and instrument choices, buyers and sellers make their platform participation decisions; (iii) the on-platform interaction between buyers and sellers unfolds according to the specified micro-foundation, captured by the functions U_i and R_i.

The solution concept is symmetric Subgame Perfect Equilibrium. Throughout, we assume the functions U_i, R_i, and SS_i are symmetric across all m platforms, continuous in a_i, and continuously

[8]Formally, we normalize each buyers' outside option to zero and assume $\underline{\epsilon}$ is high enough so that all buyers join one of the platforms for relevant prices and platform instrument choices.

differentiable in s_i. We further assume the equilibrium outcomes and welfare benchmarks all involve symmetric solutions in which all platforms choose the same P_i^B and a_i.

2.1 Microfoundations

The following microfoundation illustrates our baseline model, and we will use it extensively throughout the paper.

□ **A leading example**. Suppose there is a mass-one continuum of product categories, each with a monopolist seller facing the same downward-sloping demand function from buyers. Each product category is indexed by $(k_1, ..., k_m)$, which denotes the seller's idiosyncratic fixed cost to join each platform. The outside option of not joining any platform is valued at zero. Let $G(\cdot)$ denote the marginal distribution of k_i with support $[k_{\min}, k_{\max}]$, where $k_{\min} \geq 0$. It does not matter whether the draws k_i are correlated across platforms. The case where they are not perfectly correlated allows us to accommodate the possibility that some sellers may *choose* to singlehome, some may *choose* to multihome on a subset of platforms, and others may *choose* to multihome on all platforms (depending on their particular draws of k_1,...,k_m).

Each platform i's instrument a_i is an ad-valorem commission rate $r_i \in [0, 1]$. Given a seller's price p_i on platform i, each buyer chooses the number of units to purchase q_i to maximize their net utility; i.e., $\arg\max_{q_i}\{u(q_i) - p_i q_i\}$. As a result, each seller faces the per-buyer demand $D(p_i)$. Each seller has a constant per-unit marginal cost $c \geq 0$, where $D(c) > 0$. Define $\bar{r}$ as the lowest commission such that $D(\frac{c}{1-r_i}) = 0$ (if this is always strictly positive for all $r_i < 1$, then we define $\bar{r} = 1$). Without loss of generality, we can restrict each platform to choose $r_i \in [0, \bar{r}]$, since no sellers would join platform i if $r_i > \bar{r}$.

A seller's optimal price on platform i is

$$p(r_i) = \arg\max_{p_i} \{((1 - r_i)p_i - c)D(p_i)\}$$

and we assume $p(r_i)$ is unique and well-defined for $r_i \in [0, \bar{r}]$, and weakly increasing in r_i. These properties hold if $D(.)$ is strictly log-concave. Define $q(r_i) \equiv D(p(r_i))$. Then each seller's per-buyer profit is $\pi(r_i) = ((1 - r_i)p(r_i) - c)q(r_i)$ and each buyer's per-seller surplus is $v(r_i) = u(q(r_i)) - p(r_i)q(r_i)$, both of which are weakly decreasing in r_i.

Each seller joins platform i if and only if $k_i \leq \pi(r_i)s_i$. Thus, the mass of sellers on platform i is $G(\pi(r_i)s_i)$, which is decreasing in r_i (when holding s_i fixed). Notice a seller's choice to join platform i is independent of its choice to join any other platform j, reflecting the key requirement for this to be a competitive bottleneck. Hence, seller surplus on platform i is

$$SS_i(r_i; s_i) = \int_{k_{\min}}^{k_{\max}} \max\{\pi(r_i)s_i - k_i, 0\}dG(k_i),$$

which is decreasing in r_i. Mapping this setup into our framework yields

$$\begin{aligned} U_i(r_i; s_i) &= v(r_i)G(\pi(r_i)s_i) \\ R_i(r_i; s_i) &= r_i p(r_i)q(r_i)s_i G(\pi(r_i)s_i). \end{aligned} \tag{4}$$

Observe that each buyer's participation utility U_i is the product of (i) per-seller utility $v(r_i)$, which is weakly decreasing in commission level r_i due to potentially positive pass-through in sellers' pricing; and (ii) the mass of participating sellers $G(\pi(r_i)s_i)$ that reflects typical cross-group network effects. The latter, in turn, depends only on (r_i, s_i) and thus demonstrates how cross-group network effects can be captured by specifying U_i as a general function of (r_i, s_i). Specifically, the more buyers s_i there are on platform i, the more sellers would be willing to participate on platform i in order to reach those buyers, and the more sellers there are on platform i as a result, the more utility buyers get from interacting with sellers. Likewise, the platform's net revenue R_i is the product of the per-seller commission revenue and the mass of participating sellers, both of which just depend on (r_i, s_i).

To sharpen some results when applying this example later in the paper, assume that the distribution of seller participation cost follows the constant-elasticity form

$$G(k) = \left(\frac{k}{k_{\max}}\right)^{\varphi}, \tag{5}$$

where $\varphi \geq 1$ is the elasticity parameter.[9]

□ **Other examples.** The basic structure of our *leading example* can be amended to capture other microfoundations and platform instruments a_i. As a simple example, suppose we take the *leading example* but assume that each platform's instrument a_i is the lump-sum fee P_i^S that sellers pay to join the platform, as in Armstrong (2006). Given the absence of commissions, we can drop the function arguments in p, q, v, and π above. Then, we have $U_i = vG(\pi s_i - P_i^S)$ and $R_i = P_i^S G(\pi s_i - P_i^S)$.

Online Appendix A constructs several other microfoundations with richer specifications on the sellers' behavior and platform instruments. These include settings where: (i) platforms can enter as first-party sellers (and potentially engage in self-preferencing) to compete with third-party sellers on their own marketplaces; (ii) platforms can engage in disintermediation prevention efforts to limit sellers from inducing buyers to transact on the sellers' direct channels; (iii) platforms can impose app tracking restrictions that influence sellers' monetization choice between ads and charging buyers. It also includes a specification where the heterogeneity is in seller demand rather than their fixed joining costs, and there is direct seller competition.

In each case, we can use our general findings in Section 3.1 below to characterize how the equilibrium choice of the relevant instruments is distorted from a welfare perspective. The common theme across these microfoundations is the idea that cross-group network effects and sellers' responses to platform choices can be captured by specifying U_i and R_i as general functions of each platform i's instrument a_i and buyer-side market share s_i. These richer specifications also help illustrate the extension of our framework to the case where platforms have multiple instruments, which we address more generally in Online Appendix B.

2.2 Model discussion

□ **No spillovers across platforms.** Our baseline model implicitly assumes a "no spillover" property, meaning U_i and R_i do not directly depend on instrument choices by other platforms

[9] The assumption of $\varphi \geq 1$ is used to show the existence of a symmetric equilibrium in this example. We can also handle the degenerate case of $\varphi = 0$, in which case $k = 0$ for all sellers, and they always join all platforms.

$j \neq i$ (when holding s_i constant). In many existing competitive bottleneck models (e.g., Armstrong (2006) and those reviewed by Jullien et al. (2021)), the property follows naturally from the homing assumption in which buyers singlehome and sellers are free to multihome. In those models, each seller decides whether or not to join platform i independently of its decision to join any other platform $j \neq i$, reflecting that (i) joining one platform does not prevent a seller from joining others; and (ii) each platform grants sellers access to a distinct set of buyers. As such, each seller's decision to join i does not depend on any two-sided pricing and instrument choices by other platforms $j \neq i$ except via changes in the buyer-side market share s_i. Our no spillover property generalizes this idea to situations where sellers also make post-participation decisions independently across platforms.[10]

□ **Buyer-side assumptions.** In our baseline model, the key assumption on the buyer side is that buyers singlehome. It is necessary for the validity of the discrete-choice-based buyer participation behavior in (2). We can relax other buyer-side assumptions. In Section 4, we show that relaxing the assumptions of (i) symmetric platforms; (ii) full market coverage on the buyer side; and (iii) buyers being homogeneous in their interaction utility introduces new welfare distortions beyond those in our baseline setting.[11]

□ **Non-atomistic sellers.** Although we assume a continuum of monopoly sellers for simplicity, our framework can accommodate a finite number of sellers and/or sellers being oligopolistic (see Online Appendix A for an example). Our timing, in which buyers and sellers choose platforms simultaneously, implies that even non-atomistic sellers cannot influence buyer participation behavior. This means we rule out settings where non-atomistic sellers could publicly commit to their participation decisions prior to buyer participation decisions in order to influence them. In such settings, the arguments of U_i and R_i would need to reflect each seller's individual participation.

3 Baseline equilibrium analysis

We begin by characterizing the equilibrium. Denote each platform's buyer-side price as P^{B*}, its seller-side instrument as a^*, and its corresponding buyer-side market share as $1/m$ in a symmetric equilibrium. To pin down the equilibrium, we extend the technique of Armstrong (2006) and Tan and Zhou (2021) (developed for pure singlehoming settings) to our model as follows.

Consider a "semi-symmetric" participation equilibrium in which platform i deviates from the equilibrium and sets $(a_i, P_i^B) \neq (a^*, P^{B*})$, obtaining a market share $s_i \neq 1/m$. All other $m-1$ platforms equally absorb the resulting change in platform i's market share (due to symmetry and the market being covered), resulting in each of them having market share $s_j = \frac{1-s_i}{m-1}$. Then, the fixed-point definition of market share s_i in (2) becomes

$$s_i = \Phi\left(U_i(a_i; s_i) - U_j(a^*; \frac{1-s_i}{m-1}) - P_i^B + P^{B*}\right), \tag{6}$$

where $\Phi(\cdot)$ is the CDF of $\max_{j\neq i}\{\epsilon_j\} - \epsilon_i$, with derivative $\Phi'(\cdot)$.

[10] The analysis with cross-platform spillovers and sellers making interdependent decisions across platforms is given in Section 5.

[11] In Online Appendix C, we analyze what happens if platforms monetize on the buyer side by advertising rather than lump-sum prices.

We assume functional forms are such that a unique fixed-point in (6) always exists.[12] This requires the right-hand side of (6) has a slope less than one with respect to s_i, which holds if horizontal platform differentiation (measured by $1/\Phi' > 0$) exceeds cross-group (and potentially within-group) network effects (measured by $\partial U_i/\partial s_i + \frac{1}{m-1}\partial U_j/\partial s_j > 0$).[13] Under this condition, the resulting demand system is analogous to standard discrete choice models, where the platform i's buyer-side market share decreases in its own price, i.e., $ds_i/dP_i^B < 0$.

Platform i chooses (a_i, P_i^B) to maximize profit Π_i, taking (a^*, P^{B*}) for other platforms as given. Following the approach of Armstrong (2006) and Tan and Zhou (2021), we reframe the problem as platform i directly choosing the market share s_i it wants to implement via P_i^B, so its maximization is with respect to (a_i, s_i). Formally, we invert (6), to write P_i^B as a function of (a_i, s_i) satisfying:

$$P_i^B = P^{B*} + U_i(a_i; s_i) - U_j(a^*; \frac{1-s_i}{m-1}) - \Phi^{-1}(s_i). \tag{7}$$

Then, platform i's problem is to choose (a_i, s_i) to maximize

$$\begin{aligned} \Pi_i &= P_i^B s_i + R_i(a_i; s_i) \\ &= \left(P^{B*} + U_i(a_i; s_i) - U_j(a^*; \frac{1-s_i}{m-1}) - \Phi^{-1}(s_i)\right) s_i + R_i(a_i; s_i). \end{aligned} \tag{8}$$

To proceed, we assume that Π_i is globally strictly quasiconcave in (a_i, s_i), which ensures the existence of a unique symmetric equilibrium.[14] Consequently, each platform's optimal choice of $a_i = a^*$ in the symmetric equilibrium is the unique maximizer of $U_i(a_i; s_i)s_i + R_i(a_i; s_i)$ while holding $s_i = 1/m$.[15] We summarize the equilibrium as follow:

Proposition 1 *A unique symmetric equilibrium is characterized by all m platforms having the same market share $s^* = 1/m$, setting the same seller-side instrument*

$$a^* = \arg\max_{a_i \in \mathcal{A}} \left\{ \frac{1}{m} U_i(a_i; \frac{1}{m}) + R_i(a_i; \frac{1}{m}) \right\}, \tag{9}$$

and the same buyer-side price

$$P^{B*} = \frac{1}{m\Phi'(0)} - \left(\frac{1}{m-1}\right) \frac{\partial U_i(a^*; \frac{1}{m})}{\partial s_i} - \frac{\partial R_i(a^*; \frac{1}{m})}{\partial s_i}. \tag{10}$$

[12] As such, we rule out equilibrium multiplicity issues considered in the homogenous platform competition models by Caillaud and Jullien (2003) and, in a setting where buyer-seller interactions are modelled explicitly, by Karle et al. (2020).

[13] Formally, let $B_\Phi \equiv \sup_{x\in\mathbb{R}} \Phi'(x)$ and $B_U \equiv \sup_{a\in\mathcal{A}} \sup_{s_i\in[0,1]} |\frac{\partial}{\partial s_i} U_i(a_i, s_i)|$. Then if $2B_\Phi B_U < 1$, the right-hand side of (6) has a slope that is less than one for any $m \geq 2$. For instance, in the Hotelling specification of the *leading example* below, this sufficient condition is implied by condition (14).

[14] Without network effects (i.e., function U_i is a constant and R_i is independent of s_i), global strict quasiconcavity in s_i and equilibrium existence hold if $\Phi(\cdot)$ is log-concave (Caplin and Nalebuff, 1991). A recurring feature in the literature on platform competition is that such existence conditions hold when the extent of platform horizontal differentiation is large relative to the strength of the network effect. At the end of this section, we offer a sufficient condition for our *leading example* in terms of the model primitives.

[15] The two-dimensional quasiconcavity requirement is often stronger than necessary. As we show below, the determination of the optimal $a_i = a^*$ in our *leading example* is independent of s_i. In such cases, we simply need single-dimensional strict quasiconcavity with respect to s_i, while no such assumption would be needed for a_i (so that we could allow for cases where the optimal choice a^* is non-unique).

Proposition 1 generalizes the equilibrium characterization of existing competitive bottleneck models to allow for oligopoly platforms, arbitrary platform instruments, and various possible microfoundations involving sellers making post-participation decisions (such as setting prices). This allows us to analyze not only platform fees (such as app-store commissions) but also platform design choices (e.g., Apple's and Google's decision on whether to sell their own apps and whether to promote these over third-party apps, investments in their app stores, and various in-app transaction policies they might adopt).

A convenient feature of (9) is that the equilibrium instrument a^* can be determined by solving a simple maximization problem, without explicitly determining P^{B*}. Intuitively, each platform i adjusts P_i^B to implement its target buyer-side market share s_i, according to (7). In the equilibrium, the platform simply focuses on how a_i affects: (i) the price that platform i can earn from buyers (while maintaining its market share at $s_i = 1/m$), as captured by the extent to which a higher a_i changes $U_i(a_i; \frac{1}{m})\frac{1}{m}$; and (ii) its other net revenue sources $R_i(a_i; \frac{1}{m})$.

We discuss the welfare implications and comparative statics of Proposition 1 in the next two subsections. Before doing so, we return to the *leading example* to illustrate the equilibrium characterization.

□ **Leading example with Hotelling competition.** Continuing with our *leading example*, suppose there are $m = 2$ platforms, and the match value distribution is given by $\epsilon_1 = b - ty$ and $\epsilon_2 = b - t(1-y)$, where y is uniformly distributed on $[0, 1]$. Here, $t > 0$ is the mismatch cost parameter measuring the extent of horizontal differentiation between platforms and $b > 0$ is the standalone benefit buyers obtain (assumed high enough to ensure full market coverage). The resulting buyer-side demand for each platform is $\Phi(x) = \frac{1}{2} + \frac{x}{2t}$. Given G takes the constant-elasticity form (5), (8) becomes

$$\Pi_i = \left(P^{B*} + (v(r_i) + r_i p(r_i) q(r_i)) \left(\frac{\pi(r_i) s_i}{k_{\max}} \right)^{\varphi} - v(r^*) \left(\frac{\pi(r^*)(1-s_i)}{k_{\max}} \right)^{\varphi} - (2s_i - 1)t \right) s_i. \quad (11)$$

By Proposition 1,

$$r^* = \arg\max_{r_i \in [0,\bar{r}]} \{(v(r_i) + r_i p(r_i) q(r_i)) \pi(r_i)^{\varphi}\} \quad (12)$$

$$P^{B*} = t - (2\varphi v(r^*) + (1+\varphi) r^* p(r^*) q(r^*)) \left(\frac{\pi(r^*)}{2k_{\max}} \right)^{\varphi}. \quad (13)$$

Observe that r^* is determined by a single-variable maximization, regardless of the value of s_i. As such, any solution to (12) constitutes a symmetric equilibrium.[16]

Suppose the mismatch cost parameter t is large relative to the extent of cross-group network effects (measured by the elasticity of seller participation φ):

$$\frac{2t}{\varphi(1+\varphi)} > (v(r^*) + r^* p(r^*) q(r^*)) \left(\frac{\pi(r^*)}{k_{\max}} \right)^{\varphi}. \quad (14)$$

In Online Appendix D, we show that (14) ensures (6) has a unique fixed-point and that Π_i in (11) is globally strictly concave in $s_i \in [0, 1]$. Thus, a symmetric equilibrium exists. We also show a

[16]In Online Appendix D, we provide conditions for (12) to have a unique solution.

similar condition arises if there are $m \geq 2$ platforms and buyer-side demand follows the standard logit form that arises when ϵ_i is drawn IID from a Gumbel distribution: $\Phi(x) = \frac{1}{1+(m-1)\exp\{-x/\mu\}}$, where parameter $\mu > 0$ indicates the extent of platform differentiation.

3.1 Welfare implications for baseline setting

We are interested in comparing the equilibrium instrument a^* with second-best outcomes. Specifically, we consider a fictitious social planner who chooses the symmetric instrument $a_1 = a_2 = ...a_m = a$ to maximize its welfare objective, while allowing each platform i to optimally adjust its buyer-side price P_i^B in response to changes in a.[17]

Following Proposition 1, let $P^B(a)$ be the symmetric equilibrium buyer-side price charged by platforms in response to a. Then $P^B(a)$ is given by (10) with a^* replaced by a. By the assumptions of symmetric platforms and full market coverage, each platform has a buyer-side market share of $1/m$ in the corresponding equilibrium.[18]

We define two welfare benchmarks. The first, the "seller-excluded" (SE) benchmark, refers to the instrument value a^{SE} that maximizes total welfare excluding seller profit (i.e., the combined surplus of buyers and platforms):

$$W^{SE}(a) = \int_{\underline{\epsilon}}^{\bar{\epsilon}} \left[\hat{\epsilon} + U_i(a; \frac{1}{m}) - P^B(a) \right] d\hat{F}(\hat{\epsilon}) + P^B(a) + mR_i(a; \frac{1}{m}),$$

where $\hat{F}(\epsilon)$ is the CDF of $\hat{\epsilon} \equiv \max_{i=1,\ldots,m}\{\epsilon_i\}$. The second, the "total-welfare" (W) benchmark, refers to the instrument value a^W that maximizes total welfare:

$$W(a) = W^{SE}(a) + mSS_i(a; \frac{1}{m}).$$

To simplify the exposition, we assume throughout that both welfare functions are such that unique solutions exist.[19] As a preliminary step, we establish the following relationship between the two benchmarks.

Lemma 1 *The seller-excluded benchmark exceeds the total-welfare benchmark ($a^{SE} \geq a^W$), indicating that the seller-excluded benchmark level of instrument a is excessive.*

Lemma 1 follows immediately from seller surplus $SS_i(a)$ being decreasing in a, and does not rely on any other properties of the welfare objective functions here. As such, the lemma continues to hold in the settings of Section 4 and Section 5.

After omitting terms irrelevant to its maximization, we see that $W^{SE}(a)$ is proportional to $\frac{1}{m}U_i(a, \frac{1}{m}) + R_i(a; \frac{1}{m})$, i.e., the joint surplus of platform i and its buyers. This is exactly the objective function in Proposition 1 that characterizes a^*, and so we conclude $a^* = a^{SE}$. Together with Lemma 1, this implies:

[17] Alternatively, one could adopt a first-best approach by letting the planner choose both a and P^B. However, under symmetric platforms and full market coverage, this would not affect the characterization of a^{SE} and a^W below.

[18] This is conceptually different from fixing $s_i = 1/m$ upfront before the optimization of the welfare objective, even though fixing $s_i = 1/m$ still leads to the same solutions for the welfare benchmarks a^{SE} and a^W defined below.

[19] If these are non-unique, the results below comparing a^{SE} and a^W can be rephrased in terms of the strong set ordering of Milgrom and Shannon (1994). See Online Appendix B for further details.

Proposition 2 *The equilibrium instrument in Proposition 1 coincides with the seller-excluded benchmark, and exceeds the total-welfare benchmark:* $a^* = a^{SE} \geq a^W$.

Proposition 2 delivers our baseline welfare result: platforms' equilibrium instrument a^* is distorted in the direction of outcomes with a lower seller profit compared to the total-welfare benchmark a^W. In the *leading example*, this means the seller-excluded benchmark —and hence the equilibrium outcome — features a commission rate r that exceeds the welfare maximizing level $r^W = 0$.

In the trivial case where sellers obtain no surplus at the welfare maximizing solution (e.g., if they fully compete away all their surplus), then $a^{SE} = a^W$ so $a^* = a^{SE} = a^W$. This is one reason why we have stated the result in Proposition 2 as a weak inequality. But if sellers obtain positive surplus ($SS_i(a^W) > 0$) which strictly decreases as a increases above a^W, and a_i is a continuous variable with the equilibrium solution a^* being interior, then Proposition 2 implies that $a^* = a^{SE} > a^W$.

A few important remarks on Proposition 2 are in order.

□ **The equivalence result as a baseline distortion**. We interpret the seller-excluded benchmark a^{SE} as indicating a "baseline distortion" (i.e., as stated in Lemma 1) that arises due to the existence of a competitive bottleneck. Hence, the equivalence result $a^* = a^{SE}$ indicates that in the equilibrium of the baseline model, only the baseline distortion exists. We then operationalize our subsequent welfare analysis by showing how introducing additional modeling ingredients generates new forces in the equilibrium that either amplify (i.e., $a^* \geq a^{SE}$) or mitigate ($a^* \leq a^{SE}$) this baseline distortion.

To explain the result in Proposition 2, note that given buyers have homogenous gross participation utility, each platform i can use its lump-sum buyer-side price to capture any increase in $U_i - U_{-i}$, the difference in participation utility between platform i and its rival platforms. Here, U_{-i} is independent of a_i due to the no-spillover feature, and so platforms simply internalize buyer participation utility in addition to their residual profit, resulting in the same objective function $\frac{1}{m}U_i + R_i$ that the platform maximizes with its optimal a_i in the equilibrium analysis.[20]

It is useful to think though how this logic fails once we depart from a competitive bottleneck setting. Consider our *leading example*, but now assume a fraction of sellers cannot multihome. In this case, a higher r_i induces some sellers to switch from platform i to platform j, which makes platform j more attractive to buyers and thus raises U_j. This positive spillover causes platform i to choose a level of r_i below the level that maximizes $\frac{1}{m}U_i + R_i$ in order to make its rival less attractive, thus breaking the equivalence result. This would mitigate our baseline distortion that causes r_i to be set too high. We expand on this interpretation of singlehoming sellers as a type of positive cross-platform spillover in Section 5.2.

□ **Misalignment with buyer surplus**. Even though a^* maximizes platform-buyer joint surplus (W^{SE}), it need not maximize buyer surplus. In Online Appendix D, we analyze the *leading example* to show that the symmetric commission level r that maximizes buyer surplus

[20] This logic echoes the non-linear pricing results of Courty and Li (2000) for a monopolist and Armstrong and Vickers (2010) for duopolists. In our setting, the reasoning is less straightforward given that platforms' buyer-side pricing also takes into account the network effects generated from buyer participation, as can be seen from (10).

$BS(r) = U(r; \frac{1}{m}) - P^B(r)$ satisfies $r^{BS} \leq r^*$, with equality only holding when seller participation is completely inelastic ($\varphi = 0$) or when both r^{BS} and r^* are boundary solutions. This means that, aside from these special cases, decreasing r below its equilibrium level benefits buyers. Together with Proposition 2, this also means decreasing r increases total user surplus $BS + mSS_i$.

The general reason for this result is that the rate at which platforms pass through their additional per-buyer revenue $\frac{1}{s_i}R_i$ from a higher r into a lower per-buyer price P^B is less than one in magnitude.[21] This standard incomplete pass-through logic arises in our *leading example* because seller participation ends up decreasing in response to a higher buyer-side price.[22]

The misalignment with buyer surplus is amplified if buyers are myopic and do not fully account for their true post-participation utility U_i when deciding which platform to join (a result also highlighted by Etro (2023)). In this case, platforms will underweight U_i relative to true buyer surplus in their equilibrium choice of commissions. Indeed, the same is true with respect to the true welfare standard, leading to an additional upward distortion in the choice of r^* above the welfare benchmark. We formally show this in the Online Appendix D.2, which proves that $r^* \geq r^{SE}$ and $r^* \geq r^{BS}$ when buyers are myopic.

□ **Comparison with Armstrong (2006)'s conditional equivalence result**. Armstrong's competitive bottleneck model is a special case of our framework with $m = 2$ and a_i corresponding to a lump-sum fee charged to sellers. His Proposition 4 also shows the equivalence of the equilibrium a^* and the seller-excluded benchmark a^{SE}. An important difference is that Armstrong's result is established by conditioning each platform's buyer-side market share at some exogenously fixed level s_i^{exog} when defining a^* and a^{SE}; by contrast, in our Proposition 2 we follow a standard second-best welfare analysis and solve for the endogenous equilibrium buyer-side market share when defining a^* and a^{SE}. Note, though, in our baseline setting with full coverage and symmetric platforms, the two approaches lead to the same result when Armstrong's market shares s_i^{exog} are fixed at $1/2$. In Section 4, we show that our equivalence result does not generally hold when considering asymmetric platforms or an incompletely covered market, which is in contrast to Armstrong's conditional equivalence result that still holds in such settings. Hence, in more general settings, our approach allows us to identify additional welfare distortions (beyond the baseline distortion identified in Proposition 2) that are shut down under Armstrong's approach.

3.2 Effects of entry

Proposition 2 shows that, in our baseline setting, platforms set an excessive level of the instrument a^*, irrespective of the number of platforms $m \geq 2$. Following Tan and Zhou (2021), a natural question is whether introducing platform entry (an increase in m) amplifies or mitigates this baseline distortion. To explore this question, in this subsection we impose the following additional assumptions on the form of U_i and R_i:

[21] In general, verifying this pass-through property requires a comparison between the magnitude of pass-through from commissions to buyer-side prices (dP^B/dr) and the magnitude of pass-through from commissions to platform transaction revenue (dR_i/dr).

[22] Our result is different from those obtained by Etro (2023) and Jeon and Rey (2024), whereby this pass-through rate always equals one and so buyers do not benefit from a decrease in r from the equilibrium level (in their respective baseline models). This difference is due to their sequential participation timing assumptions: in these papers, platform i takes seller participation as given when choosing its buyer-side price P_i^B, and so the pass-through rate of one is implied by the fully covered buyer-side market.

$$\begin{aligned} U_i(a_i;s_i) &= v(a_i)G(\pi(a_i)s_i) \\ R_i(a_i;s_i) &= w(a_i)s_iG(\pi(a_i)s_i) \end{aligned}, \tag{15}$$

which nests our *leading example* as a special case.

This setup reflects two core assumptions: (i) platforms' residual profit R_i is transaction-based; and (ii) there is a continuum of sellers deciding whether to join each platform independently. Here, $v(a_i) \geq 0$ is a buyer's net benefit from interacting with *each* seller on platform i (incorporating any seller pricing response to fees); $\pi(a_i) \geq 0$ is a seller's profit from interacting with *each* buyer on platform i; $w(a_i) \geq 0$ is the platform's revenue for each buyer-seller interaction; $G(\cdot)$ is the CDF of seller participation cost $k_i \in [0, k_{\max}]$, which is incurred on a per-platform basis.

We assume G is weakly log-concave and its density function is denoted as $g(\cdot)$. It is useful to define seller participation elasticity as

$$\varphi(k) \equiv \frac{kg(k)}{G(k)} \geq 0.$$

We assume that $v(a_i)$, $w(a_i)$, and $\pi(a_i)$ are twice continuously differentiable. Lastly, we assume $\pi(a_i)$ is strictly decreasing in a_i, consistent with our baseline assumption that a higher a_i corresponds to a lower seller surplus.

To fix ideas, we interpret a_i as the platform's transaction-based fee (which can be per-unit or ad-valorem). However, (15) can also capture other platform instruments and microfoundations such as those discussed at the end of Section 2.1.

Given (15), Proposition 1 implies that the equilibrium instrument is

$$a^* = \arg\max_{a_i \in \mathcal{A}} \left\{ \frac{v(a_i) + w(a_i)}{m} G(\frac{\pi(a^*)}{m}) \right\}.$$

We then obtain the following comparative statics for platform entry:

Proposition 3 *Suppose we specialize Proposition 1 to the functional form in (15).*

- *If seller participation elasticity $\varphi(\cdot)$ is constant, a^* does not change with the number of platforms m.*
- *If seller participation elasticity $\varphi(\cdot)$ is decreasing (increasing), a^* weakly decreases (increases) with m, strictly so if a^* is interior.*

Interpreting Proposition 3 in the *leading example* (where platforms set commissions), the first result says that if seller participation elasticity is constant, the equilibrium commission level remains unchanged as the number of platforms increases. More platforms lower commissions only if seller participation elasticity is decreasing. This differs from the standard notion of entry lowering equilibrium prices. In competitive bottleneck settings, platforms do not compete for sellers given each seller's decision to join a platform i is made independently of its decisions to join other platforms. Hence, a higher m does not intensify platform competition for sellers.

The intuition for the remaining results in Proposition 3 is easiest to see if we state the equilibrium a^* in terms of the first-order condition:

$$\underbrace{\left(\frac{\partial v}{\partial a_i}+\frac{\partial w}{\partial a_i}\right)\frac{G(\pi(a_i)/m)}{g(\pi(a_i)/m)}}_{\text{additional revenue from inframarginal sellers}} \quad + \underbrace{(v(a^*)+w(a^*))\frac{1}{m}\frac{\partial \pi}{\partial a_i}}_{\text{loss in seller participation}} = 0. \tag{16}$$

When m increases, the equilibrium mass of buyers $1/m$ on each platform decreases, which has two effects on platforms' incentives in (16). First, it dilutes the seller participation profit $\pi(a_i)/m$ on each platform, and the reduced mass of inframarginal sellers implies a smaller revenue gain from raising commissions. Second, at the same time, it makes seller participation profit $\pi(a_i)/m$ less sensitive to changes in a_i, which implies a smaller participation loss from raising commissions. Proposition 3 says that the downward (upward) effect on commissions dominates when seller participation, as described by the CDF function G, has a decreasing (increasing) elasticity. Both of these possibilities can be consistent with log-concave G.[23]

In standard applications, platform transaction fees create deadweight losses in the post-participation transaction surplus generated on the platforms. In our current context, this feature can be captured by assuming that the sum $v(a_i)+w(a_i)+\pi(a_i)$ is decreasing in a_i. Under this assumption, $W(a_i)$ is monotonically decreasing in a_i, and so the welfare-maximizing fee is $a^W=0$ (assuming the lowerbound is zero) and thus is independent of m. This was true in our *leading example*. As a result, Proposition 3 also applies to the gap a^*-a^W, which can increase or decrease in m, depending on the properties of $\varphi(\cdot)$.

To supplement the analysis above, we also consider how platform entry affects the surplus of buyers and sellers. We write

$$\begin{aligned} mSS_i &= m\int_0^{\frac{\pi(a^*)}{m}}[\frac{\pi(a^*)}{m}-k_i]dG \\ BS &= \mathbb{E}[\max_{i=1,\dots,m}\{\epsilon_i\}]+v(a^*)G(\frac{\pi(a^*)}{m})-P^B(a), \end{aligned}$$

where $\mathbb{E}[\max_{i=1,\dots,m}\{\epsilon_i\}]$ is the expected match value of buyers. To sharpen the exposition, the formal result below focuses on constant seller participation elasticity φ.

Corollary 1 *Continue from Proposition 3 and suppose seller participation elasticity φ is constant.*

- *Total seller surplus mSS_i always decreases with the number of platforms m.*
- *Suppose that φ is sufficiently small and that match values $\{\epsilon_i\}_{i=1,\dots,m}$ are IID with a log-concave density function, then total buyer surplus BS increases with m.*

By Proposition 3, a^* does not change with the number of platforms. Therefore, the decrease in seller surplus as m increases is due to the same total pool of buyers becoming more fragmented

[23] One instrument not covered by (15) is a seller-side lump-sum fee $a_i=P_i^S$ as in Armstrong (2006). That corresponds to $U_i=vG(\pi s_i-a_i)$ and $R_i=a_iG(\pi s_i-a_i)$, with the interaction benefits of buyers and sellers, v and π, independent of a_i. Online Appendix D shows that the effect of m on a^* again depends on the properties of φ.

across an increased number of platforms. This fragmentation leads to: (i) sellers incurring additional per-platform entry costs to access buyers, and (ii) reduced seller participation on existing platforms, thus harming sellers. Buyers are similarly harmed by the reduction in sellers on their chosen platform, but they gain from increased platform variety ($\mathbb{E}[\max_{i=1,\ldots,m}\{\epsilon_i\}]$) and a potentially lower buyer-side price P^{B*}. When seller participation elasticity φ is small, the former harm is dominated and so BS increases. Therefore, platform entry impacts buyers and sellers in opposing directions, meaning that the implications for total user surplus $BS + mSS_i$ and total welfare are ambiguous.

4 New competitive bottleneck welfare distortions

In this section, we examine new welfare distortions that can arise in competitive bottleneck settings (in which sellers still make independent participation and post-participation decisions across platforms) once we relax key buyer-side assumptions from the baseline model. Readers primarily interested in how cross-platform spillovers (which can make sellers' decisions interdependent across platforms) lead to new welfare distortions can skip directly to Section 5.

To keep things tight, we return to our *leading example*, showing how generalizing buyer-side payoffs introduces new forces that break the equivalence result (Proposition 2) in one direction or the other. The results that follow focus on comparing a^* with a^{SE}. Since Lemma 1 still applies ($a^{SE} \geq a^W$), the results indicate how the baseline welfare distortion is either amplified (when $a^* > a^{SE}$) or mitigated (when $a^* < a^{SE}$). The full details of the analysis, along with more general versions of the results (where applicable), are contained in Online Appendix E.

4.1 Heterogeneous interaction benefits

When buyers have heterogeneous gross utility U_i, a new welfare distortion can arise that either amplifies or mitigates the baseline distortion, depending on the nature of the correlation between buyers' horizontal preference (across platforms) and vertical preference (for interaction benefits).[24]

To proceed, we augment our *leading example* with a mass-λ of "loyal buyers", each of whom has a preferred platform that is drawn with an equal probability across the m platforms (so each platform has λ/m of loyal buyers). Neither sellers nor platforms can price-discriminate between loyal and regular buyers. A buyer loyal to platform i only compares the surplus $b + U_i - P_i^B$ on platform i and the outside option (valued at zero).[25] Regular and loyal buyers can vary in how much they want to buy from sellers. A buyer of type $\tau \in \{reg, loyal\}$ chooses how many units to purchase q to solve $\arg\max_q \{u_\tau(q) - p_i q\}$. Let $D_\tau(p_i)$ be the resulting per-buyer demand. To avoid the complicated second-order conditions discussed in Online Appendix E.1 for the more general case, here we impose the following linear-quadratic specification:

$$u_\tau(q) = Vq - \frac{1}{2\theta_\tau}q^2, \text{ such that } D_\tau(p_i) = \theta_\tau(V - p_i).$$

[24] See also Anderson and Bedre-Defolie (2024), who provide welfare results for a monopoly platform that charges a device price to the buyer side and commissions to the seller side, in which sellers differ in their quality and buyers have heterogeneous preferences for quality.

[25] Formally, the idiosyncratic match value of such a buyer is $\epsilon_i = b$ on platform i and $\epsilon_i = -\infty$ otherwise.

We assume b is high enough that loyal buyers always participate (which implies only regular buyers are marginal from a given platform's perspective), and λ is small enough so second-order conditions for platform profit maximization hold.

Proposition 4 *Consider the above model of heterogeneous buyers. Suppose $\theta_{loyal} > (<)\theta_{reg}$ so that loyal buyers correspond to a higher (lower) interaction value than regular consumers. Then $r^* \geq (\leq) r^{SE}$, strictly so for interior solutions.*

The result has a "Spence distortion" flavor: platforms focus on marginal buyers (regular buyers), while a planner aiming to maximize seller-excluded welfare focuses on average buyers (including loyal buyers). If loyal buyers have higher interaction values ($\theta_{loyal} > \theta_{reg}$) so that there is a positive correlation between the horizontal and vertical preference of buyers, then the usual Spence logic implies there is an added upward distortion to equilibrium commissions. On the other hand, if loyal buyers have lower interaction values ($\theta_{loyal} < \theta_{reg}$) so that there is a negative correlation between the horizontal and vertical preference of buyers, then we get an inverse Spence logic with r^* lower than r^{SE}, which mitigates the upward distortion in equilibrium commissions. Finally, if $\theta_{loyal} = \theta_{reg}$ so that regulars and loyals have equal interaction benefits, there is no correlation between the horizontal and vertical preference of buyers, and we get the usual equivalence result of Proposition 2.

4.2 Partial market coverage

When the buyer-side market is not fully covered, welfare objectives must account for how changes in the commission level influence total market coverage. This creates a new distortion that further amplifies the baseline distortion of excessive equilibrium commission levels.

To show this, we modify our *leading example with Hotelling competition* by adding hinterlands to the standard Hotelling model. In addition to the measure one of regular buyers, each platform $i \in \{1, 2\}$ has a measure L of loyal buyers located a distance x away from i who are only interested in purchasing from platform i, where $x \sim U[0, L]$.[26] A loyal buyer's utility is

$$b + U_i - P_i^B - t_L x.$$

We assume L is large enough, so some loyal buyers choose the outside option. We assume $\varphi = 1$ (i.e., G is linear) so that the model can be solved explicitly, and t and t_L are high enough for market shares to be uniquely defined and the second-order condition for platform profit maximization to hold.

Proposition 5 *Consider the above model of two platforms with partial market coverage. Then $r^* \geq r^{SE}$, strictly so for interior solutions.*

To understand Proposition 5, note that similar to the usual monopoly deadweight loss logic with elastic demand, here too the planner wants to increase the measure of loyal buyers joining

[26] As in Section 4.1, sellers and platforms cannot discriminate between the two types of buyers.

each platform. The less obvious part of the result is that this is achieved by decreasing commissions r below the equilibrium level r^*.

To explain this latter part of the result, note the effect of a decrease in r is to increase the user-interaction benefit U_i (through more sellers participating and through lower prices if commissions are passed through in prices). When buyer demand increases via higher interaction benefits, the platforms will raise their buyer-side prices, a feedback effect which will tend to limit the increase in buyer participation. Without any regular buyers, it is easy to check that this feedback effect fully offsets the increase in U_i, and a decrease in r does not change the measure of loyal buyers participating, so that the equivalence result in Proposition 2 still holds. However, competition for the regular buyers constrains any such price increase, and ensures that a decrease in commissions does indeed lead to more loyal buyers participating.

4.3 Asymmetric platforms

With asymmetric platforms, welfare objectives must also consider how changes in commissions affect each platform's market share.[27] As we will show, accounting for this leads to an additional upward distortion in the setting of commissions.

We modify our *leading example with Hotelling competition* by adding $\beta > 0$ to ϵ_1 so that the idiosyncratic match value of platform 1 is shifted up by β. Although platform 1 obtains a larger market share as a result, the determination of r^* in (12) remains unchanged, and in particular is still equal across platforms. For this reason, we focus on what happens when the planner changes this common commission level. We assume $\varphi = 1$ (i.e., G is linear) so that the model can be solved explicitly, and t is high enough for market shares to be uniquely defined and the second-order condition for platform profit maximization to hold.

Proposition 6 *Consider the above model of two asymmetric platforms. If buyers get an additional benefit of $\beta > 0$ on platform 1, then $r^* \geq r^{SE}$, strictly so for interior solutions.*

Proposition 6 provides another reason why a competitive bottleneck setting leads commissions to be set too high. The logic involves two steps. First, starting from r^*, seller-excluded welfare W^{SE} can be increased by changing the common commission r to increase platform 1's market share s_1^*.[28] Second, a decrease in r increases s_1^*. To see this latter result, note a decrease in r has two positive effects on buyer surplus on each platform: (i) an increase in seller participation (due to an increase in seller-profit *per buyer*), and (ii) an increase in buyer-surplus *per seller* (if the decrease in r is passed through into a decrease in seller prices). Both of these positive effects are larger on platform 1 which has more buyers and sellers. Thus, a decrease in r will shift buyers onto platform 1 from platform 2. As demand for platform 1 increases, platform 1 responds by increasing P_1^B relative to P_2^B, a feedback effect which we show offsets some of the initial shift in buyers towards it but never dominates it.

[27] Peitz and Sato (forthcoming) adopt an aggregative game approach to model asymmetric oligopoly platform competition in a setting without multihoming.

[28] This is the standard logic of differentiated competition, that too few buyers purchase from the more favored firm because it sets its price higher than that of the less favored firm.

5 Beyond competitive bottlenecks: cross-platform spillovers

So far we have highlighted the welfare distortions that arise in competitive bottleneck settings. In this section, we identify additional welfare distortions that arise due to "cross-platform spillovers". As we will see, such spillovers typically mean sellers no longer make independent decisions across platforms — a key feature of competitive bottleneck settings — thus, breaking the equivalence result in Proposition 2.

Throughout this section, we write the gross utility function as $U_i(\boldsymbol{a}; \boldsymbol{s})$ and the platform's residual profit function as $R_i(\boldsymbol{a}, \boldsymbol{s})$, both of which can depend on the profile of platform instruments $\boldsymbol{a} = (a_1, ..., a_m) \in \mathcal{A}^m$ and the profile of buyer-side market share $\boldsymbol{s} = (s_1, ..., s_m) \in [0, 1]^m$ due to spillovers. We define cross-platform spillovers as:

- **Negative (Positive) cross-platform spillovers:** For all platforms i, $U_i(\boldsymbol{a}; \boldsymbol{s})$ and $R_i(\boldsymbol{a}, \boldsymbol{s})$ are weakly decreasing (increasing) in each rival platform j's instrument a_j (for every $j \neq i$), holding $\boldsymbol{s}$ constant.

In the proof of the next proposition, we first characterize the equilibrium outcome in the presence of spillovers based on the same technique that establishes Proposition 1. In any symmetric equilibrium, each platform chooses

$$a^* = \arg\max_{a_i \in \mathcal{A}} \left\{ \frac{1}{m} U_i(\hat{\boldsymbol{a}}; \frac{1}{m}\mathbf{1}) - \frac{1}{m} U_j(\hat{\boldsymbol{a}}; \frac{1}{m}\mathbf{1}) + R_i(\hat{\boldsymbol{a}}; \frac{1}{m}\mathbf{1}) \right\}, \tag{17}$$

where $\hat{\boldsymbol{a}} = (a_i, a^*, ..., a^*)$ and $\mathbf{1} = (1, ..., 1)$ is a $1 \times m$ vector of ones. Compared to the equilibrium in Proposition 1, each platform i now takes into account how its choice of a_i affects the gross utility offered by rival platforms $j \neq i$, and so its relative attractiveness to buyers. We then compare the equilibrium outcomes with the unique seller-excluded benchmark.[29]

Proposition 7 *Suppose the negative (positive) cross-platform spillovers condition holds. Then, in any symmetric equilibrium,* $a^* \geq (\leq)\, a^{SE}$.

Given that Lemma 1 continues to hold in the presence of cross-platform spillovers, Proposition 7 says that negative spillovers create an additional distortion (in the same direction) to the baseline distortion result in Proposition 2. Meanwhile, positive spillovers mitigate the distortions identified in Proposition 2, leading to ambiguous welfare implications in this case.

There are three possible channels by which distortions arise in Proposition 7, which we illustrate in the case of negative spillovers. First, each platform i does not internalize the negative spillover on the utility buyers get from other platforms when it increases a_i, so a^* is set too high (from a welfare perspective). Second, each platform i wants to exploit the negative spillover to lower the utility buyers obtain on rival platforms, so a^* is set too high. Third, each platform i does not internalize the negative spillover on the revenue other platforms obtain when it increases a_i, so a^* is set too high.

[29] If negative or positive spillovers are strict, the inequalities in Proposition 7 are also strict, as long as a_i is a continuous variable with an interior equilibrium.

We illustrate Proposition 7 through three distinct types of seller-driven cross-platform spillovers. Each departs from the competitive bottleneck setting, where sellers previously made independent decisions across platforms.[30] These are: (i) spillovers from seller participation; (ii) spillovers when sellers are restricted to singlehome; and (iii) spillovers from sellers' post-participation decisions. Throughout, we adapt the *leading example* to illustrate each type, but similar spillovers can easily be incorporated into the other examples noted in Section 2.1. All omitted derivation details are given in Online Appendix F.

5.1 Spillovers from seller participation

Cross-platform spillovers naturally arise when sellers face economies of scale or scope in their participation decisions. One key example is when sellers only need to incur participation costs once (e.g., for product or app development) in order to sell on all platforms, as in Jeon and Rey (2024). In this case, a seller's participation decision depends on the total post-participation profit it earns on all platforms, so that any instrument that decreases the seller's net profit on one platform (e.g., a platform's fees) generates negative cross-platform spillovers. More generally, a negative spillover arises whenever a seller's marginal participation cost of joining an additional platform decreases in the number of platforms already joined.

To formalize this within our *leading example*, assume fixed participation costs are perfectly correlated across platforms and only incurred once, so a seller has no additional fixed cost to participate on additional platforms once it has participated on one platform. Let $k = k_i$ for all $i = 1, ..., m$, where k follows the CDF G. Thus, in equilibrium, a type-k seller either joins no platforms or all platforms. The latter occurs if and only if

$$k \leq \sum_{i=1}^{m} s_i \pi(r_i).$$

Since the post-participation behavior of sellers remains the same, we continue from (4) to get

$$\begin{aligned} U_i &= v(r_i) G\left(\sum_{i=1}^{m} s_i \pi(r_i)\right) \\ R_i &= r_i p(r_i) q(r_i) s_i G\left(\sum_{i=1}^{m} s_i \pi(r_i)\right). \end{aligned}$$

Holding $(s_1, .., s_m)$ fixed, both U_i and R_i are decreasing in r_j for $j \neq i$ given $\pi(\cdot)$ is a decreasing function. By Proposition 7, this implies $r^* \geq r^{SE} \geq r^W$, with strict inequalities for interior solutions. Thus, within-seller economies of scale in seller participation yield negative spillovers in platform commissions, creating an additional upward distortion in equilibrium commission rates.

5.2 Spillovers from seller singlehoming

Positive spillovers arise when sellers treat platforms as substitutes. This occurs if sellers face frictions that prevent multihoming (e.g., contract exclusivity) or are otherwise incentivized to singlehome (e.g., market share discounts), so that their only effective choice is between participating

[30] Spillovers can also arise from platform decisions or via the buyer side without breaking the independence of sellers' decisions across platforms. For example, a successful platform investment that is imitated by other platforms may make all platforms more attractive to buyers. Anderson and Peitz (2023) provides another example.

on one platform or none at all. In such cases, any increase in a_i on platform i can induce them to substitute to other platforms, thereby generating positive spillovers.[31]

To illustrate, we modify our *leading example* by restricting each seller to join at most one platform. Let n_i be the mass of participating sellers on platform i:

$$n_i = \Pr\left(\pi(r_i)s_i - k_i \geq \max_{j\neq i}\{\pi(r_j)s_j - k_j, 0\}\right).$$

Then, from (4) we have:

$$U_i = v(r_i)n_i$$
$$R_i = r_i p(r_i)q(r_i)s_i n_i,$$

where n_i depends on the full profiles $(r_1, .., r_n)$ and $(s_1, .., s_m)$. Holding $(s_1, .., s_m)$ constant, n_i increases in r_j for $j \neq i$, implying positive spillovers. Proposition 7 then implies $r^{SE} \geq r^*$, with equality holding only when r^* and r^{SE} coincide at the same corner solution. Meanwhile, by the standard deadweight loss logic, the welfare-maximizing commission is $r^W = 0$.

In Online Appendix F.2, we provide closed-form solutions for r^* and r^{SE} assuming the seller-side market is fully covered. We show conditions for positive spillovers to mitigate the baseline distortion partially ($r^{SE} > r^* > r^W$) and completely ($r^{SE} > r^* = r^W$). In particular, the latter occurs when heterogeneity in sellers' idiosyncratic draws of participation costs $(k_1, ..., k_m)$ is low. In sum, restricting sellers to singlehome gives rise to positive spillovers in platform commissions, thus mitigating the tendency towards excessive commissions found in the baseline setting.

5.3 Spillovers from seller-side post-participation decisions

In this section we consider different ways seller-side post-participation decisions can lead to spillovers. One set of mechanisms operates through seller pricing. For example, suppose each seller benefits from a positive product-level network effect, as in online multiplayer games, dating platforms, and social networks. If a higher fee on platform i leads a seller to raise its price there, its demand on platform i falls. Due to within-seller network effects, this reduction also lowers the seller's demand on other platforms, resulting in negative cross-platform spillovers.

Another spillover via seller pricing arises in case some sellers adopt uniform pricing across all the channels they sell on (called price coherence in Edelman and Wright (2015)).[32] We modify our *leading example* so that each product category has probability $\omega > 0$ of being subject to price coherence. To focus on sellers' post-participation decisions, we assume that all sellers have zero participation costs $k_i = 0$ (i.e., distribution G is degenerate), so they will always join at least one of the platforms.

A seller subject to price coherence that multihomes on a subset $\phi \subseteq \{1, 2, ..., m\}$ of at least two platforms faces an effective commission of

$$r^{avg} = \frac{\sum_{i\in\phi} s_i r_i}{\sum_{i\in\phi} s_i}.$$

[31] This mechanism does not arise if sellers *voluntarily* choose to singlehome due to it only being profitable to join a single platform, something we allowed for in the baseline model.

[32] Price coherence can be the result of price-parity contracts imposed by platforms, or some more behavioral-type factors on the part of buyers that leads to the same outcome.

Using the same notation as in the *leading example*, such sellers set their uniform price at $p(r^{avg})$ and obtain a per-buyer profit of $\pi(r^{avg})$ on each platform. The corresponding transaction quantity and buyer surplus are $q(r^{avg})$ and $v(r^{avg})$. All such sellers will multihome on all platforms as long as the difference in commission levels $\max_{i,j} |r_i - r_j|$ is not too large. Moreover, no platform has an incentive to deviate and induce large commission differences if ω is small enough. Combining the fraction ω of sellers subject to price coherence with the $1-\omega$ who are not, gives:

$$U_i = \omega v(r^{avg}) + (1-\omega)v(r_i)$$
$$R_i = r_i(\omega p(r^{avg})q(r^{avg}) + (1-\omega)p(r_i)q(r_i))s_i.$$

Observe that provided $c > 0$, higher commissions imply higher seller prices, and lower q and v. In this case, an increase in r_j for any platform $j \neq i$ decreases U_i (given $v(r^{avg})$ decreases) and R_i (given it can be shown that $p(r^{avg})q(r^{avg})$ must decrease). Thus, price coherence gives rise to negative spillovers in platform commissions, and we conclude from Proposition 7 that $r^* \geq r^{SE} \geq r^W$.

Cross-platform spillovers can also arise from sellers' post-participation investments and marketing decisions that affect their demand across all platforms. In Online Appendix F, we explore three such examples, which we summarize here:

☐ **Seller investment that applies to all platforms.** If sellers invest in broad marketing or quality improvements that apply across all platforms they sell on, an increase in transaction-based fees on any platform will lower the return on such investments, resulting in less investment and lower demand across all platforms — another negative spillover in fees.

☐ **Platform and seller investment.** When we add platform investment that is either complementary or substitutable to sellers' investments, cross-platform spillovers in platform investment arise. When investments are complementary, if one platform curtails its own investment this causes sellers to reduce their investments causing negative spillovers for the rival platforms that these sellers operate on. Conversely, when investments are substitutable, a platform's decision to curtail investment prompts sellers to invest more, creating positive spillovers for rival platforms.

☐ **Promotion of sellers' direct channel.** An increase in one platform's transaction fees can prompt sellers to steer buyers more aggressively towards their direct channel, potentially via advertising it more. This can also generate negative spillovers for rival platforms: as more buyers become aware of sellers' direct channels, rival platforms lose sales volume as well. A spillover also arises when platforms can invest in disintermediation-prevention efforts. The more a platform tries to stop its sellers informing their buyers of their direct channel through the platform, the more sellers will want to promote their direct channels themselves. This again shifts transactions off rival platforms onto sellers' direct channel and creates a negative cross-platform spillover.

6 Policy implications and conclusions

We have developed a general framework for studying platforms that compete to attract buyers on one side, while acting as gatekeepers with respect to sellers seeking to access their unique buyers on the other side. We provided a wide ranging analysis of the resulting welfare distortions. To conclude, we illustrate several policy implications of our analysis through the lens of mobile app platforms, where Apple (iOS and the App Store) and Google (Android and the Play Store) dominate in most major OECD countries.

Regardless of how much competition there is for consumers (buyers), our baseline findings indicate that platform commissions and other design choices — such as first-party entry, self-preferencing, and restrictions on disintermediation — are set at excessive levels from a total welfare perspective. This shifts surplus from app developers (sellers) to platforms and, in part, to consumers. We also show that even when policymakers focus on other objectives — either consumer surplus only or the total surplus of customers on both sides (consumers and developers) — platform commissions remain excessive.

Applying the features of mobile app platforms to the additional results in Sections 4 and 5 further reinforces our baseline welfare distortions. This includes that the correlation between buyers' horizontal preference across mobile platforms and vertical preference for interaction benefits on apps is likely positive, and that cross-platform spillovers are likely negative given economies of scale in app development, app developers' non-platform-specific investment in the quality and marketing of their apps, complementarity between app developer and platform investments, network effects for certain apps, and the potential for price coherence.

Our results suggest that policies focused solely on increasing competition for consumers (e.g., reducing the costs of switching operating systems) may not help address the underlying distortions. Similarly, policies that induce entry by mobile platforms will not necessarily address the underlying distortions. We show that adding more platforms can increase or decrease equilibrium commissions depending on whether developer participation elasticity is increasing or decreasing.

Given these findings, one possible policy response is to regulate platform fees. Recent studies show how such regulation can improve outcomes in the context of a monopoly platform (Gomes and Mantovani, 2025; Bisceglia and Tirole, 2024; Wang and Wright, 2025). Yet, while fee regulation may increase welfare if designed appropriately, our analysis highlights that distortions extend beyond platform fees. Lowering fees alone does not address other sources of distortion inherent in platform design and behavior.

This suggests a more effective approach may be to remove the source of the bottleneck problem. This can be done in two ways. One way is by promoting consumer multihoming — lowering the cost for consumers to use multiple platforms — thereby providing developers multiple channels to reach the same users. Preventing platforms from imposing barriers to users' multihoming would support this goal (Athey and Morton, 2022). However, sometimes there are inherent costs for consumers to multihome (e.g., buying a second mobile device), so achieving significant multihoming at the device level on the consumer side may be unrealistic.

Alternatively, or in addition, policymakers can ensure developers are not denied other ways to reach and transact with a platform's users. This could include forbidding platforms from banning or limiting alternative app stores or direct app downloads, and allowing developers to steer users toward cheaper alternatives stores or their own websites. Notably, Articles 5(4), 5(5), 5(7), and 6(4) of the European Union's Digital Markets Act, which came into force in 2024, impose exactly these types of obligations on Apple and Google.

While we have focused on the application to mobile app platforms here, future work could apply our framework to other sectors. For instance, media platforms and prioritized internet service providers — which may exhibit different platform instruments and spillovers — can be usefully analyzed using our general framework and results.

7 Appendix

Proof. (Proposition 1). The derivation of a^* in (9) is described in the text. It remains to establish (10). In equilibrium, all other platforms $j \neq i$ set P^{B*} and a^*. The derivative of the deviating platform i's profit function in (8) with respect to s_i is

$$\begin{aligned}\frac{d\Pi_i}{ds_i} &= \left(\frac{\partial U_i(a^*;s_i)}{\partial s_i} - \frac{1}{m-1}\frac{\partial U_j(a^*;\frac{1-s_i}{m-1})}{\partial s_i} - \frac{1}{\Phi'}\right)s_i + \frac{\partial R_i(a_i;s_i)}{\partial s_i} \\ &\quad + \left(P^{B*} + U_i(a_i;s_i) - U_j(a^*;\frac{1-s_i}{m-1}) - \Phi^{-1}(s_i)\right).\end{aligned}$$

Imposing symmetry and using $\Phi^{-1}(1/m) = 0$, the first-order condition implies

$$P^{B*} + \left(\frac{m}{m-1}\frac{\partial U_i(a^*;\frac{1}{m})}{\partial s_i} - \frac{1}{\Phi'(0)}\right)\frac{1}{m} + \frac{\partial R_i(a^*;\frac{1}{m})}{\partial s_i} = 0,$$

which can be rearranged to give (10). To establish uniqueness of the symmetric equilibrium, we note that the only symmetric demand profile is $s_i = 1/m$; this observation and the strict quasiconcavity assumption imply that the single-variable maximization problem in (9) has a unique solution a^*, which then imply the uniqueness of P^{B*} in (10). ■

Proof. (Lemma 1). By contradiction, suppose $a^{SE} < a^W$, which implies $W(a^{SE}) = W^{SE}(a^{SE}) + mSS_i(a^{SE}) > W(a^W)$ by the definition of a^{SE}, thus violating the definition of a^W. ■

Proof. (Proposition 3). We utilize the technique of monotone comparative statics. Dropping the redundant multiplicative coefficient, we express the objective function of a^* as

$$H(a_i;m) = (v(a_i) + w(a_i))G\left(\frac{\pi(a_i)}{m}\right),$$

such that $a^* = \arg\max_{a_i \in \mathcal{A}} H(a_i;m)$. Its derivative is

$$\begin{aligned}\frac{dH(a_i;m)}{da_i} &= \left(\frac{\partial v}{\partial a_i} + \frac{\partial w}{\partial a_i}\right)G\left(\frac{\pi(a_i)}{m}\right) + \frac{v(a_i)+w(a_i)}{m}\frac{\partial \pi}{\partial a_i}g\left(\frac{\pi(a_i)}{m}\right) \\ &= \left[\frac{\partial v}{\partial a_i} + \frac{\partial w}{\partial a_i} + \underbrace{\frac{v(a_i)+w(a_i)}{\pi(a_i)}\frac{\partial \pi}{\partial a_i}}_{<0}\varphi\left(\frac{\pi(a_i)}{m}\right)\right] \times G\left(\frac{\pi(a_i)}{m}\right). \qquad (18)\end{aligned}$$

When $\varphi(\cdot) > 0$ is constant, then the sign of $\frac{dH(a_i;m)}{da_i}$ is independent of m, and so the maximizer is independent of m. When $\varphi(\cdot)$ is increasing, then $\frac{dH(a_i;m)}{da_i}$ is single-crossing in m: for any m' and m'' such that $m'' > m'$, we have

$$\frac{dH(a_i;m')}{da_i} \geq 0 \implies \frac{dH(a_i;m'')}{da_i} > 0.$$

Hence, the family of functions $\{H(a_i;m)\}_{a_i \in \mathcal{A}, m \geq 2}$ obey single crossing differences, and so its set of maximizers $\mathcal{A}^*$ is increasing in m in terms of strong set order as defined by Milgrom and Shannon (1994). In the special case of $\mathcal{A}^*$ being degenerate, this implies a^* is weakly increasing in m, and strictly so if a^* is an interior solution pinned down by the first-order condition. When $\varphi(\cdot)$ is decreasing, the analogous proof applies. ■

Proof. (Corollary 1). With a constant φ, a^* does not change with m by Proposition 3. So,

$$\begin{aligned} mSS_i &= \int_0^{\frac{\pi(a^*)}{m}} [\pi(a^*) - mk_i]dG \\ \frac{d}{dm}(mSS_i) &= -\int_0^{\frac{\pi(a^*)}{m}} k_i dG < 0. \end{aligned}$$

For buyer surplus, we first specialize Proposition 1 to this setting:

$$P^{B*} = \frac{1}{m\Phi'(0)} - \left(\frac{m\varphi}{m-1}u(a^*) + (1+\varphi)w(a^*)\right)\left(\frac{\pi(a^*)}{mk_{\max}}\right)^{\varphi}$$

Its derivative is

$$\frac{dP^{B*}}{dm} = \frac{d}{dm}\left(\frac{1}{m\Phi'(0)}\right) + \underbrace{\left(\frac{m\varphi - \varphi + 1}{(m-1)^2}u(a^*) + \frac{1+\varphi}{m}w(a^*)\right)\left(\frac{\pi(a^*)}{mk_{\max}}\right)^{\varphi}\varphi}_{=0 \text{ if } \varphi \to 0}.$$

By the standard results of discrete-choice demand systems, $\frac{d}{dm}\left(\frac{1}{m\Phi'(0)}\right) \leq 0$ because $\{\epsilon_i\}_{i=1,\ldots,m}$ are IID with a log-concave densitiy (see, e.g., Tan and Zhou (2021)). Therefore,

$$\begin{aligned} BS &= \mathbf{E}[\max_{i=1,\ldots,m}\{\epsilon_i\}] - P^{B*} + u(a^*)\left(\frac{\pi(a^*)}{mk_{\max}}\right)^{\varphi} \\ \frac{d}{dm}BS &= \underbrace{\frac{d}{dm}\mathbf{E}[\max_{i=1,\ldots,m}\{\epsilon_i\}]}_{>0} - \underbrace{\frac{dP^{B*}}{dm}}_{\leq 0 \text{ if } \varphi \to 0} - \underbrace{\frac{\varphi}{m}u(a^*)\left(\frac{\pi(a^*)}{mk_{\max}}\right)^{\varphi}}_{=0 \text{ if } \varphi \to 0}, \end{aligned}$$

and so $\frac{d}{dm}BS > 0$ if $\varphi \to 0$. By continuity, this holds for $\varphi > 0$ that are sufficiently small. ■

Proof. (**Proposition 7**). In Online Appendix F, we show that any symmetric equilibrium in this setting is similar to Proposition 1: all m platforms have the same market share $s^* = 1/m$ and set the same instrument a^* that satisfies the fixed-point relation

$$a^* \in \arg\max_{a_i \in \mathcal{A}} \left\{\frac{1}{m}U_i(\hat{\boldsymbol{a}}; \frac{1}{m}\mathbf{1}) - \frac{1}{m}U_j(\hat{\boldsymbol{a}}; \frac{1}{m}\mathbf{1}) + R_i(\hat{\boldsymbol{a}}; \frac{1}{m}\mathbf{1})\right\},$$

where $\hat{\boldsymbol{a}} = (a_i, a^*, ..., a^*)$ and $\mathbf{1} = (1, ..., 1)$ is a $1 \times m$ vector of ones, and the same buyer-side price

$$P^{B*} = \frac{1}{m\Phi'(0)} - \left(\frac{1}{m-1}\right)\left(\frac{\partial U_i(\boldsymbol{a}; \boldsymbol{s})}{\partial s_i} - \frac{\partial U_j(\boldsymbol{a}; \boldsymbol{s})}{\partial s_i}\right) - \left(\frac{\partial R_i(\boldsymbol{a}; \boldsymbol{s})}{\partial s_i} - \frac{\partial R_i(\boldsymbol{a}; \boldsymbol{s})}{\partial s_j}\right),$$

where the derivatives are evaluated at the symmetric outcome $(\boldsymbol{a}; \boldsymbol{s}) = (a^*\mathbf{1}; \frac{1}{m}\mathbf{1})$.

We are now ready to prove the statement of Proposition 7. We will focus on the case of negative cross-platform spillovers (the case of positive spillovers can be proven similarly). In what follows we omit the market share profile argument when expressing functions $U_i(\boldsymbol{a}; \boldsymbol{s})$ and $R_i(\boldsymbol{a}; \boldsymbol{s})$ given that we always set $\boldsymbol{s} = \frac{1}{m}\mathbf{1}$.

Denote $(a_i, a\mathbf{1}_{m-1}) \in \mathcal{A}^m$ as a profile such that platform i is choosing instrument $a_i \in \mathcal{A}$ while all other platforms $j \neq i$ are choosing the same instrument $a \in \mathcal{A}$. Denote auxiliary functions

$$\begin{aligned} Z(a_i; a) &\equiv U_i\left(a_i, a\mathbf{1}_{m-1}\right) - U_j\left(a_i, a\mathbf{1}_{m-1}\right) + mR_i\left(a_i, a\mathbf{1}_{m-1}\right) \\ Z_0(a_i; a) &\equiv U_j\left(a_i, a\mathbf{1}_{m-1}\right) + U_i(a_i, a_i\mathbf{1}_{m-1}) - U_i\left(a_i, a\mathbf{1}_{m-1}\right) + mR_i\left(a_i, a_i\mathbf{1}_{m-1}\right) - mR_i\left(a_i, a\mathbf{1}_{m-1}\right). \end{aligned}$$

In the presence of negative spillovers, function Z_0 has a convenient property:

$$a_i > a \Rightarrow Z_0(a_i; a) \leq U_j\left(a_i, a\mathbf{1}_{m-1}\right) \leq U_j\left(a, a\mathbf{1}_{m-1}\right) = Z_0(a; a). \tag{19}$$

Moreover, by definition, in any symmetric equilibrium

$$a^* \in \arg\max_{a_i \in \mathcal{A}} Z(a_i; a^*).$$

Meanwhile, our uniqueness assumption on the welfare solutions implies

$$a^{SE} = \arg\max_{a_i \in \mathcal{A}} \{U_i\left(a_i, a_i\mathbf{1}_{m-1}\right) + mR_i\left(a_i, a_i\mathbf{1}_{m-1}\right)\},$$

which is equivalent to

$$a^{SE} = \arg\max_{a_i \in \mathcal{A}} \{Z(a_i; a^*) + Z_0(a_i; a^*)\}, \tag{20}$$

where the equivalence can be verified by cancelling the redundant plus-minus terms in $Z(a_i; a^*) + Z_0(a_i; a^*)$. By contradiction, suppose there exists a symmetric equilibrium with $a^* < a^{SE}$. Then,

$$\begin{aligned} & Z(a^{SE}; a^*) + Z_0(a^{SE}; a^*) \\ \leq\ & Z(a^{SE}; a^*) + Z_0(a^*; a^*) \text{ by (19)} \\ \leq\ & Z(a^*; a^*) + Z_0(a^*; a^*) \text{ by the definition of } a^*, \end{aligned}$$

which contradicts the definition of a^{SE} being a unique maximizer to (20). Hence, we must have $a^* \geq a^{SE}$ in any symmetric equilibrium. ■

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Online Appendix: Competitive bottlenecks and platform spillovers

Tat-How Teh[1] and Julian Wright[2]

In the following sections we provide additional workings and results referred to but not included in the main paper.

A Other microfoundations

In this section, we first describe Examples 1 - 3 that are similar to the *leading example* in the main paper in that they assume a continuum of atomistic product categories, with a single (monopoly) seller in each. We will then describe Example 4, which presents a microfoundation with sellers that behave oligopolistically.

Examples 1 - 3 are written in a way to accommodate each platform i choosing a multi-dimensional instrument vector a_i. To recover the single-dimensional case, we can simply fix all but component of the vector for all platforms. Online Appendix B explains in our general baseline model how the analysis and results extend to this multi-dimensional setting.

As will be shown below, the functions U_i and R_i that correspond to Examples 1 - 3, also satisfy the special functional form (15) imposed in Section 3.2 when holding all but one of the multi-dimensional instruments as fixed. Therefore, the entry results (Proposition 3 and Corollary 1) hold for these examples, as claimed in Section 3.2.

In Examples 1 - 3, we impose a simplifying assumption of $c = 0$. This means that seller's optimal price $p(r_i)$, the resulting consumer demand $q(r_i)$, and the corresponding utility $v(r_i)$ are all independent of commission rate r_i, and so we denote them as p^m, q^m, and v^m respectively. Then, denote $\pi(r_i) = (1 - r_i)\,\pi^m \equiv (1 - r_i)\,p^m q^m$.

□ **Example 1 (First-party entry and self-preferencing).** Continue from the *leading example*, but suppose now each platform chooses $a_i = (r_i, e_i, l_i)$, where $e_i \in \{0, 1\}$ indicates whether platform i operates as a dual-mode marketplace or not and $l_i \in \{0, 1\}$ indicates whether platform i engages in self-preferencing or not.[3] When it operates in dual mode, it introduces a first-party product whenever a third-party seller has entered in any product category.

With probability $1 - \alpha$, the first-party entry fails, and the third-party seller (in the relevant category) remains a monopolist (with corresponding gross profit π^m and buyer utility v^m). With probability α, the first-party entry succeeds. The resulting duopolistic competition results in two possible outcomes. When the platform doesn't engage in self-preferencing, the first-party profit is π^{fp} and the third-party seller profit is $(1 - r_i)\pi^d$, where $0 < \pi^d < \pi^m$, while the corresponding buyer utility is $v^d > v^m$. When the platform engages in self-preferencing, the first-party profit is $\pi^{sp} > \pi^{fp}$ and, for expositional simplicity, the third-party seller profit is set to zero, while the corresponding buyer utility is v^{sp}, where $v^{sp} < v^d$. We assume that first-party products do not "cross-list" on rival platforms.

Following the same steps in our *leading example*, we have

$$\bar{k}_i \equiv (1 - r_i)(\pi^m - \alpha e_i(\pi^m - (1 - l_i)\,\pi^d))s_i,$$

and

$$\begin{aligned} U_i &= (v^m + \alpha e_i(l_i v^{sp} + (1 - l_i)\,v^d - v^m))G\left(\bar{k}_i\right) \\ R_i &= (r_i \pi^m + \alpha e_i(l_i \pi^{sp} + (1 - l_i)\left(r_i \pi^d + \pi^{fp}\right) - r_i \pi^m))G\left(\bar{k}_i\right) s_i. \end{aligned}$$

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[3]A literature has recently emerged to address whether the choice of dual-mode marketplace is desirable in the context of a single platform, either absent the possibility of self-preferencing (see, for example, Etro (2021)) or also allowing for the possibility of self-preferencing (see, for example, Hagiu et al. (2022) and Anderson and Bedre-Defolie (2023)).

Here, e_i and l_i directly affect buyers' utility on platform i, as well as indirectly via how many sellers participate on platform i. Finally, seller surplus is $SS_i = \int_{k_{\min}}^{k_{\max}} \max\{\bar{k}_i - k_i, 0\} dG(k_i)$, where $\bar{k}_i$ is clearly decreasing in r_i, e_i, and l_i.

By Proposition 2, if we restrict platforms to choose only one of the instruments from (r_i, e_i, l_i) (while exogenously fixing the remaining two instruments), then each of the following holds in isolation: $r^* = r^{SE} \geq r^W$, $e^* = e^{SE} \geq e^W$, and $l^* = l^{SE} \geq l^W$. Moreover, as shown in Online Appendix B, under a relatively mild quasi-supermodularity condition, we can establish similar results even when platforms choose all instruments together, so that $(r^*, e^*, l^*) = (r^{SE}, e^{SE}, l^{SE}) \geq (r^W, e^W, l^W)$. That is, the equilibrium levels of commission, first-party entry, and self-preferencing are excessive.

□ **Example 2 (Preventing disintermediation).** Suppose sellers continue to be monopolists as in our *leading example*, but now they also have direct sales channels (e.g., their own websites). In order for buyers to transact on a seller's direct channel, buyers must first discover them through a platform. A direct channel allows a seller to avoid a platform's transaction-based fees if buyers switch from the platform to purchase from the seller through their direct channel, which we call disintermediation.[4] A fraction $\lambda_i \geq 0$ of buyers are unaware of this option to buy from the seller directly, with the remaining fraction $1 - \lambda_i$ aware of the option. Buyers have heterogeneous costs to switch to the direct channel. Specifically, with probability ζ buyers who are aware of the direct channel are assumed to be able to costlessly switch (and so buy from whichever channel is cheapest), while with probability $1 - \zeta$ buyers face a sufficiently high switching cost such they will never use the direct channel regardless of the price difference. Buyers realize which situation they are in after participating on a platform.

Each platform chooses $a_i = (r_i, \lambda_i)$, where $\lambda_i \in [\lambda_{\min}, \lambda_{\max}]$ reflects that the platform can influence the probability any given buyer will be aware of a seller's direct-channel option via its design choices. For example, a platform could take steps to prevent communication by sellers on the platform which would make it more difficult for them to inform buyers of their direct channel.

Participating sellers set prices p_i (on platforms $i = 1, ..., m$) and p_d (their price when selling directly). Buyers on platform i who are informed and able to switch would buy directly if and only if $p_i \geq p_d$. Moreover, given $r_i \geq 0$, each seller would always want to induce disintermediation. Therefore, a seller that joins a non-empty set of platform(s) $\phi \subseteq \{1, 2, ..., m\}$ sets its prices to maximize

$$\sum_{i \in \phi} \left[(1 - r_i)\, p_i D(p_i)(1 - (1 - \lambda_i)\zeta) + p_d D(p_d)(1 - \lambda_i)\zeta\right] s_i$$
$$\text{subject to } p_d \leq p_i, i \in \phi.$$

Given the pricing problem across channels is additively separable, the optimal price is

$$p_d = p_i = \arg\max_{p_i} \{p_i q(p_i)\} \equiv p^m$$

for all $i \in \phi$, so the standard profit and utility terms π^m and v^m still apply in this case (given $c = 0$). Each seller participates on platform i if and only if

$$k_i \leq (1 - r_i + (1 - \lambda_i)\zeta r_i)\, \pi^m s_i \equiv \bar{k}_i,$$

so

$$U_i = G\left(\bar{k}_i\right) v^m$$
$$R_i = (1 - (1 - \lambda_i)\zeta) r_i G\left(\bar{k}_i\right) \pi^m s_i.$$

Finally, seller surplus is $SS_i = \int_{k_{\min}}^{k_{\max}} \max\{\bar{k}_i - k_i, 0\} dG(k_i)$, where $\bar{k}_i$ is clearly decreasing in r_i and λ_i.

[4] Hagiu and Wright (2023) study disintermediation (or leakage in their terminology) in the case of a monopoly platform.

By Proposition 2, if we restrict platforms to choose only one of the instruments from (r_i, λ_i) (while exogenously fixing the remaining instrument), then each of the following holds in isolation: $r^* = r^{SE} \geq r^W$ and $\lambda^* = \lambda^{SE} \geq \lambda^W$. Moreover, as shown in Online Appendix B, under a relatively mild quasi-supermodularity condition, we can establish similar results even when platforms choose both instruments together, so that $(r^*, \lambda^*) = (r^{SE}, \lambda^{SE}) \geq (r^W, \lambda^W)$. That is, the equilibrium levels of commission and disintermediation-prevention effort are excessive.

□ **Example 3 (App tracking restriction).** Similar to Example 2, buyers must first discover sellers through a platform before transacting. Buyers on platform i can obtain (e.g., unlock) q units of content from sellers by either: (i) paying the seller price p_i per unit; or (ii) watching ads, which results in ad disutility z per unit to buyers and generates per-unit ad revenue $\pi_a (1 - \kappa_i) > 0$ to sellers. Here $\kappa_i \in [0, \kappa_{\max}]$ with $\kappa_{\max} < 1$ measures how restrictive platform i's app tracking policy is, which can limit the ad revenue of sellers, which is at most π_a. Suppose seller's revenue from (i) can be taxed by the platform through its commission r_i, while its ad revenue in (ii) cannot. We assume $z \geq 0$ is IID across buyers and sellers, drawn from the weakly log-concave CDF H.

Each platform chooses $a_i = (r_i, \kappa_i)$. Then, a typical seller that joins a non-empty set of platform(s) $\phi \subseteq \{1, 2, ..., m\}$ sets its prices to maximize its profit[5]

$$\sum_{i \in \phi} \left((1 - r_i) p_i q(p_i)(1 - H(p_i)) + \pi_a (1 - \kappa_i) \int_0^{p_i} q(z) dH(z) \right) s_i.$$

Observe that the pricing problems are separable, and so each seller's optimal price p on platform i is independent of the (r_j, κ_j) (when holding s_i) constant. Each seller would participate on i if and only if

$$k_i \leq \left((1 - r_i) p q(p)(1 - H(p)) + \pi_a (1 - \kappa_i) \int_0^{p} q(z) dH(z) \right) s_i \equiv \bar{k}_i,$$

and so

$$\begin{aligned} U_i &= \left(\int_0^\infty v(q(\min(p, z)) - \min(p, z) q(\min(p, z)) dH(z) \right) G\left(\bar{k}_i\right) \\ R_i &= r_i p q(p)(1 - H(p_i^*)) s_i G\left(\bar{k}_i\right). \end{aligned}$$

Finally, seller surplus is $SS_i = \int_{k_{\min}}^{k_{\max}} \max\{\bar{k}_i - k_i, 0\} dG(k_i)$, where $\bar{k}_i$ is clearly decreasing in r_i and κ_i.

By Proposition 2, if we restrict platforms to choose only one of the instruments from (r_i, κ_i) (while exogenously fixing the remaining instrument), then each of the following holds in isolation: $r^* = r^{SE} \geq r^W$ and $\kappa^* = \kappa^{SE} \geq \kappa^W$. Moreover, as shown in Online Appendix B, when all sellers have zero participation costs $k_i = 0$ (i.e., the distribution G is degenerate), we can establish similar results even when platforms choose both instruments together, so that $(r^*, \kappa^*) = (r^{SE}, \kappa^{SE}) \geq (r^W, \kappa^W)$. That is, the equilibrium levels of commission and app-tracking restriction effort are excessive.

□ **Example 4 (Demand-side heterogeneity and competing sellers).** This example is constructed independently of our leading example and those above (and so CDF $G(\cdot)$ has a different interpretation here). Each platform chooses its commission $a_i = r_i$. There is a continuum of product categories with mass 1 indexed by the buyers' interaction benefit parameter V, where V is drawn from some distribution G on $[0, V_{\max}]$. There are $n \geq 1$ potential competing sellers in each product category. A representative buyer's gross utility function for purchasing q_l units from each seller $k = 1, .., n$ in a particular product category is

$$u(q_1, ..., q_n) = V \sum_{k=1}^{n} q_k - \frac{n}{2} \left((1 - \theta) \sum_{k=1}^{n} q_k^2 + \frac{\theta}{n} \left(\sum_{k=1}^{n} q_k \right)^2 \right),$$

[5] We assume the profit function is strictly quasiconcave, a sufficient condition for which is that $q(p_i)$ has an elasticity (in magnitude) that is non-decreasing and is no lower than one over the relevant range.

and $\theta \in [0,1)$ is a measure of seller differentiation within the category. This is the model by Shubik and Levitan (1980). Then, buyer demand for seller k in category V is

$$D_{V,k} = \frac{1}{n}\left(V - \frac{p_k}{1-\theta} + \frac{\theta}{1-\theta}\sum_{k'=1}^{n}\frac{p_{k'}}{n}\right).$$

We assume sellers face no fixed costs of participating on a platform, but face a positive marginal cost per unit of sales $c > 0$.

Solving for the symmetric equilibrium between sellers yields the equilibrium price on platform i, which is denoted $p_V(r_i)$,

$$p_V(r_i) = \frac{(1-\theta)\,nV}{2n-\theta\,(n+1)} + \frac{(n-\theta)\,c}{(2n-\theta\,(n+1))\,(1-r_i)}.$$

This is increasing in V, and in r_i because $c > 0$. The demand and profit an individual seller gets in product category V from a representative buyer is denoted $q_V(r_i) = \frac{1}{n}(V - p_V(r_i))$ and

$$\begin{aligned}\pi_V(r_i) &= ((1-r_i)\,p_V(r_i) - c)\,q_V(r_i)\\ &= (1-r_i)\,\frac{(1-\theta)\,(n-\theta)}{(2n-\theta\,(n+1))^2}\left(V - \frac{c}{1-r_i}\right).\end{aligned}$$

The corresponding per-buyer utility in product category V is $v_V(r_i) = \frac{n^2}{2}q_V(r_i)^2$. Once it has joined platform i, each participating seller in product category V sets the price $p_V(r_i)$ on platform i and transacts with each buyer on that platform once, with the representative buyer consuming $q_V(r_i)$ units from each such seller.

Notice there is an equilibrium where the n sellers of type V can operate (make positive sales) and obtain a profit if and only if $(1-r_i)\,p_V(r_i) > c$. But the highest price that sellers can charge and obtain positive demand is V. Therefore, in the absence of any seller fixed costs of participation, if $r_i < 1 - \frac{c}{V}$, all n sellers in category V participate on platform i and make positive sales; while if $r_i \geq 1 - \frac{c}{V}$, none of them participate on platform i since in equilibrium they would not make a profit while making positive sales. The measure of product categories where sellers participate on platform i is $1 - G\left(\frac{c}{1-r_i}\right)$. Therefore,

$$\begin{aligned}U_i &= \textstyle\int_{\frac{c}{1-r_i}}^{V_{\max}} v_V(r_i)dG(V)\\ R_i &= s_i r_i n \textstyle\int_{\frac{c}{1-r_i}}^{V_{\max}} p_V(r_i)q_V(r_i)dG(V).\end{aligned}$$

Finally, seller surplus is

$$SS_i = \int_{\frac{c}{1-r_i}}^{V_{\max}} \pi_V(r_i)dG(V),$$

where $\pi_V(r_i)$ is clearly decreasing in r_i. By Proposition 2, we conclude $r^* = r^{SE} \geq r^W$. That is, the equilibrium level of commission is excessive in this oligopolistic seller model.

B Multi-dimensional instruments

We now extend the baseline model in Section 2 by allowing each platform's instrument choice $a_i \in \mathcal{A} \subseteq \mathbb{R}^N$ be a multi-dimensional vector, where $N \geq 1$. Our ordering that a higher a_i corresponds to a lower seller surplus means that $SS_i(a_i; s_i)$ is *decreasing in every dimension* of a_i, holding s_i constant, and denote $SS(a) = mSS_i(a; 1/m)$. The analysis below admits the possibility of non-unique equilibrium instruments a^* and non-unique solutions to welfare benchmarks a^{SE} and a^W (where we denote the sets of solutions a^{SE} and a^W as $\mathcal{A}^{SE}$ and $\mathcal{A}^W$ respectively.).

It is straightforward to verify that the analysis in Section 2 holds as it is. In particular, the definition of the equilibrium object (9) always applies regardless of whether a_i is single-dimensional or multi-dimensional. Denote set

$$\mathcal{A}^* = \arg\max_{a_i \in \mathcal{A}} \left\{ \frac{1}{m} U_i(a_i; \frac{1}{m}) + R_i(a_i; \frac{1}{m}) \right\}$$

We focus on the case where every $a^* \in \mathcal{A}^*$ constitute an equilibrium (this is true in, e.g., our leading example in Section 3). Then, given that $\mathcal{A}^{SE}$ is defined by the exact same condition, we have $\mathcal{A}^{SE} = \mathcal{A}^*$.

Lemma 1 now requires additional conditions. One well-known complication of multi-dimensional comparative statics is the cross-dimension effects, whereby distortions in one of the dimensions may reinforce or diminish distortions in other dimensions. To proceed, we define the following concepts as introduced by Milgrom and Shannon (1994):

- **Quasi-supermodularity** (QSM). A function $W : \mathcal{A} \to \mathbb{R}$ is *quasi-supermodular* in its argument $a_i \in \mathcal{A}$ if, for any pair of vectors $a_i' \in \mathcal{A}$ and $a_i'' \in \mathcal{A}$, we have

  $$W(a') - W(a' \wedge a'') \geq (>) 0 \Rightarrow W(a' \vee a'') - W(a'') \geq (>) 0.$$

  Here, $a' \vee a''$ is the dimension-wise maxima of the two vectors and $a' \wedge a''$ is the dimension-wise minima of the two vectors.

Intuitively, quasi-supermodularity expresses a weak kind of complementarity between each dimension of vector a. That is, if an increase in some dimensions has a positive marginal return at some level of the remaining dimensions, then the marginal return will also be positive at any higher level of those remaining dimensions. Clearly, it is implied by the standard weak supermodularity condition. More generally, by Milgrom and Shannon (1994), there are several easy-to-check sufficient conditions for $W(a_i)$ to be QSM: (i) $W(a_i)$ is monotone in a_i; (ii) if a is one-dimensional then QSM trivially holds; (iii) if a is two-dimensional, then QSM is equivalent to $W(a)$ obeying single-crossing difference in a pairwise manner.[6]

To compare $\mathcal{A}^{SE}$ and $\mathcal{A}^W$, we adopt the following notion by Milgrom and Shannon (1994):

- **Strong set order**. A set $\mathcal{A}''$ is higher than set $\mathcal{A}'$ in *strong set order* (denoted as $\mathcal{A}'' \geq_{sso} \mathcal{A}'$) if for any pairs of vectors $a' \in \mathcal{A}'$ and $a'' \in \mathcal{A}''$, we have $a' \vee a'' \in \mathcal{A}''$ and $a' \wedge a'' \in \mathcal{A}'$.

Then, the following is analogous to Lemma 1 and Proposition 2. It shows that the baseline distortion persists under multi-dimensional platform instruments.

Proposition 8 *Suppose function $W(a)$ (or $W^{SE}(a)$) is quasi-supermodular. The seller-excluded benchmark exceeds the total-welfare benchmark ($\mathcal{A}^{SE} \geq_{sso} \mathcal{A}^W$), indicating that the seller-excluded benchmark level of instrument is excessive. Consequently, $\mathcal{A}^* = \mathcal{A}^{SE} \geq_{sso} \mathcal{A}^W$.*

Proof. (Proposition 8). We first verify that $W(a)$ single-crossing dominates $W^{SE}(a)$: for any $a'' > a'$, whenever $W(a'') - W(a') \geq (>) 0$ holds, we must have

$$\begin{aligned} & W^{SE}(a'') - W^{SE}(a') \\ = \quad & W(a'') - W(a') + \underbrace{SS(a') - SS(a'')}_{\geq 0} \geq (>) 0 \end{aligned}$$

because $SS(\cdot)$ is decreasing. Then, we apply Theorem 1 of Amir and Rietzke (2023), which implies $\mathcal{A}^{SE} \geq_{sso} \mathcal{A}^W$, as required. ■

[6]That is, if we assume continuous choice and differentiability, and let $N = 2$ so that a platform's instrument vector is $a_i = (z_1, z_2) \in \mathbb{R}^2$, then this is equivalent to $\partial \hat{W} / \partial z_k$ being single-crossing in z_l for each dimension $k \neq l$, $k = 1, 2$. That is, if $\partial \hat{W} / \partial z_k \geq (>) 0$ at $z_l = z_l'$, then $\partial \hat{W} / \partial z_k \geq (>) 0$ for all $z_l > z_l'$.

It is useful to verify that QSM holds in Examples 1-3 presented in Section A, all of which involve multi-dimensional instruments (Example 4 has a single-dimensional instruments and so QSM trivially holds).

□ **Example 1 (First-party entry and self-preferencing).** Dropping the redundant constant terms, the total welfare objective function is

$$W(r,e,l) = \left(v^m + \pi^m + \alpha e\left(l\Delta^{sp} + (1-l)\,\Delta^{fp}\right)\right) G\left(\bar{k}_i\right) - m\int_{k_{\min}}^{\bar{k}} kdG\left(k\right),$$

where $\bar{k} = (1-r)(\pi^m - \alpha e(\pi^m - (1-l)\,\pi^d))\frac{1}{m}$. Observe that $\bar{k}$ is decreasing in r, e, and l.

Define $\Delta^{sp} = \pi^{sp} + v^{sp} - \pi^m - v^m$ and $\Delta^{fp} = \pi^{fp} + \pi^d + v^d - \pi^m - v^m$ as the ex-post efficiency gain from first-party entry with and without self-preferencing. Suppose $\Delta^{fp} > \Delta^{sp}$. Then W is decreasing in r, decreasing in e regardless of l provided Δ^{fp} is not too large, and decreasing in l:

$$\frac{dW}{dr} = \underbrace{(v^m + \pi^m + \alpha e\left(l\Delta^{sp} + (1-l)\,\Delta^{fp}\right) - m\bar{k})}_{>0 \text{ because } m\bar{k}_i<(1-r_i)\pi^*} g\left(\bar{k}\right)\frac{d\bar{k}}{dr} < 0;$$

$$\frac{dW}{dl} = \left(v^m + \pi^m + \alpha e\left(l\Delta^{sp} + (1-l)\,\Delta^{fp}\right) - m\bar{k}\right) g\left(\bar{k}\right)\frac{d\bar{k}}{dl} + \alpha e_i\left(\Delta^{sp} - \Delta^{fp}\right) G\left(\bar{k}\right) < 0$$

$$\frac{dW}{de} = \left(v^m + \pi^m + \alpha e\left(l\Delta^{sp} + (1-l)\,\Delta^{fp}\right) - m\bar{k}\right) g\left(\bar{k}\right)\frac{d\bar{k}}{de} + \alpha\left(l\Delta^{sp} + (1-l)\,\Delta^{fp}\right) G\left(\bar{k}\right) < 0$$

As such, W is QSM when these conditions hold. Proposition 8 then implies $(r^*, e^*, l^*) = (r^{SE}, e^{SE}, l^{SE}) \geq (r^W, e^W, l^W)$.

□ **Example 2 (Preventing disintermediation).** Dropping the redundant constant terms,

$$W(r,\lambda) = (v+\pi)G\left(\bar{k}\right) - m\int_{k_{\min}}^{\bar{k}} kdG\left(k\right),$$

where $\bar{k} = (1-r+(1-\lambda)\zeta r)\,\frac{\pi}{m}$. Clearly, $W(r,\lambda)$ is decreasing in platform fee r and disintermediation prevention effort λ by the standard deadweight loss logic (a higher λ can be seen as amplifying the effective fees paid by sellers). Thus, $W(r,\lambda)$ is QSM, and Proposition 8 then implies $(r^*, \lambda^*) = (r^{SE}, \lambda^{SE}) \geq (r^W, \lambda^W)$.

□ **Example 3 (App tracking).** Assuming the seller objective function is strictly quasiconcave, then by additive separability, the optimal price p satisfies the first-order condition (FOC)

$$p = \frac{\pi_a\left(1-\kappa_i\right)}{1-r_i} + \left(1 + p\frac{q'(p)}{q(p)}\right)\frac{1-H(p)}{h(p)}.$$

Observe that p is an increasing function of $\frac{1-\kappa_i}{1-r_i}$. That is, sellers set a higher price for their apps (to divert buyers to watch ads) when ads becomes more profitable relative to their share of transaction revenue $1-r_i$. To check strict quasiconcavity of the seller objective function, notice $d\pi/dp_i$ has the same sign as

$$-p_i + \frac{\pi_a\left(1-\kappa_i\right)}{1-r_i} + \left(1+e_q\right)\frac{1-H(p_i)}{h(p_i)}, \tag{21}$$

where $e_q \equiv p_i\frac{q'(p_i)}{q(p_i)} < 0$ is elasticity of $q(.)$. By standard results, e_q is weakly decreasing in p_i if $q(.)$ is weakly log-concave or admits constant-elasticity. Therefore, as long as $(1+e_q) > 0$ then we know $(1+e_q)\,\frac{1-H(p_i)}{h(p_i)}$ is decreasing in p_i by log-concavity of $1-H$, and so (21) is always decreasing in p_i, which establishes strict-quasiconcavity.

Imposing symmetry and dropping constant terms, the total welfare objective function that is relevant for determining (r^W, κ^W) is

$$W(r,\kappa) = U_0(p)G\left(\bar{k}\right) + r_i R_0(p) G\left(\bar{k}\right) + m\int_0^{\bar{k}} (\bar{k} - k_i) dG,$$

where

$$\begin{aligned} U_0(p) &= \int_0^p u(q(z)) - zq(z)dH(z) + \int_p^\infty u(q(p)) - pq(p)dH(z) \\ R_0(p) &= pq(p)(1 - H(p)) \\ \bar{k} &= \frac{(1-r)}{m} pq(p)(1-H(p)) + \frac{\pi_a(1-\kappa)}{m}\int_0^p q(z)dH(z). \end{aligned}$$

To establish quasi-supermodularity, we reframe the maximization problem as choosing $a = (r, -p)$, where

$$\kappa = \kappa(r,p) = 1 + \psi(p)\left(\frac{1-r}{\pi_a}\right)$$

and

$$\psi(p) \equiv (1+e_q)\frac{1-H(p)}{h(p)} - p < 0$$

is strictly decreasing in p by the properties on (21) as established above. Then

$$\frac{1}{G\left(\bar{k}\right)}\frac{dW}{dr} = \left(U_0(p) + r_i R_0(p)\right)\frac{g\left(\bar{k}\right)}{G\left(\bar{k}\right)}\frac{d\bar{k}}{dr} + m\frac{d\bar{k}}{dr} < 0$$

for all p because $\frac{d\bar{k}}{dr} = -\frac{1}{1-r}\bar{k} < 0$. Thus, dW/dr is single-crossing in p, as required. Likewise,

$$\frac{1}{G\left(\bar{k}\right)}\frac{dW}{dp} = \left(\frac{dU_0}{dp} + \frac{dR_0}{dp}r\right) + \left(U_0(p) + rR_0(p)\right)\varphi\left(\bar{k}\right)\frac{d\bar{k}/dp}{\bar{k}} + m\frac{d\bar{k}}{dp},$$

where $\varphi(x) \equiv \frac{xg(x)}{G(x)}$ is the elasticity of G with respect to its argument. If we impose constant-elasticity $G(k) = \left(\frac{k}{k_{\max}}\right)^{\varphi}$ on $[0, k_{\max}]$, and let $\varphi \to 0$, then

$$\frac{1}{G\left(\bar{k}_i\right)}\frac{d^2W}{dpdr} \to \frac{dR_0}{dp} + m\frac{d^2\bar{k}}{dpdr} < 0$$

because $\frac{dR_0}{dp} < 0$ by (21), and

$$\frac{d^2\bar{k}}{dpdr} = -\frac{1}{1-r}\frac{d\bar{k}}{dp} = \frac{1}{m}\psi'(p)\int_0^p q(z)dH(z) < 0.$$

Thus, dW/dp is single-crossing in r, as required. Proposition 8 implies $(r^*, -p^*) = (r^{SE}, -p^{SE}) \geq (r^W, -p^W)$. Given p is an increasing function of $\frac{1-\kappa_i}{1-r_i}$, we conclude that $p^{SE} \leq p^W$ and $r^{SE} \geq r^W$ together imply $\kappa^{SE} \geq \kappa^W$. Hence, $(r^*, \kappa^*) = (r^{SE}, \kappa^{SE}) \geq (r^W, \kappa^W)$.

C Advertising on the buyer-side

Suppose instead of setting lump-sum prices on the buyer side, each platform i chooses its advertising intensity A_i and gets an associated payoff A_i per buyer. At the same time, buyers incur an associated disutility of γA_i, where $\gamma > 0$ captures a nuisance cost. Here, $\gamma = 1$ implies that raising advertising intensity reduces buyer utility by the same amount it increases platform revenue — just like a lump-sum price. More generally though, ad monetization may be more efficient than using lump-sum prices (i.e., one dollar of extra revenue can be extracted from a buyer with less than a one dollar reduction in utility, so $\gamma < 1$), or less efficient ($\gamma > 1$).

To understand the new welfare distortion in this setting, consider the case of inefficient ad monetization ($\gamma > 1$) and consider a decrease in commission r below r^*. Fixing the level of ad monetization, the decrease in r leads to higher seller-excluded welfare because the inefficient revenue extraction means that platforms do not fully internalize buyer utility in their choice of r^*, resulting in an excessive equilibrium commission. In our *leading example*, an incomplete pass-through argument shows that this direct effect dominates any feedback effect from platforms reoptimizing their level of ad monetization. Formally, we get:

Proposition 9 *Consider the above model with advertising on the buyer side. Suppose $\gamma > (<)1$ so that advertising is inefficient (efficient). Then $r^* \geq (\leq) r^{SE}$, strictly so for interior solutions.*

Proof. (Proposition 9). We first state the equilibrium in this case without invoking the *leading example.* By the same reframing technique used to establish Proposition 1, we get

$$a^* = \arg\max_{a_i \in \mathcal{A}} \left\{ \frac{1}{\gamma m} U_i \left(a_i; \frac{1}{m} \right) + R_i \left(a_i; \frac{1}{m} \right) \right\}.$$

Meanwhile for each given instrument, we first solve for the equilibrium ad intensity $A(a)$. Following the steps in the proof of Proposition 1, solving for the symmetric FOCs gives

$$A(a) = \frac{1/m}{\Phi'(0)} - \frac{1}{m-1} \frac{\partial U_i(a; \frac{1}{m})}{\partial s_i} - \frac{\partial R_i(a; \frac{1}{m})}{\partial s_i} = 0.$$

Therefore, when the (common) instrument a changes, we have

$$A'(a) = -(\frac{1}{m-1}) \frac{\partial^2 U_i(a; \frac{1}{m})}{\partial s_i \partial a_i} - \frac{\partial^2 R_i(a; \frac{1}{m})}{\partial s_i \partial a_i}.$$

We now specialize the expressions above to the *leading example*, where we know

$$r^* = \arg\max_{r_i \in [0, \overline{r}]} \left\{ \frac{\gamma}{m} U_i \left(r_i; \frac{1}{m} \right) + R_i \left(r_i; \frac{1}{m} \right) \right\},$$

the FOC of which is

$$\frac{1}{\gamma m} \frac{\partial U_i \left(r^*; \frac{1}{m} \right)}{\partial r_i} + \frac{\partial R_i \left(r^*; \frac{1}{m} \right)}{\partial r_i} = 0.$$

Meanwhile, using

$$\begin{aligned} U_i(r_i; s_i) &= v(r_i) \left(\frac{\pi(r_i)}{k_{\max}} \right)^{\varphi} s_i^{\varphi} \\ R_i(r_i; s_i) &= r_i p(r_i) q(r_i) \left(\frac{\pi(r_i)}{k_{\max}} \right)^{\varphi} s_i^{1+\varphi}, \end{aligned}$$

it is clear that $\frac{\partial U_i}{\partial s_i} = \frac{\varphi}{s_i} U_i$, and so $\frac{\partial^2 U_i}{\partial s_i \partial r_i} = \frac{\varphi}{s_i} \frac{\partial U_i}{\partial r_i}$; likewise, $\frac{\partial R_i}{\partial s_i} = \frac{1+\varphi}{s_i} R_i$, and so $\frac{\partial^2 R_i}{\partial s_i \partial r_i} = \frac{1+\varphi}{s_i} \frac{\partial R_i}{\partial r_i}$. Then

$$A'(r) = -\left(\frac{m}{m-1} \right) \frac{\varphi}{\gamma} \frac{\partial U_i \left(r; \frac{1}{m} \right)}{\partial r_i} - (1+\varphi) m \frac{\partial R_i \left(r; \frac{1}{m} \right)}{\partial r_i}.$$

Evaluating this at $r = r^*$ and using the FOC associated with r^*, we get

$$\begin{aligned} A'(r^*) &= -\left(\frac{m}{m-1}\right)\frac{\varphi}{\gamma}\frac{\partial U_i}{\partial r_i} + \frac{1+\varphi}{\gamma}\frac{\partial U_i}{\partial r_i} \\ &= \left(\frac{m-1-\varphi}{m-1}\right)\frac{1}{\gamma}\frac{\partial U_i}{\partial r_i}. \end{aligned}$$

From the expression of the seller-excluded welfare, we have

$$\frac{dW^{SE}(r)}{dr} = \frac{\partial U_i}{\partial r_i} + m\frac{\partial R_i}{\partial r_i} + (1-\gamma)A'(r).$$

Evaluating the above at $r = r^*$, we have

$$\begin{aligned} \frac{dW^{SE}(r^*)}{dr} &= -\left(\frac{1-\gamma}{\gamma}\right)\frac{\partial U_i}{\partial r_i} + (1-\gamma)A'(r^*) \\ &= -\left(\frac{1-\gamma}{\gamma}\right)\frac{\partial U_i}{\partial r_i} + \left(\frac{m-1-\varphi}{m-1}\right)\left(\frac{1-\gamma}{\gamma}\right)\frac{\partial U_i}{\partial r_i} \\ &= -\left(\frac{1-\gamma}{\gamma}\frac{\varphi}{m-1}\right)\frac{\partial U_i}{\partial r_i}, \end{aligned}$$

which is negative if and only if $\gamma > 1$ (because $\frac{\partial U_i}{\partial r_i} < 0$). ■

D Details for Section 3

D.1 The leading example

□ ***Leading example*** **with Hotelling competition**. We first check our claim on global concavity: for any given r_i, if (14) holds, Π_i is concave in $s_i \in [0,1]$. Let $z(r_i) = v(r_i) + r_i p(r_i) q(r_i)$. Then we can rewrite (11) as:

$$\Pi_i = \left(P^{B*} + z(r_i)\left(\frac{\pi(r_i)}{k_{\max}}\right)^{\varphi} s_i^{\varphi} - v(r^*)\left(\frac{\pi(r^*)}{k_{\max}}\right)^{\varphi}(1-s_i)^{\varphi} - (2s_i-1)t\right)s_i.$$

The derivatives are

$$\begin{aligned} \frac{d\Pi_i}{ds_i} &= P^{B*} + (1+\varphi)z(r_i)\left(\frac{\pi(r_i)}{k_{\max}}\right)^{\varphi} s_i^{\varphi} \\ &\quad -v(r^*)\left(\frac{\pi(r_i)}{k_{\max}}\right)^{\varphi}(1-s_i)^{\varphi-1}[1-s_i-\varphi s_i] - 4ts_i + t \\ \frac{d^2\Pi_i}{ds_i^2} &= \varphi(1+\varphi)z(r_i)\left(\frac{\pi(r_i)}{k_{\max}}\right)^{\varphi} s_i^{\varphi-1} + 2\varphi v(r^*)\left(\frac{\pi(r_i)}{k_{\max}}\right)^{\varphi}(1-s_i)^{\varphi-1} \\ &\quad -(\varphi-1)v(r^*)\left(\frac{\pi(r_i)}{k_{\max}}\right)^{\varphi}(1-s_i)^{\varphi-2}\varphi s_i - 4t. \end{aligned}$$

Among the terms in $d^2\Pi_i/ds_i^2$, only the first two components are positive, and we note $s_i^{\varphi-1}$ and $(1-s_i)^{\varphi-1}$ are both bounded below one given $\varphi \geq 1$. Recalling from (12) that $r^* = \arg\max_{r_i \in [0,\bar{r}]}\{z(r_i)\pi(r_i)^{\varphi}\}$, a sufficient condition for $d^2\Pi_i/ds_i^2 < 0$ to hold for any s_i and r_i is

$$2t > \varphi(1+\varphi)z(r^*)\left(\frac{\pi(r^*)}{k_{\max}}\right)^{\varphi}, \tag{22}$$

which coincides with the condition in (14). Notice this condition implies $2t \geq 2\varphi v(r^*)\left(\frac{\pi(r_i)}{k_{\max}}\right)^{\varphi}$ because $\varphi \geq 1$ and $z(r^*) \geq v(r^*)$. Meanwhile, the condition for there to be a unique fixed-point in (6) is equivalent to requiring (7) to be strictly decreasing in s_i, i.e.,

$$\frac{dP_i^B}{ds_i} = \varphi z(r_i)\left(\frac{\pi(r_i)}{k_{\max}}\right)^{\varphi} s_i^{\varphi-1} + \varphi v\left(r^*\right)\left(\frac{\pi(r_i)}{k_{\max}}\right)^{\varphi}(1-s_1)^{\varphi-1} - 2t < 0,$$

which holds given (14).

Next, we provide two sets of conditions under which the objective function in (12) is strictly quasiconcave (hence has a unique maximizer).

One condition is to impose $c = 0$, which recall means $\overline{r} = 1$. Then using the same notation as in Section A, we have

$$\left(z(r_i)\pi(r_i)^{\varphi}\right)|_{c=0} = (v^m + r_i\pi^m)((1-r_i)\pi^m)^{\varphi}.$$

The derivative with respect to r_i has the same sign as

$$\left(\pi^m - \frac{\varphi\left(v^m + r_i\pi^m\right)}{1-r_i}\right)(1-r_i)^{\varphi}.$$

Observe the expression in the first (large) brackets is monotonically decreasing in r_i, and so there exists a (possibly negative) threshold $\hat{r} < 1$ such that the expression is strictly negative if and only if $r_i > \hat{r}$. Hence, $z(r_i)\pi(r_i)^{\varphi}$ is strictly single-peaked and so strictly quasiconcave.

Suppose instead $c > 0$. Then another set of conditions is $\varphi = 1$ and a linear-quadratic utility specification

$$u\left(q\right) = Vq - \frac{1}{2}q^2, \text{ such that } D(p_i) = V - p_i,$$

with $V > c$. This implies $q(r_i) = \frac{1}{2}\left(V - \frac{c}{1-r_i}\right)$, $p(r_i) = \frac{1}{2}\left(V + \frac{c}{1-r_i}\right)$, $\pi(r_i) = \frac{1-r_i}{4}\left(V - \frac{c}{1-r_i}\right)^2$, and $v(r_i) = \frac{1}{8}\left(V - \frac{c}{1-r_i}\right)^2$, where recall $\overline{r} = 1 - \frac{c}{V} < 1$. Then, the objective function defining r^* can be rewritten

$$z(r_i)\pi(r_i) = \frac{1}{32}\left(V - \frac{c}{1-r_i}\right)^3 B(r_i)$$

for $r_i \in [0, \overline{r}]$, where $V - \frac{c}{1-r_i}$ and $B(r_i) = V - c + (V + 2c)\,r_i - 2r_i^2 V$ which are both strictly concave and positive on $r_i \in [0, \overline{r})$ and $V - \frac{c}{1-\overline{r}} = 0$. This implies the maximum must occur on $[0, \overline{r})$. Within this range, the objective $z(r_i)\pi(r_i)$ can therefore be written as the product of positive and strictly concave functions, so must itself be strictly quasiconcave.

□ **Leading example with logit specification**. Suppose we impose the logit demand system $\Phi(x) = \frac{1}{1+(m-1)\exp\{-x/\mu\}}$, with scale parameter $\mu > 0$. Then

$$\Phi^{-1}\left(s_i\right) = \mu \ln\left(\frac{(m-1)s_i}{1-s_i}\right)$$

and $\frac{\partial}{\partial s_i}\Phi^{-1}\left(s_i\right) = \frac{\mu}{(1-s_i)s_i}$. By the same calculation as before

$$\begin{aligned}\frac{d\Pi_i}{ds_i} &= P^{B*} + (1+\varphi)z(r_i)\left(\frac{\pi(r_i)}{k_{\max}}\right)^{\varphi} s_i^{\varphi} \\ &\quad - v\left(r^*\right)\left(\frac{\pi(r_i)}{k_{\max}}\right)^{\varphi}(1-s_i)^{\varphi-1}\left[1 - s_i - \varphi s_i\right] - \left[\frac{\mu}{1-s_i} + \mu\ln\left(\frac{(m-1)s_i}{1-s_i}\right)\right]\end{aligned}$$

$$\begin{aligned}\frac{d^2\Pi_i}{ds_i^2} &= \varphi(1+\varphi)z(r_i)\left(\frac{\pi(r_i)}{k_{\max}}\right)^{\varphi} s_i^{\varphi-1} + 2\varphi v\left(r^*\right)\left(\frac{\pi(r_i)}{k_{\max}}\right)^{\varphi}(1-s_i)^{\varphi-1} \\ &\quad - (\varphi-1)v\left(r^*\right)\left(\frac{\pi(r_i)}{k_{\max}}\right)^{\varphi}(1-s_i)^{\varphi-2}\varphi s_i - \left[\frac{\mu}{(1-s_i)^2 s_i}\right].\end{aligned}$$

We know $\max_{s_i\in[0,1]}(1-s_i)^2 s_i = 4/27$. Hence, using the equivalent condition to (22), a sufficient condition for strict concavity is

$$\mu > \frac{8}{27}\varphi(1+\varphi)z(r^*)\left(\frac{\pi(r^*)}{k_{\max}}\right)^{\varphi}. \tag{23}$$

This condition also ensures (7) is strictly decreasing in s_i. To see this, note

$$\begin{aligned}\frac{dP_i^B}{ds_i} &= \varphi z(r_i)\left(\frac{\pi(r_i)}{k_{\max}}\right)^{\varphi} s_i^{\varphi-1} + \varphi v(r^*)\left(\frac{\pi(r_i)}{k_{\max}}\right)^{\varphi}(1-s_i)^{\varphi-1} - \frac{\mu}{(1-s_i)s_i} \\ &\leq \varphi z(r^*)\left(\frac{\pi(r^*)}{k_{\max}}\right)^{\varphi} s_i^{\varphi-1} + \varphi z(r^*)\left(\frac{\pi(r^*)}{k_{\max}}\right)^{\varphi}(1-s_i)^{\varphi-1} - \frac{\mu}{(1-s_i)s_i} \\ &\leq 2\varphi z(r^*)\left(\frac{\pi(r^*)}{k_{\max}}\right)^{\varphi} - 4\mu \\ &< 0,\end{aligned}$$

where the first inequality is due to the definitions of $z(r_i)$ and r^*; the second inequality is due to $s_i^{\varphi-1} \leq 1$, $(1-s_i)^{\varphi-1} \leq 1$, and $\max_{s_i\in[0,1]}(1-s_i)s_i = 1/4$; and the last inequality is due to the stated sufficient condition and $\varphi \geq 1$.

□ **Buyer surplus**. Buyer surplus is given by $BS(r) = U(r;\frac{1}{m}) - P^B(r)$. Continuing from Proposition 1, we have

$$P^B(r) = \frac{1}{m\Phi'(0)} - \left(\frac{m\varphi v(r)}{m-1} + (1+\varphi)rp(r)q(r)\right)\left(\frac{\pi(r)}{mk_{\max}}\right)^{\varphi}$$

so that

$$BS(r) = \left(v(r) + \frac{m\varphi v(r)}{m-1} + (1+\varphi)rp(r)q(r)\right)\left(\frac{\pi(r)}{mk_{\max}}\right)^{\varphi} - \frac{1}{m\Phi'(0)}.$$

Hence, the maximizer can be simplified as

$$r^{BS} = \arg\max_{r\in[0,\overline{r}]}\left(\frac{m-1+m\varphi}{m-1+m\varphi-\varphi}v(r) + rp(r)q(r)\right)\pi(r)^{\varphi},$$

which we now compare with $r^* = r^{SE} = \arg\max_{r\in[0,\overline{r}]}(v(r) + rp(r)q(r))\pi(r_i)^{\varphi}$. Using the observations that $\frac{m-1+m\varphi}{m-1+m\varphi-\varphi} > 1$ and that $v(r) > 0$ is decreasing, it follows that $r^{BS} \leq r^{SE} = r^*$, with strictly inequality for interior solutions.

To see the more general pass-through logic discussed in the text, in what follows we assume that $BS(r)$ is strictly quasiconcave in r and that r^* is an interior solution. We note $\frac{d}{dr}BS(r) = \frac{\partial}{\partial r}U(r;\frac{1}{m}) - \frac{d}{dr}P^B(r)$. Using the equilibrium condition for r^*, this becomes

$$\frac{d}{dr}BS(r^*) = -m\frac{\partial R_i(r^*;\frac{1}{m})}{\partial r} - \frac{dP^B(r^*)}{dr}$$

and so $\frac{d}{dr}BS(r^*)(\leq) < 0$ if and only if

$$\frac{dP^B(r^*)}{dr}(\geq) > -m\frac{\partial R_i(r^*;\frac{1}{m})}{\partial r}. \tag{24}$$

That is, starting from the equilibrium value r^*, for any increase in r that raises the per-buyer revenue $\frac{1}{1/m}R_i$ by one unit, the per-buyer price P^B does not decrease by more than one unit. We now prove that inequality (24) always hold in strict inequality in our leading example. Continuing from Proposition 1,

$$\frac{dP^B(r)}{dr} = -\left(\frac{1}{m-1}\right)\frac{\partial U_i(r;\frac{1}{m})}{\partial r\partial s_i} - \frac{\partial R_i(r;\frac{1}{m})}{\partial r\partial s_i},$$

where the constant-elasticity $G(\cdot)$ in the leading example means

$$\frac{\partial R_i(r;\frac{1}{m})}{\partial s_i \partial r} = m(1+\varphi)\frac{\partial R_i(r;\frac{1}{m})}{\partial r} \quad \text{and} \quad \frac{\partial U_i(r;\frac{1}{m})}{\partial s_i \partial r} = m\varphi\frac{\partial U_i(r;\frac{1}{m})}{\partial r}$$

and so

$$\begin{aligned} \frac{dP^B(r)}{dr} &= -\left(\frac{m\varphi}{m-1}\right)\frac{\partial U_i(r;\frac{1}{m})}{\partial r} - m(1+\varphi)\frac{\partial R_i(r;\frac{1}{m})}{\partial r} \\ \frac{dP^B(r^*)}{dr} &= \left(\frac{\varphi}{m-1}-1\right)m\frac{\partial R_i(r^*;\frac{1}{m})}{\partial r} \\ &> -m\frac{\partial R_i(r^*;\frac{1}{m})}{\partial r}. \end{aligned}$$

For a general platform instrument a_i beyond the leading example, the same pass-through logic of inequality (24) works. Specifically, suppose the equilibrium outcome a^* is an interior solution. Then, we get $\frac{d}{dr}BS(a^*) \leq 0$ if and only if

$$\frac{dP^B(a^*)}{da} \geq -m\frac{\partial R_i(a^*;\frac{1}{m})}{\partial a}.$$

That is, the pass-through rate of extra per-buyer revenue from a higher level of a_i onto a lower buyer-side price is no more than one. When this is true, we can conclude $a^{BS} \leq a^*$ if $BS(a)$ is quasiconcave (strictly so, if the inequality above is strict), and $a^{TUS} \equiv \arg\max_{a\in\mathcal{A}} BS(a) + mSS_i(a;\frac{1}{m}) < a^*$ if $BS(a)+mSS_i(a;\frac{1}{m})$ is quasiconcave.

D.2 Buyer myopia

Suppose buyers' perceived U_i when making their platform choice is discounted by δ, where $0 \leq \delta < 1$. We first characterize the equilibrium before specializing to the *leading example*. By the same steps that establish Proposition 1, we get

$$a^* = \arg\max_{a_i\in\mathcal{A}}\left\{\frac{\delta}{m}U_i\left(a_i;\frac{1}{m}\right) + R_i\left(a_i;\frac{1}{m}\right)\right\}$$

and $P^{B*} = P^B(a)$, where

$$P^B(a) = \frac{1}{m\Phi'(0)} - \left(\frac{\delta}{m-1}\right)\frac{\partial U_i(a;\frac{1}{m})}{\partial s_i} - \frac{\partial R_i(a;\frac{1}{m})}{\partial s_i}.$$

Meanwhile, the seller-excluded benchmark remains the same as in the baseline model because the planner takes into account the actual utility of buyers, so

$$a^{SE} = \arg\max_{a_i\in\mathcal{A}}\left\{\frac{1}{m}U_i\left(a_i;\frac{1}{m}\right) + R_i\left(a_i;\frac{1}{m}\right)\right\}.$$

Finally, $a^{BS} = \arg\max_{a_i\in\mathcal{A}}\left\{U(a_i;\frac{1}{m}) - P^B(a_i)\right\}$.

We now specialize to the *leading example*, where $a_i = r_i$. We claim that $r^* \geq r^{SE}$ and $r^* \geq r^{BS}$. In the leading example $U_i(r_i) = v(r_i)\left(\frac{\pi(r_i)}{mk_{\max}}\right)^{\varphi}$, so

$$\begin{aligned} r^* &= \arg\max_{r\in[0,\overline{r}]}\left(\delta v(r) + rp(r)q(r)\right)\pi(r)^{\varphi} \\ r^{SE} &= \arg\max_{r\in[0,\overline{r}]}\left(v(r) + rp(r)q(r)\right)\pi(r)^{\varphi} \\ r^{BS} &= \arg\max_{r\in[0,\overline{r}]}\left(\frac{m-1+m\delta\varphi}{(m-1)(1+\varphi)}v(r) + rp(r)q(r)\right)\pi(r)^{\varphi}, \end{aligned}$$

where we used

$$BS(r) = \left(v(r) + \frac{\delta m\varphi v(r)}{m-1} + (1+\varphi)rp\,(r)\,q(r)\right)\left(\frac{\pi\,(r)}{mk_{\max}}\right)^{\varphi} - \frac{1}{m\Phi'(0)}.$$

Given $v(r) > 0$ is decreasing, it follows that $\delta < 1$ implies $r^* \geq r^{SE}$. If r^* or r^{SE} is pinned down as an interior solution, the associated FOC then implies $r^* > r^{SE}$. Likewise,

$$\frac{m-1+m\delta\varphi}{(m-1)\,(1+\varphi)} > \delta \implies r^{BS} \leq r^*,$$

with strictly inequality holds for interior solutions. Moreover, $\frac{m+m\delta\varphi-1}{(m-1)(1+\varphi)} > 1$ is equivalent to $(m-1)(1-\delta)+\delta\varphi > 0$, which always holds by assumption.

D.3 Effect of entry

Proof. (Corollary 1). With a constant φ, a^* does not change with m by Proposition 3. So,

$$\begin{aligned} mSS_i &= \int_0^{\frac{\pi(a^*)}{m}} [\pi(a^*) - mk_i]dG \\ \frac{d}{dm}(mSS_i) &= -\int_0^{\frac{\pi(a^*)}{m}} k_i dG < 0. \end{aligned}$$

For buyer surplus, we first specialize Proposition 1 to this setting:

$$P^{B*} = \frac{1}{m\Phi'(0)} - \left(\frac{m\varphi}{m-1}u(a^*) + (1+\varphi)w(a^*)\right)\left(\frac{\pi(a^*)}{mk_{\max}}\right)^{\varphi}$$

Its derivative is

$$\frac{dP^{B*}}{dm} = \frac{d}{dm}\left(\frac{1}{m\Phi'(0)}\right) + \underbrace{\left(\frac{m\varphi-\varphi+1}{(m-1)^2}u(a^*) + \frac{1+\varphi}{m}w(a^*)\right)\left(\frac{\pi(a^*)}{mk_{\max}}\right)^{\varphi}\varphi}_{=0 \text{ if } \varphi\to 0}.$$

By the standard results of discrete-choice demand systems, $\frac{d}{dm}\left(\frac{1}{m\Phi'(0)}\right) \leq 0$ because $\{\epsilon_i\}_{i=1,\ldots,m}$ are IID with a log-concave density (see, e.g., Tan and Zhou (2021)). Therefore,

$$\begin{aligned} BS &= \mathbf{E}[\max_{i=1,\ldots,m}\{\epsilon_i\}] - P^{B*} + u(a^*)\left(\frac{\pi(a^*)}{mk_{\max}}\right)^{\varphi} \\ \frac{d}{dm}BS &= \underbrace{\frac{d}{dm}\mathbf{E}[\max_{i=1,\ldots,m}\{\epsilon_i\}]}_{>0} - \underbrace{\frac{dP^{B*}}{dm}}_{\leq 0 \text{ if } \varphi\to 0} \underbrace{-\,m\varphi u(a^*)\left(\frac{\pi(a^*)}{mk_{\max}}\right)^{\varphi}}_{=0 \text{ if } \varphi\to 0}, \end{aligned}$$

and so $\frac{d}{dm}BS > 0$ if $\varphi \to 0$. By continuity, this holds for $\varphi > 0$ that are sufficiently small. ■

□ **When platforms charge seller-side lump-sum fees.** Suppose we interpret instrument a_i as a seller-side lump-sum fee ($a_i = P_i^S$). Then, $U_i = vG(\pi s_i - a_i)$ and $R_i = a_i G(vs_i - a_i)$, where the interaction benefits of buyers and sellers, v and π, are independent of the level a_i on sellers. Applying Proposition 1, in the equilibrium

$$a^* = \arg\max_{a_i\in\mathcal{A}}\left\{(\frac{v}{m}+a_i)G(\frac{\pi}{m}-a_i)\right\}.$$

Given G is log-concave, the objective function is quasiconcave and so the FOC gives

$$a^* = -\frac{v}{m} + \frac{G(\frac{\pi}{m} - a^*)}{g(\frac{\pi}{m} - a^*)} \tag{25}$$
$$= -\frac{v}{m} + \frac{\frac{\pi}{m} - a^*}{\varphi(\frac{\pi}{m} - a^*)}.$$

Log-concavity implies $\frac{x}{\varphi(x)}$ is an increasing function in $x \geq 0$, i.e., $\varphi(x) - x\varphi'(x) > 0$. Totally differentiating

$$\frac{da^*}{dm} = \frac{1}{m^2}\left(v - \frac{\pi}{\varphi(x)} - \frac{\pi x}{\varphi(x)^2}\varphi'(x)\right)_{x=\frac{\pi}{m}-a^*}.$$

Therefore,

$$\frac{da^*}{dm} < 0 \Leftrightarrow \frac{v}{\pi} < \left(\frac{1}{\varphi(x)} + \frac{x\varphi'(x)}{\varphi(x)^2}\right)_{x=\frac{\pi}{m}-a^*}.$$

In the special case of constant-elasticity G, $\varphi'(\cdot) = 0$, and so

$$\frac{da^*}{dm} < 0 \Leftrightarrow \frac{v}{\pi} < \frac{1}{\varphi}.$$

Intuitively, a higher m means sellers get a lower surplus from joining each individual platform, which induces platforms to set a lower seller-side fee a^*; at the same time, a higher m also means that each platform extracts less buyer utility, and so platforms are less incentivized to attract sellers and raise buyer utility. The fee-decreasing (fee-increasing) effect dominates when the elasticity of seller participation φ is relatively large (small).

Nonetheless, if we denote $\bar{k}^* \equiv \frac{\pi}{m} - a^*$ as the marginal participating seller in the equilibrium, then (25) becomes

$$\bar{k}^* + \frac{G(\bar{k}^*)}{g(\bar{k}^*)} = \frac{\pi + v}{m}.$$

So, $\bar{k}^*$ is always decreasing in m. That is, platform entry results in less sellers participating on platforms in equilibrium, although each seller that does participate, participates on a greater number of platforms.

E Details for Section 4

E.1 Heterogeneous interaction benefits

□ **Preliminaries.** We first state the equilibrium pricing by the sellers. Facing the commission rate r_i, a seller's optimal price on platform i is then

$$p(r_i) = \arg\max_{p_i}\left\{((1 - r_i)\,p_i - c)(V - p_i)\left(\frac{\lambda}{m}\theta_{reg} + s_i\theta_{loyal}\right)\right\},$$

where $\frac{\lambda}{m}\theta_{reg} + s_i\theta_{loyal}$ is the sum of buyers on platform i (weighted according to their interaction value). Then, define $q(r_i) = V - p(r_i)$. The linear demand form implies $q(r_i) > 0$ for all $r_i < \bar{r} = 1 - \frac{c}{V}$ and $q(r_i) = 0$ otherwise. Seller total profit from platform i is $\left(\frac{\lambda}{m}\theta_{reg} + s_i\theta_{loyal}\right)\pi(r_i)$, where $\pi(r_i) = ((1 - r_i)\,p(r_i) - c)(V - p(r_i))$, and the per-seller surplus of the buyer is

$$v_\tau(r_i) = V\theta_\tau q(r_i) - \frac{\theta_\tau^2}{2\theta_\tau}q(r_i)^2 - p(r_i)\theta_\tau q(r_i)$$
$$= \frac{\theta_\tau}{2}(V - p(r_i))^2.$$

We have

$$U_i^\tau = v_\tau(r_i)\left(\frac{\pi(r_i)\left(\frac{\lambda}{m}\theta_{reg}+s_i\theta_{loyal}\right)}{k_{\max}}\right)^{\varphi}$$
$$R_i = r_i p(r_i) q(r_i)\left(\frac{\lambda}{m}\theta_{reg}+s_i\theta_{loyal}\right)\left(\frac{\pi(r_i)\left(\frac{\lambda}{m}\theta_{reg}+s_i\theta_{loyal}\right)}{k_{\max}}\right)^{\varphi}.$$

Platform profit is $\left(\frac{\lambda}{m}+s_i\right)P_i^B + R_i$.

□ **Equilibrium existence.** We now use the *leading example with Hotelling competition* to demonstrate the conditions for equilibrium existence. Recall that loyal buyers have no transportation costs for their preferred platform and infinite transportation costs for the other platform, and their outside option is valued at zero.

Clearly, if $\lambda = 0$, the model reduces to the leading example with Hotelling competition, in which case the existence condition (14) immediately applies. Hence, our strategy here is to show equilibrium existence for sufficiently small $\lambda \to 0$. We focus on $\theta_{reg} = 0$.

Each platform i chooses r_i and s_i to maximize

$$\begin{aligned}\Pi_i &= \left(P^{B*} + U_i^{reg}(r_i; s_i) - U_{-i}^{reg}(r^*; 1-s_i) - (2s_i - 1)t\right)\left(\frac{\lambda}{2}+s_i\right) + R_i(a_i; s_i)\\ &= \left(P^{B*} - (2s_i-1)t\right)s_i + r_i p(r_i) q(r_i)\left(\frac{\pi(r_i)}{k_{\max}}\right)^{\varphi}\left(\frac{\lambda}{2}\theta_{loyal}\right)^{1+\varphi},\end{aligned}$$

where we used $\theta_{reg} = 0$. Observe that r^* is determined by a single-variable maximization, regardless of the value of s_i:

$$r^* = \arg\max_{r_i\in[0,\bar{r}]} r_i p(r_i) q(r_i))\pi(r_i)^{\varphi}.$$

Meanwhile, maximization with respect to s_i is a standard Hotelling problem and so local concavity holds and the FOC gives $P^{B*} = (1+\lambda)\,t$. The equilibrium profit is

$$\Pi^{eqm} = \frac{t}{2}(1+\lambda)^2 + r^* p(r^*) q(r^*)\left(\frac{\pi(r^*)}{k_{\max}}\right)^{\varphi}\left(\frac{\lambda}{2}\theta_{loyal}\right)^{1+\varphi}.$$

Full coverage of the regular type requires

$$b > \left(\frac{3}{2}+\lambda\right)t,$$

which also ensures full coverage of the loyal type.

It remains to rule out a global deviation where each platform just fully exploits its loyal buyers by setting

$$P^{dev} = b + v_{loyal}\left(r^{dev}\right)\left(\frac{\pi\left(r^{dev}\right)}{k_{\max}}\right)^{\varphi}\left(\frac{\lambda}{2}\theta_{loyal}\right)^{\varphi} > (1+\lambda)\,t = P^{B*}$$

together with the optimal deviation commission r^{dev} that is the maximizer of

$$\Pi^{dev} = \max_{r_i\in[0,\bar{r}]}\left\{\left(b + v_{loyal}(r_i)\left(\frac{\pi(r_i)\frac{\lambda}{2}\theta_{loyal}}{k_{\max}}\right)^{\varphi}\right)\left(\frac{\lambda}{2}+s_i^{dev}\right) + r_i p(r_i) q(r_i)\left(\frac{\pi(r_i)}{k_{\max}}\right)^{\varphi}\left(\frac{\lambda}{2}\theta_{loyal}\right)^{1+\varphi}\right\},$$

where

$$s_i^{dev} = \frac{1}{2} + \frac{1}{2t}\left((1+\lambda)\,t - b - v_{loyal}(r_i)\left(\frac{\pi(r_i)\frac{\lambda}{2}\theta_{loyal}}{k_{\max}}\right)^{\varphi}\right).$$

Using an envelope theorem argument, it is easy to verify that $\lim_{\lambda\to 0}\Pi^{dev} < \Pi^{eqm}$ by definition. Hence, the equilibrium exists for λ sufficiently small.

□ **Proof of Proposition 4.** Notice that only regular buyers are marginal because loyal buyers always purchase from their respective preferred platform. We apply the same reframing technique used in Proposition 1: each platform's optimal r_i (for given s_i) maximizes

$$\begin{aligned} & \left(\frac{\lambda}{m}+s_i\right) U_i^{reg}+R_i \\ = & \left(v_{reg}(r_i)\left(\frac{\lambda}{m}+s_i\right)+r_i p(r_i) q(r_i)\left(\frac{\lambda}{m} \theta_{reg}+s_i \theta_{loyal}\right)\right)\left(\frac{\pi(r_i)\left(\frac{\lambda}{m} \theta_{reg}+s_i \theta_{loyal}\right)}{k_{\max}}\right)^{\varphi} . \end{aligned}$$

After imposing symmetry and removing the multiplicative coefficients that are irrelevant for the maximization problem, we conclude that in the equilibrium r^* is

$$r^*=\arg \max_{r_i \in[0, \bar{r}]}\left\{\left(v_{reg}(r)(1+\lambda)+r p(r) q(r)\left(\theta_{reg}+\lambda \theta_{loyal}\right)\right) \pi(r)^{\varphi}\right\},$$

whereas

$$P^{B*}=\frac{1}{m \Phi^{\prime}(0)}-\left(\frac{m}{m-1} \varphi v_{reg}(r^*)+(1+\varphi) r^* p(r^*) q(r^*)\left(\theta_{reg}+\lambda \theta_{loyal}\right)\right)\left(\frac{\pi(r^*)\left(\theta_{reg}+\lambda \theta_{loyal}\right)}{m k_{\max}}\right)^{\varphi} .$$

Now consider W^{SE}:

$$\begin{aligned} W^{SE}(r) & = U_i^{reg}+m R_i+\lambda\left(b+U_i^{loyal}\right)+\int_{\underline{\epsilon}}^{\bar{\epsilon}} \hat{\epsilon} d \hat{F}(\hat{\epsilon}) \\ & = \lambda b+(1+\lambda) U_i^{reg}+m R_i+\lambda\left(U_i^{loyal}-U_i^{reg}\right)+\int_{\underline{\epsilon}}^{\bar{\epsilon}} \hat{\epsilon} d \hat{F}(\hat{\epsilon}), \end{aligned}$$

where $(1+\lambda) U_i^{reg}+m R_i$ is proportional to the objective of r^* and so it is maximized at r^*. Meanwhile,

$$\begin{aligned} U_i^{loyal}-U_i^{reg} & = \left(v_{loyal}(r)-v_{reg}(r)\right)\left(\frac{\pi(r)\left(\theta_{reg}+\lambda \theta_{loyal}\right)}{m k_{\max}}\right)^{\varphi} \\ & = \frac{\theta_{loyal}-\theta_{reg}}{2}(V-p(r))^2\left(\frac{\pi(r)\left(\theta_{reg}+\lambda \theta_{loyal}\right)}{m k_{\max}}\right)^{\varphi}, \end{aligned}$$

which is monotonically decreasing (increasing) in r if $\theta_{loyal}>(<)\theta_{reg}$. A simple proof by contradiction then shows $r^{SE} \leq (\geq) r^*$ if $\theta_{loyal}>(<)\theta_{reg}$, thus completing the proof.

□ **General demand specification.** We now consider a more general demand specification rather than the linear-quadratic specification in the main text. We denote seller total profit on i as

$$\bar{\pi}(r_i ; s_i)=\max_{p_i}\left\{\left((1-r_i) p_i-c\right)\left[\frac{\lambda}{m} D_{loyal}(p_i)+s_i D_{reg}(p_i)\right]\right\},$$

where $p(r_i)$ is its maximizer. The corresponding total transaction quantity is

$$\bar{q}(r_i ; s_i)=\frac{\lambda}{m} D_{loyal}(p(r_i ; s_i))+s_i D_{reg}(p(r_i ; s_i)),$$

while the per-seller surplus of the buyer is $v_\tau(r_i ; s_i)=u_\tau\left(D_\tau\left(p\left(r_i ; s_i\right)\right)\right)-p\left(r_i ; s_i\right) D_\tau\left(p\left(r_i ; s_i\right)\right)$ for each type $\tau \in\{reg, loyal\}$. We have

$$\begin{aligned} U_i^\theta & =v_\theta\left(r_i ; s_i\right)\left(\tfrac{\bar{\pi}(r_i ; s_i)}{k_{\max}}\right)^{\varphi} \\ R_i & =r_i p(r_i) \bar{q}(r_i ; s_i)\left(\tfrac{\bar{\pi}(r_i ; s_i)}{k_{\max}}\right)^{\varphi} . \end{aligned}$$

Platform profit is $\left(\frac{\lambda}{m}+s_i\right) P_i^B+R_i$. By the same reframing technique used in Proposition 1, each platform's optimal r_i (for given $s_i = 1/m$) maximizes

$$\begin{aligned} r^* &= \arg\max_{r_i \in [0,\bar{r}]} \left\{ \frac{1+\lambda}{m} U_i^{reg} + R_i \right\} \\ &= \arg\max_{r_i \in [0,\bar{r}]} \left\{ \left(v_{reg}(r;\frac{1}{m}) \frac{1+\lambda}{m} + rp(r;\frac{1}{m})\bar{q}(r;\frac{1}{m}) \right) \bar{\pi}(r;\frac{1}{m})^{\varphi} \right\}, \end{aligned}$$

while

$$W^{SE}(r) = \lambda b + (1+\lambda) U_i^{reg} + mR_i + \lambda \left(U_i^{loyal} - U_i^{reg} \right) + \int_{\underline{\epsilon}}^{\bar{\epsilon}} \hat{\epsilon} d\hat{F}(\hat{\epsilon}),$$

where $(1+\lambda) U_i^{reg} + mR_i$ is proportional to the objective of r^* and so it is maximized at r^*. Meanwhile,

$$U_i^{loyal} - U_i^{reg} = \left(v_{loyal}(r;\frac{1}{m}) - v_{reg}(r;\frac{1}{m}) \right) \left(\frac{\bar{\pi}(r;\frac{1}{m})}{k_{\max}} \right)^{\varphi},$$

and so $r^{SE} \le (\ge) r^*$ holds if $\frac{d}{dr}(U_i^{loyal} - U_i^{reg}) \le (\ge) 0$.

Suppose the utility function satisfies the standard Spence-Mirrlees single-crossing condition $\frac{d}{dq}u_{loyal} > \frac{d}{dq}u_{reg}$ (for all $q \ge 0$) and boundary condition $u_{loyal}(0) = u_{reg}(0)$. It implies $v_{loyal}(r;\frac{1}{m}) > v_{reg}(r;\frac{1}{m})$ and $\frac{d}{dr}\left[v_{loyal}(r;\frac{1}{m}) - v_{reg}(r;\frac{1}{m})\right] < 0$, so that $r^{SE} \le r^*$. In the opposite case of $\frac{d}{dq}u_{loyal} < \frac{d}{dq}u_{reg}$, the same reasoning implies $r^{SE} \ge r^*$.

E.2 Partial market coverage

□ **Preliminaries.** In this setting, the interaction benefit U_i of all buyers on platform i is the same, and it just depends on the total measure s_i of (regular and loyal) buyers on platform i. Hence, we can employ our standard technique for solving for the equilibrium. Recall, in our *leading example*, the solution for the equilibrium commission r^* is determined by a single-variable maximization, regardless of the value of s_i, as shown in (12). Thus, the determination of r^* remains unchanged even when the market is only partially covered.

Denote the total mass of buyers (both regulars and loyals) on platform i as $s_i = s_i^{reg} + s_i^{loyal}$. Continuing from the *leading example with Hotelling competition*, we know the market shares of regular buyers (i.e., those between the Hotelling line)

$$s_1^{reg} = \frac{1}{2} + \frac{U_1 - U_2 + P_2^B - P_1^B}{2t},$$

with $s_2^{reg} = 1 - s_1^{reg}$; whereas the market shares of loyal buyers (i.e., those in the hinterlands) is

$$s_i^{loyal} = \frac{b + U_i - P_i^B}{t_L}.$$

Combining,

$$\begin{aligned} s_1 &= \frac{1}{2} + \frac{U_1 - U_2 + P_2^B - P_1^B}{2t} + \frac{b + U_1 - P_1^B}{t_L} \\ s_2 &= \frac{1}{2} - \frac{U_1 - U_2 + P_2^B - P_1^B}{2t} + \frac{b + U_2 - P_2^B}{t_L}. \end{aligned}$$

It is useful to define

$$\begin{aligned} y(r) &\equiv v(r) \frac{\pi(r)}{k_{\max}} \\ z(r) &\equiv \frac{1}{k_{\max}} \left(v(r) + rp(r)q(r) \right) \pi(r), \end{aligned}$$

where we note $z(r) > y(r)$ for all $r \in [0, \bar{r}]$. Throughout, we assume

$$\min\{t, t_L\} > \max_{r \in [0,\bar{r}]} \{2z(r)\} \equiv 2z(r^*). \tag{26}$$

As will be shown below, condition (26) ensures that the market share expressions below are well-behaved, and that the second-order conditions for the platform's profit-maximizing pricing choices hold.

To express s_1 and s_2 explicitly in terms of prices, we substitute $U_i = y(r_i)s_i$ (given $\varphi = 1$) to get

$$\begin{aligned} s_1 &= \frac{1}{2} + \frac{y(r_1)s_1 - y(r_2)s_2 + P_2^B - P_1^B}{2t} + \frac{b + y(r_1)s_1 - P_1^B}{t_L} \\ s_2 &= \frac{1}{2} - \frac{y(r_1)s_1 - y(r_2)s_2 + P_2^B - P_1^B}{2t} + \frac{b + y(r_2)s_2 - P_2^B}{t_L}, \end{aligned}$$

which upon solving implies

$$s_1 = \frac{1}{2} + \frac{(2P_2^B - 2P_1^B + y(r_1) - y(r_2))t_L^2 + 2(tt_L - ty(r_2) - t_L y(r_2))\left(2b - 2P_1^B + y(r_1)\right)}{4tt_L^2 + 4(t + t_L)y(r_1)y(r_2) - (2t_L^2 + 4tt_L)(y(r_1) + y(r_2))}.$$

The denominator of s_1 is positive because

$$\begin{aligned} & 4tt_L^2 + 4(t + t_L)y_1 y_2 - (2t_L^2 + 4tt_L)(y_1 + y_2) \\ > \ & 4tt_L^2 + 4(t + t_L)t_L^2 - 2(2t_L^2 + 4tt_L)t_L = 0, \end{aligned}$$

where the inequality uses that the denominator is decreasing in y_1 and y_2 and that $y(r) < t_L$ by (26).

Note that s_i is decreasing in P_i^B, and so the reframing technique used to establish Proposition 1 continues to apply. Then following the derivation associated with (12), we know that each platform's optimal r_i is independent of its market share s_i, and the equilibrium r^* maximizes $z(r)$. Hence, in what follows, we focus on the symmetric commission $r_1 = r_2 = r$. In this case, the market share expressions simplify to

$$\begin{aligned} s_1 &= \frac{1}{2} + \frac{t_L^2\left(P_2^B - P_1^B\right)}{2(t_L - y(r))(tt_L - (t + t_L)y(r))} + \frac{2\left(b - P_1^B\right) + y(r)}{2(t_L - y(r))} \\ s_2 &= \frac{1}{2} + \frac{t_L^2\left(P_1^B - P_2^B\right)}{2(t_L - y(r))(tt_L - (t + t_L)y(r))} + \frac{2\left(b - P_2^B\right) + y(r)}{2(t_L - y(r))}, \end{aligned} \tag{27}$$

where the denominators are positive due to (26) as noted above.

□ **Proof of Proposition 5.** Platform profit functions are $P_1^B s_1 + R_1$ and $P_2^B s_2 + R_2$ respectively, where recall

$$R_i = \frac{1}{k_{\max}} rp(r)q(r)\pi(r)s_i^2.$$

For any given r, solving the symmetric FOCs with respect to P_i^B gives

$$P^{B*} = \frac{\left(tt_L^2 + (t + t_L)y(r)(2z(r) - y(r)) - t_L(2t + t_L)z(r)\right)(t_L + 2b)}{t_L^3 + 4tt_L^2 + 4(t + t_L)y(r)z(r) - t_L(4t + 3t_L)y(r) - 2t_L(2t + t_L)z(r)}.$$

Condition (26) implies denominator of P^{B*} expression is positive given $z(r^*) > y(r)$ for all r. Substituting P^{B*} back into the expressions for s_i given by 27, the symmetric equilibrium measure of buyers on each platform will be

$$s^* = \left(b + \frac{t_L}{2}\right)\frac{2tt_L + t_L^2 - 2(t + t_L)y(r)}{t_L^3 + 4tt_L^2 + 4(t + t_L)y(r)z(r) - t_L(4t + 3t_L)y(r) - 2t_L(2t + t_L)z(r)}.$$

Note s^* is increasing in $y(r)$; and it is also increasing in $z(r)$ if $t_L(2t+t_L) > 2(t+t_L)\,y(r)$, which holds due to (26).

Now consider W^{SE}, which is equal to

$$W^{SE}(r) = 2bs^* + 2z(r)\,(s^*)^2 - \frac{t}{4} - t_L\left(s^* - \frac{1}{2}\right)^2.$$

Given $z(r)$ is maximized at r^*, at r^* a small change in r only changes W^{SE} via s^*. Then

$$\frac{dW^{SE}}{dr}|_{r=r^*} = \frac{\partial W^{SE}}{\partial s^*}\frac{ds^*}{dr}|_{r=r^*},$$

where

$$\begin{aligned}
\frac{\partial W^{SE}}{\partial s^*} &= 2b + 4z(r)s^* - 2t_L\left(s^* - \frac{1}{2}\right) \\
&= 2b + t_L - 2\left(t_L - 2z(r)\right)s^* \\
&= 2t_L\left(b + \frac{t_L}{2}\right)\frac{2t\left(t_L - y\left(r\right)\right) - t_L y(r)}{t_L^3 + 4tt_L^2 + 4\left(t+t_L\right)y(r)z(r) - t_L\left(4t+3t_L\right)y(r) - 2t_L\left(2t+t_L\right)z\left(r\right)} \\
&= 2t_L s^*\frac{2t\left(t_L - y(r)\right) - t_L y(r)}{2tt_L + t_L^2 - 2\left(t+t_L\right)y\left(r\right)} > 0,
\end{aligned}$$

where the last inequality holds given (26) as it implies $tt_L > (t+t_L)\,y(r)$; whereas $\frac{ds^*}{dr}|_{r=r^*}$ has the same sign as $\frac{\partial}{\partial r}y(r) < 0$. We conclude $\frac{dW^{SE}}{dr}|_{r=r^*} < 0$.

□ **Equilibrium existence.** Computing the second derivative of platform profit with respect to P_i^B, concavity holds if

$$-\left(t_L^2 + 2tt_L - 2\left(t_L+t\right)y(r)\right)\left(2tt_L^2 + 2\left(t_L+t\right)y(r)z(r) - t_L\left(2t+t_L\right)\left(y(r)+z(r)\right)\right) < 0.$$

Condition (26) implies $tt_L > (t+t_L)\,y(r)$, and so the first bracketed term is positive. Thus, we require

$$2tt_L^2 + 2\left(t+t_L\right)y(r)z(r) > t_L\left(2t+t_L\right)\left(y(r)+z(r)\right). \tag{28}$$

Note since the expression is linear in $y(r)$, for it to be true for all $0 \leq y(r) \leq z(r)$, it just needs to be true when $y(r) = 0$ and when $y(r) = z\,(r)$. When $y(r) = 0$ it requires $2tt_L > (2t+t_L)\,z(r)$, which is true given $tt_L > (t+t_L)\,z(r)$. When $y(r) = z(r)$, it requires $2tt_L^2 > 2t_L\left(2t+t_L\right)z - 2\left(t+t_L\right)z^2$, which follows from (26).

E.3 Asymmetric platforms

□ **Preliminaries.** Continuing from the *leading example with Hotelling competition*, when platform 1 offers an additional standalone benefit $\beta > 0$, we have

$$s_1 = \frac{1}{2} + \frac{U_1 - U_2 + P_2^B - P_1^B + \beta}{2t},$$

with $s_2 = 1 - s_1$. It is useful to define

$$\begin{aligned}
y(r) &\equiv v\left(r\right)\frac{\pi\left(r\right)}{k_{\max}} \\
z(r) &\equiv \frac{1}{k_{\max}}\left(v\left(r\right) + rp(r)q(r)\right)\pi\left(r\right).
\end{aligned}$$

Throughout, we assume

$$t > \max_{r\in[0,\overline{r}]} \{z(r)\} \equiv z(r^*). \tag{29}$$

As will be shown below, condition (29) ensures that the market share expression below is well-behaved, and that the second-order conditions for the platform's profit-maximizing pricing choices hold.

To express s_1 and s_2 explicitly in terms of prices, we substitute $U_i = y(r_i)s_i$ (given $\varphi = 1$) to get

$$s_1 = \frac{1}{2} + \frac{y(r_1)s_1 - y(r_2)s_2 + P_2^B - P_1^B + \beta}{2t},$$

which implies

$$s_1 = \frac{1}{2} + \frac{P_2^B - P_1^B + \beta}{(2t - y(r_1) - y(r_2))},$$

and $s_2 = 1 - s_1$. Notice the denominator is positive due to (29). Note that s_i is decreasing in P_i^B, and so the reframing technique used to establish Proposition 1 continues to apply. Then following the derivation associated with (12), we know that each platform's optimal r_i is independent of its market share s_i, and the equilibrium r^* maximizes $z(r)$.

□ **Proof of Proposition 6.** Platform profit functions are $P_1^B s_1 + R_1$ and $P_2^B s_2 + R_2$ respectively, where recall

$$R_i = \frac{1}{k_{\max}} rp(r)q(r)\pi(r) s_i^2.$$

For any given r, solving the FOCs gives

$$P_2^{B*} - P_1^{B*} = -2\beta\frac{t - z(r)}{3t - y(r) - 2z(r)}$$

so that

$$s_1^* = \frac{1}{2} + \frac{\beta}{6t - 2y(r) - 4z(r)}. \tag{30}$$

Since r^* is the maximizer of $z(r)$ and given $y(r)$ is decreasing in r, we have $\frac{ds_1^*}{dr}|_{r=r^*} < 0$.

Now consider W^{SE}, which is equal to

$$\begin{aligned} W^{SE} &= b + U_1 s_1^* + U_2 s_2^* + R_1 + R_2 - \frac{t}{2}(s_1^*)^2 - \frac{t}{2}(s_2^*)^2 + \beta s_1^* \\ &= b + \left(z(r) - \frac{t}{2}\right)(s_1^{*2} + s_2^{*2}) + \beta s_1^*. \end{aligned}$$

Given $z(r)$ is maximized at r^*, at r^* a small change in r only changes W^{SE} via s^*. Then

$$\frac{dW^{SE}}{dr}|_{r=r^*} = \frac{\partial W^{SE}}{\partial s_1^*}\frac{ds_1^*}{dr}|_{r=r^*} < 0$$

because

$$\begin{aligned} \frac{\partial W^{SE}}{\partial s^*} &= 2(2s_1 - 1)\left(z(r) - \frac{t}{2}\right) + \beta \\ &= \beta\left(\frac{2t - y(r)}{3t - y(r) - 2z(r)}\right) > 0, \end{aligned}$$

where the last inequality holds given (29) as it implies $t > y(r)$.

F Details for Section 5

We first verify the equilibrium construction stated in the proof of Proposition 7, and then provide the omitted details corresponding to Sections 5.2 and 5.3.

F.1 Equilibrium with spillovers

To characterize any symmetric equilibrium (a^*, P^{B*}), we consider an off-path "semi-symmetric" participation equilibrium when one of the platforms (say platform $i = 1$) deviates and sets $\left(a_i, P_i^B\right) \neq (a^*, P^{B*})$, resulting in an off-equilibrium path instrument vector profile

$$\hat{\boldsymbol{a}} = (a_i, a^* \mathbf{1}_{m-1}) = (a_i, a^*, ..., a^*) \in \mathcal{A}^m,$$

buyer-side price profile $(P_i^B, P^{B*}, ..., P^{B*})$ and buyer-side market share profile:

$$(s_i, \frac{1-s_i}{m-1}\mathbf{1}_{m-1}) = \left(s_i, \frac{1-s_i}{m-1}, \cdots, \frac{1-s_i}{m-1}\right),$$

where $\mathbf{1}_{m-1}$ is a $1 \times (m-1)$ vector of ones. That is, all other $m-1$ platforms $j \neq i$ equally absorb the resulting change in market share (due to symmetry and the market being covered), resulting in $s_j = \frac{1-s_i}{m-1}$.

Then, the fixed-point definition of market share s_i in (2) becomes

$$s_i = \Phi\left(U_i(\hat{\boldsymbol{a}}; s_i, \frac{1-s_i}{m-1}\mathbf{1}_{m-1}) - U_j(\hat{\boldsymbol{a}}; s_i, \frac{1-s_i}{m-1}\mathbf{1}_{m-1}) - P_i^B + P^{B*}\right).$$

Notice we are expressing U_i and U_j as functions of $(a_1, a_2, ..., a_m; s_1, s_2, ..., s_m)$ in the exact stated order. Therefore, we let $\partial U_i/\partial s_i$ and $\partial U_j/\partial s_i$ (likewise, $\partial U_i/\partial s_j$ and $\partial U_j/\partial s_j$) denote the partial derivative of U_i and U_j with respect to their $m+i$-th argument (likewise, $m+j$-th argument). Then, the slope of the right-hand-side with respect to s_i is

$$\begin{aligned} &\Phi' \times \left(\frac{\partial U_i}{\partial s_i} - \frac{\partial U_j}{\partial s_i} - \frac{1}{m-1}(\frac{\partial U_i}{\partial s_j} - \frac{\partial U_j}{\partial s_j}) - \frac{1}{m-1}\sum_{l\neq i,j}(\frac{\partial U_i}{\partial s_l} - \frac{\partial U_j}{\partial s_l})\right) \\ < \quad & B_\Phi \times \left(\frac{m}{m-1}B_{Uown} + \frac{m}{m-1}B_{Ucross} + \frac{m-2}{m-1}2B_{Ucross}\right), \end{aligned}$$

where

$$\begin{aligned} B_\Phi &\equiv \sup_{x\in\mathbb{R}} \Phi'(x) \\ B_{Uown} &\equiv \sup_{\boldsymbol{a}\in\mathcal{A}^m} \sup_{\boldsymbol{s}\in[0,1]^m} |\frac{\partial}{\partial s_i}U_i(\boldsymbol{a}, \boldsymbol{s})| \\ B_{Ucross} &\equiv \sup_{\boldsymbol{a}\in\mathcal{A}^m} \sup_{\boldsymbol{s}\in[0,1]^m} |\frac{\partial}{\partial s_j}U_i(\boldsymbol{a}, \boldsymbol{s})|. \end{aligned}$$

Therefore, to ensure the existence of a fixed point, a formal sufficient condition is $2B_\Phi \times (B_{Uown} + 2B_{Ucross}) < 1$. Under this condition, the resulting demand system is analogous to standard discrete choice models.

Platform i chooses $\left(a_i, P_i^B\right)$ to maximize profit Π_i, taking as given (a^*, P^{B*}) set by each other platform. Following the approach of Armstrong (2006) and Tan and Zhou (2021), to solve this maximization problem, we reframe the problem as platform i directly choosing the target market share s_i implementable by its buyer-side price P_i^B, i.e., maximization with respect to (a_i, s_i). Formally, this is done by inverting (6), so that P_i^B becomes a function of (a_i, s_i) satisfying:

$$P_i^B = P^{B*} + U_i(\hat{\boldsymbol{a}}; s_i, \frac{1-s_i}{m-1}\mathbf{1}_{m-1}) - U_j(\hat{\boldsymbol{a}}; s_i, \frac{1-s_i}{m-1}\mathbf{1}_{m-1}) - \Phi^{-1}(s_i).$$

Then, platform i's problem is to choose (a_i, s_i) to maximize

$$\begin{aligned}\Pi_i(a_i; s_i) &= P_i^B s_i + R_i(a_i; s_i) \\ &= \left(P^{B*} + U_i(\hat{\boldsymbol{a}}; s_i, \frac{1-s_i}{m-1}\mathbf{1}_{m-1}) - U_j(\hat{\boldsymbol{a}}; s_i, \frac{1-s_i}{m-1}\mathbf{1}_{m-1}) - \Phi^{-1}(s_i)\right) s_i + R_i(\hat{\boldsymbol{a}}; s_i, \frac{1-s_i}{m-1}\mathbf{1}_{m-1}).\end{aligned}$$

To ensure the existence of a symmetric equilibrium, we assume that Π_i is globally strictly quasiconcave in (a_i, s_i), as in the baseline model. In any symmetric equilibrium, each platform's optimal choice of $a_i = a^*$ is a maximizer of $\Pi_i(a_i; s_i)$ while holding $s_i = 1/m$ and the instrument choices of other platforms constant at a^*. That is,

$$a^* \in \arg\max_{a_i \in \mathcal{A}} \left\{\frac{1}{m}U_i(\hat{\boldsymbol{a}}; \frac{1}{m}\mathbf{1}) - \frac{1}{m}U_j(\hat{\boldsymbol{a}}; \frac{1}{m}\mathbf{1}) + R_i(\hat{\boldsymbol{a}}; \frac{1}{m}\mathbf{1})\right\},$$

which is exactly (17). Meanwhile, the derivative of Π_i with respect to s_i (using $s_j = \frac{1-s_i}{m-1}$, as noted above) is

$$\begin{aligned}\frac{d\Pi_i}{ds_i} = &\left(\frac{\partial U_i}{\partial s_i} - \frac{\partial U_j}{\partial s_i} - \frac{1}{m-1}(\frac{\partial U_i}{\partial s_j} - \frac{\partial U_j}{\partial s_j}) - \frac{1}{m-1}\sum_{l\neq i,j}(\frac{\partial U_i}{\partial s_l} - \frac{\partial U_j}{\partial s_l}) - \frac{1}{\Phi'}\right) s_i + \left(\frac{\partial R_i}{\partial s_i} - \frac{1}{m-1}\sum_{l\neq i}\frac{\partial R_i}{\partial s_l}\right) \\ &+ P^{B*} + U_i - U_j - \Phi^{-1}(s_i),\end{aligned}$$

where we have omitted function arguments. Imposing symmetry, that is, $\frac{\partial U_i}{\partial s_i} = \frac{\partial U_j}{\partial s_j}$, $\frac{\partial U_i}{\partial s_j} = \frac{\partial U_j}{\partial s_i}$ for $i \neq j$ and $\frac{\partial U_i}{\partial s_l} = \frac{\partial U_j}{\partial s_l}$, and $\frac{\partial R_i}{\partial s_l} = \frac{\partial R_i}{\partial s_j}$ for $l \neq i, j$, we get

$$\frac{d\Pi_i}{ds_i} = \left(\frac{m}{m-1}(\frac{\partial U_i}{\partial s_j} - \frac{\partial U_j}{\partial s_j}) - \frac{1}{\Phi'}\right)\frac{1}{m} + (\frac{\partial R_i}{\partial s_i} - \frac{\partial R_i}{\partial s_j}) + P^{B*} - \Phi^{-1}(\frac{1}{m}).$$

So the FOC gives

$$P^{B*} = \frac{1}{m\Phi'(0)} - \frac{1}{m-1}(\frac{\partial U_i(\boldsymbol{a}; \boldsymbol{s})}{\partial s_i} - \frac{\partial U_i(\boldsymbol{a}; \boldsymbol{s})}{\partial s_j}) - (\frac{\partial R_i(\boldsymbol{a}; \boldsymbol{s})}{\partial s_i} - \frac{\partial R_i(\boldsymbol{a}; \boldsymbol{s})}{\partial s_j}), \tag{31}$$

where the derivatives are evaluated at the symmetric outcome $(\boldsymbol{a}; \boldsymbol{s}) = (a^*\mathbf{1}; \frac{1}{m}\mathbf{1})$.

Meanwhile, the welfare objectives are given by

$$\begin{aligned}W^{SE}(a) &= \int_{\underline{\epsilon}}^{\bar{\epsilon}} \left[\hat{\epsilon} + U_i(a\mathbf{1}; \frac{1}{m}\mathbf{1})\right] d\hat{F}(\hat{\epsilon}) + mR_i(a\mathbf{1}; \frac{1}{m}\mathbf{1}), \\ W(a) &= W^{SE}(a) + mSS_i(a\mathbf{1}; \frac{1}{m}\mathbf{1}).\end{aligned}$$

Given SS_i is decreasing in a, it is immediately clear that $a^{SE} \geq a^W$ (as claimed in Lemma 1).

F.2 Spillovers from seller singlehoming

We continue from the *leading example* and assume that sellers' outside option is zero, and each seller is indexed by $(k_1, ..., k_m) \in [k_{\min}, k_{\max}]^m$. We add a standalone benefit b_S to seller's participation utility from joining platform i, which is now

$$b_S + \pi(r_i)s_i - k_i.$$

We assume b_S is sufficiently high to ensure full coverage of the seller-side market. Denote $\Psi(\cdot)$ as the CDF of $k_i - \max_{j\neq i}\{k_j\}$ and the corresponding derivative is denoted as $\Psi'(\cdot)$. To ensure that seller participation is well behaved, as we did on the buyer side, we assume that the extent of heterogeneity in sellers' idiosyncratic draws of participation costs $(k_1, ..., k_m)$, as measured by $1/\Psi' > 0$, is large enough.

We know from (17) that

$$r^* = \arg\max_{r_i\in[0,\overline{r}]} \left\{ \frac{1}{m}U_i - \frac{1}{m}U_j + R_i \right\}.$$

Due to the semi-symmetry structure in the off-equilibrium path when one platform i deviates, we have $n_j = \frac{1-n_i}{m-1}$ for $j \neq i$, and so

$$r^* = \arg\max_{r_i\in[0,\overline{r}]} \left\{ \frac{1}{m}\left(v(r_i)n_i - v(r^*)\frac{1-n_i}{m-1} \right) + \frac{r_i p(r_i)q(r_i)}{m} n_i \right\},$$
$$\text{where } n_i = \Psi\left(\frac{\pi(r_i) - \pi(r^*)}{m} \right).$$

It is useful to define

$$z(r_i) \equiv v(r_i) + r_i p(r_i)q(r_i).$$

Then, ignoring boundary conditions, the corresponding FOCs for r^* is

$$\left(\frac{mv(r^*)}{m-1} + r^* p(r^*)q(r^*) \right) \frac{\Psi'(0)}{m^2}\frac{d\pi(r^*)}{dr_i} + \frac{dz(r^*)}{dr_i}\frac{1}{m^2} = 0. \tag{32}$$

Meanwhile, the welfare benchmarks have

$$\begin{aligned} r^{SE} &= \arg\max_{r_i\in[0,\overline{r}]} \{v(r_i) + r_i p(r_i)q(r_i)\} = \arg\max_{r_i\in[0,\overline{r}]} z(r_i) \\ r^W &= \arg\max_{r_i\in[0,\overline{r}]} \{v(r_i) + (p(r_i) - c)q(r_i)\} = 0. \end{aligned}$$

Next, given $d\pi/dr_i < 0$, it is clear that $r^{SE} \geq r^*$, with strict inequality if r^* or r^{SE} is an interior solution, or if $r^* = 0$ and $r^{SE} = \bar{r}$.

□ **A closed-form solution.** To proceed further, suppose seller marginal cost is $c = 0$, so that $v(r_i) = v$, and $p(r_i)q(r_i) = pq$ are now constants that are independent of the commission rate r_i. Then, $dz/dr_i = -d\pi/dr_i = pq > 0$, and so (32) simplifies to

$$-\left(\frac{vm}{m-1} + r^* pq \right) \Psi'(0) + 1 = 0$$

$$\Longrightarrow r^* = \frac{1}{pq}\left(\frac{1}{\Psi'(0)} - \frac{vm}{m-1} \right),$$

whereas $r^{SE} = \bar{r}$ (where $\bar{r} = 1$ due to $c = 0$). Therefore, we have

$$r^* < r^{SE} = \bar{r} \quad \text{if} \quad \frac{vm}{m-1} > \frac{1}{\Psi'(0)} - pq$$

and

$$r^* > r^W = 0 \quad \text{if} \quad \frac{vm}{m-1} < \frac{1}{\Psi'(0)}.$$

In particular, in the equilibrium the baseline distortion is completely mitigated (i.e., $r^* = r^W < r^{SE}$) if $\frac{vm}{m-1} \geq \frac{1}{\Psi'(0)}$. This holds when the extent of heterogeneity in sellers' idiosyncratic draws of participation costs $(k_1, ..., k_m)$ is low (provided the symmetric equilibrium still exists — see, e.g., the two-sided Hotelling specification below). If we allow platforms to choose negative commissions $r_i < 0$, then it is straightforward to show that $r^W = 0$ continues to hold, so that $\frac{vm}{m-1} > \frac{1}{\Psi'(0)}$ implies a reversion of the sign of distortion in equilibrium (i.e., $r^* < r^W$). Intuitively, the reversion reflects that platforms are overly focused on attracting sellers and thus subsidize sellers by too much relative to the socially optimal level.

As an illustration, we consider the following *two-sided Hotelling specification* with $m = 2$ platforms. That is, the buyer-side participation demand is $\Phi(x) = \frac{1}{2} + \frac{x}{2t_B}$ whereas the seller-side participation demand is $\Psi(x) = \frac{1}{2} + \frac{x}{2t_S}$, where t_B and t_S are the respective mismatch cost parameters. Then,

$$r^* = \frac{2}{pq}(t_S - v).$$

Meanwhile, the pricing equation (31) and

$$\begin{aligned} U_i &= v\left(\tfrac{1}{2} + \tfrac{(1-r_i)pqs_i-(1-r_j)pqs_j}{2t_S}\right) \\ R_i &= r_i pqs_i\left(\tfrac{1}{2} + \tfrac{(1-r_i)pqs_i-(1-r_j)pqs_j}{2t_S}\right), \end{aligned}$$

imply

$$P^{B*} = t_B - \left(v + \frac{r^*pq}{2}\right)\frac{(1-r^*)pq}{t_S} - \frac{r^*pq}{2}.$$

Suppose $p = q = 1$, and $t_B = t_S = 2$. We can verify that the symmetric equilibrium exists for v in the range $[1.5, 2]$ which maps out $r^* = 1$ down to $r^* = 0$. Note that at $v = 2$, we have $r^* = 0$, illustrating that the outcome of $r^* = r^W < r^{SE}$ does not necessarily violate equilibrium existence.

□ **Seller-side lump-sum fees**. We can apply our formula (17) to the case of seller-side lump-sum fees P_i^S considered by Armstrong (2006) and Tan and Zhou (2021). Given the absence of commissions, we can drop the function arguments in v and π. By the same analysis as above, P^{S*} is the maximizer of

$$\begin{aligned} P^{S*} &= \arg\max_{P_i^S}\left\{\frac{1}{m}U_i - \frac{1}{m}U_j + R_i\right\} \\ &= \arg\max_{P_i^S}\left\{\frac{v}{m}\left(n_i - \frac{1-n_i}{m-1}\right) + P_i^S n_i\right\}, \end{aligned}$$

where $n_i = \Psi\left(P^{S*} - P_i^S\right)$. Note we do not need the domain of feasible P_i^S to be compact for this maximization problem to be well-defined. The corresponding FOC is

$$P^{S*} = \underbrace{\frac{1/m}{\Psi'(0)}}_{\text{market power}} - \underbrace{\frac{v}{m-1}}_{\text{cross-subsidization due to benefits enjoyed by buyers}},$$

which is a special micro-founded case of the equilibrium pricing formula obtained by Tan and Zhou (2021).

F.3 Spillovers from seller-side post-participation decisions

Throughout this subsection, we assume all sellers have zero fixed costs and zero participation costs $k_i = 0$ (i.e., the distribution G is degenerate) in order to show spillovers can arise absent any fixed participation cost.

□ **Price coherence.** We first prove the claim on $p(r^{avg})\,q(r^{avg})$ being decreasing in r^{avg}. Whenever a seller is subjected to price coherence, the seller chooses its common price p to maximize

$$\left(\sum\nolimits_{i\in\phi} s_i\left((1-r_i)p - c\right)\right)D(p),$$

which can be rewritten as

$$\left((1-r^{avg})p - c\right)D(p)\sum\nolimits_{i\in\phi} s_i,$$

where $r^{avg} = \frac{1}{\sum_{i\in\phi} s_i} \sum_{i\in\phi} s_i r_i$. We denote the optimal price as $p(r^{avg})$. Given that $D(p)$ is strictly log-concave, $p(r^{avg})$ is given by the FOC:

$$\begin{aligned} p &= \frac{c}{1-r^{avg}} - \frac{D(p)}{D'(p)} \\ &\implies pD'(p) < -D(p). \end{aligned}$$

The last inequality implies

$$\begin{aligned} \frac{d}{dr^{avg}} p\left(r^{avg}\right) q(r^{avg}) &= \frac{d}{dr^{avg}} p\left(r^{avg}\right) D(p(r^{avg})) \\ &= \underbrace{\left(D(p) + pD'(p)\right)}_{<0} \underbrace{\frac{dp}{dr^{avg}}}_{>0} < 0. \end{aligned}$$

Similar to the *leading example*, we denote $\pi(r) = \max_p \left((1-r)p - c\right) D(p)$. Then, the seller has joined a set ϕ of platforms (and subjected to price coherence) earns profit $\sum_{i\in\phi} s_i \pi(r^{avg})$.

Next, we verify the claims that all sellers will multihome on all platforms as long as the commission difference $\max_{j\neq i} |r_i - r_j|$ is not too large, and that the platforms have no incentive to deviate and induce large commission differences if β is small enough. Without loss of generality, it suffices to focus on the case where platform i sets $r_i \leq r^*$ while all other platforms $j \neq i$ set $r_j = r^*$. Consider an individual seller's decision on whether to multihome. Clearly, all sellers who are not subjected to price coherence would prefer to multihome. For the sellers subjected to price coherence, multihoming on all platforms is always better than joining only the higher-commission platforms (platform $j \neq i$) because

$$\begin{aligned} \pi_{all} &= \pi(r^*(1-s_i) + r_i s_i) \\ &> \pi(r^*) \\ &> \pi(r^*)(1-s_i) = \pi_{j \text{ only}}, \end{aligned}$$

since $\pi(.)$ is a decreasing function. Meanwhile, multihoming is better than singlehoming on the lower-commission platform (platform i) if and only if

$$\pi_{all} = \pi(r^*(1-s_i) + r_i s_i) \geq \pi(r_i)s_i,$$

which holds if and only if the commission difference $r^* - r_i$ is small enough.

We now verify that platforms have no incentive to set a large difference in commission as long as ω is sufficiently small. Let us pin down the equilibrium commission level r^*. Recall

$$\begin{aligned} &U_i = \omega v(r^{avg}) + (1-\omega)v(r_i) \\ &R_i = r_i \left(\omega p\left(r^{avg}\right) q(r^{avg}) + (1-\omega)p(r_i)q(r_i)\right) s_i. \end{aligned}$$

Assuming all sellers multihome on all platforms in the equilibrium, the FOC satisfies:

$$\left(\frac{\partial U_i}{\partial r_i} - \frac{\partial U_{-i}}{\partial r_i}\right)\frac{1}{m} + \frac{\partial R_i}{\partial r_i} = 0$$

$$\iff \frac{(1-\omega)v'(r^*)}{m} + \frac{p\left(r^*\right) q(r^*)}{m} + \frac{r^*}{m}\left(\frac{\omega}{m} + 1 - \omega\right)\left(p'\left(r^*\right) q(r^*) + p\left(r^*\right) q'(r^*)\right) = 0.$$

Observe that r^* is increasing in ω because the derivative of the left-hand-side with respect to ω is

$$-\frac{v'(r^*)}{m} - \frac{r^*}{m}\left(\frac{m-1}{m}\right)\underbrace{\left(p'\left(r^*\right) q(r^*) + p\left(r^*\right) q'(r^*)\right)}_{<0} > 0.$$

Suppose platform i wants to deviate by choosing $(r_i, P_i^B) \neq (r^*, P^{B*})$ to induce some sellers to single-home. Recall this necessarily requires $r_i < r^*$. This is applicable only to the mass ω of sellers that are subjected to price coherence. A successful deviation requires

$$\pi(s_i r_i + (1 - s_i)r^*) < \pi(r_i)s_i.$$

Let us denote the maximum deviation fee as r^{dev}, which we know is *strictly* below r^* as long as $s_i < 1$ (i.e., buyer-side heterogeneity is not too small), for any $\omega \geq 0$. With this undercutting strategy, buyers expect utility difference

$$U_i - U_{-i} = v(r^{dev}) - (1 - \omega)v(r^*) + P_i^B - P^{B*}$$

and the deviation platform profit is

$$\Pi^{dev} = \max_{P_i^B; r_i \leq r^{dev}} \left\{ \begin{array}{c} (P_i^B + r_i p(r_i)q(r_i)) \\ \times \Phi(v(r^{dev}) - (1 - \omega)v(r^*) + P_i^B - P^{B*}) \end{array} \right\}.$$

Furthermore, observe that the equilibrium platform profit can be expressed as

$$\begin{aligned} \Pi^* &= (P^{B*} + r^* p(r^*)q(r^*))\frac{1}{m} \\ &= \max_{P_i^B; r_i} \left\{ \begin{array}{c} (P_i^B + \omega r_i p(r^{avg})q(r^{avg}) + (1 - \omega)r_i p(r_i)q(r_i)) \\ \times \Phi((1 - \omega)(v(r_i) - v(r^*)) + P_i^B - P^{B*}) \end{array} \right\}. \end{aligned}$$

Therefore, if $\omega \to 0$, then the two objective functions coincide. Therefore, the constraint of $r^{dev} < r^*$ implies $\Pi^{dev} < \Pi^*$.

□ **Seller investment that applies to all platforms.** Consider our *leading example.* Suppose in addition to setting prices, sellers can choose how much to invest to raise their product demand in ways that are not platform-specific (e.g., this could include investments in broad marketing efforts or quality improvements).

Specifically, each buyer chooses the number of units to purchase q_i to maximize their net utility; i.e., $\arg\max_{q_i} \{u(q_i)B(I_s) - p_i q_i\}$, where $B(I_s) > 0$ indicates the utility enhancement due to seller investment and I_s is a seller's investment level. We assume $B(\cdot)$ is differentiable, and the derivative $B'(\cdot) > 0$. We assume sellers face the associated corresponding cost function $K(I_s)$, where K is increasing and strictly convex, with boundary conditions $\lim_{I_s \to \infty} K'(I_s) = \infty$ and $K'(0) = 0$. Sellers are assumed to set I_s at the same time as their prices on the different platforms. All sellers participate given the absence of participation fixed cost.

Suppose each platform chooses $r_i \in [0, \bar{r}]$. We let $c = 0$ to simplify seller pricing. Then we define a seller's quality-adjusted price $\hat{p}_i = \frac{p_i}{B(I_s)}$, and denote the optimal quality-adjusted price as

$$\hat{p} \equiv \arg\max_{\hat{p}_i} (1 - r_i) B(I_s) \hat{p}_i D_i(\hat{p}_i),$$

which does not depend on either r_i or I_s. The per-buyer gross profit (not including investment costs) of each seller is $(1 - r_i) B(I_s) \pi^m$ and the per-seller surplus of the buyer is $B(I_s) v^m$, where $\pi^m = \hat{p} D(\hat{p})$ and $v^m = u(D(\hat{p})) - \hat{p} D(\hat{p})$.

Each seller's optimal investment maximizes

$$\sum_{i=1}^{m} (1 - r_i) B(I_s) s_i \pi^m - K(I_s).$$

The above conditions ensures a seller's optimal investment I_s^* is uniquely defined, strictly positive, and satisfies the FOC

$$\sum_{i=1}^{m} (1 - r_i) B' (I_s^*) s_i \pi^m = K'(I_s^*).$$

Moreover, I_s^* is decreasing in r_i on each platform i. As a result, both $U_i = B(I_s^*) v^m$ and $R_i = r_i B(I_s^*) \pi^m s_i$ are decreasing in r_j for $j \neq i$. Therefore, there are negative spillovers and $r^* \geq r^{SE} \geq r^W$.

□ **Platform and seller investment.** Continue from the setting immediately above (which we refer to as the *seller-only investment* application) and suppose now that each platform chooses $a_i = -I_i$, where I_i is platform i's level of investment with associated convex cost $C(I_i)$. We keep the commission rate $r_i = r \in [0, \bar{r}]$ fixed and equal across all platforms $i = 1, ..., m$. Note that we define the platform instrument in terms of the negative of I_i to maintain the order of a_i, which recall was defined so that a higher a_i corresponds to a lower seller surplus.

Platform i's investment I_i scales up the buyer's gross utility obtained from transacting with any seller. The gross utility of buyers is now $u(q_i)B(I_s, I_i)$, where I_s is a seller's investment with the corresponding cost function $K(I_s)$, with the properties defined in the *seller-only investment* application above. We assume B is differentiable and increasing in both its arguments, with $B_1(I_s, I_i) > 0$ when evaluated at $I_s = 0$, and $B_1(I_s, I_i)$ weakly decreasing in I_s. This combination of assumptions ensures that each seller's optimal investment is unique, strictly positive, and finite. We say the two types of investments are complements (substitutes) if $B_1(I_s, I_i)$ is everywhere increasing (decreasing) in I_i. The timing is that platforms set their investments first (at the same time as their prices to buyers), before sellers set their investments and prices.

Defining the seller's quality-adjusted price

$$\hat{p}_i = \frac{p_i}{B(I_s, I_i)}$$

each seller sets $\hat{p}_i$ to maximize $(1 - r) B(I_s, I_i) \hat{p}_i q_i(\hat{p}_i)$. Let the resulting profit maximizing price be denoted $\hat{p}$, which does not depend on either r, I_s or I_i. The per-buyer gross profit (not including investment costs) of each seller is $(1 - r) B(I_s, I_i) \pi^m$ and the per-seller surplus of the buyer is $B(I_s, I_i)v^m$, where π^m and v^m are defined in the *seller-only investment* application above.

Each seller's optimal investment maximizes

$$\sum_{i=1}^{m} (1 - r) B(I_s, I_i) s_i \pi^m - K(I_s).$$

The above conditions ensures a seller's optimal investment I_s^* is uniquely defined, strictly positive, and satisfies the FOC

$$\sum_{i=1}^{m} (1 - r) B_1 (I_s^*, I_i) s_i \pi^m = K'(I_s^*).$$

Moreover, I_s^* is decreasing (increasing) in $a_i = -I_i$ on each platform i if the two types of investments are complements (substitutes). As a result, both $U_i = B(I_s^*, I_i) v^m$ and $R_i = rB(I_s^*, I_i) \pi^m s_i - C(I_i)$ are decreasing (increasing) in $a_j = -I_j$ for $j \neq i$ if the two types of investments are complements (substitutes).

Therefore, there are negative spillovers and $I^* \leq I^{SE} \leq I^W$ (since $a^* \geq a^{SE} \geq a^W$) if the two types of investments are complements, and there are positive spillovers and $I^* \geq I^{SE}$ (since $a^* \leq a^{SE}$) which mitigates the baseline distortion that $I^{SE} \leq I^W$ if the two types of investments are substitutes.

□ **Promotion of sellers' direct channel.** We continue from Example 2 in Online Appendix A and modify it by allowing sellers to promote their direct channels. Specifically, suppose each seller chooses the amount to spend on promoting their direct channel (say spending on an advertising campaign on it), denoted as κ. Then, each buyer will become aware of the seller's direct channel with some positive probability $0 \leq Y(\kappa) \leq 1$, where $Y(0) = 0$, $Y(\infty) = 1$, $Y' > 0$ and $Y'' < 0$. Thus, if λ_i of a seller's buyers on platform i are initially uninformed of its direct channel, after promoting its direct channel, only $\lambda_i (1 - Y(\kappa))$ of its buyers on platform i will remain uninformed.

Given $G(\cdot)$ is degenerate, we know all sellers will always choose to multihome on all platforms due to the fact that sellers do not face any restrictions in setting the on-platform prices, face no other costs, and still keep a fraction of their revenues. Meanwhile, their pricing problem remains the same as Example 2. Therefore,

$$U_i = v^m.$$

Meanwhile, a seller's total profit is $\sum_{i=1}^{m}(1 - r_i + (1 - \lambda_i (1 - Y(\kappa)))\zeta r_i)\pi^m s_i - \kappa$, and the maximization with respect to κ leads to the optimal promotion spending κ^* satisfying

$$\zeta\pi^* \sum_{i=1}^{m} \lambda_i r_i s_i = \frac{1}{Y'(\kappa^*)},$$

where κ^* is increasing in $\sum_{i=1}^{m} \lambda_i r_i s_i$ given $Y'' < 0$. Moreover,

$$R_i = (1 - (1 - \lambda_i (1 - Y(\kappa^*)))\zeta) r_i \pi^m s_i.$$

Observe that R_i decreases when the "disintermediation-adjusted effective commission" $r_j\lambda_j$ on platform j increases, because a higher effective commission on platform j induces more sellers to invest in promoting their direct channels, i.e., a higher κ^*. Therefore, this direct channel mechanism results in negative spillovers in platform fees r_j and disintermediation prevention efforts λ_j through platform i revenues. We can immediately conclude from Proposition 7 that $r^* \geq r^{SE} \geq r^W$ or $\lambda^* \geq \lambda^{SE} \geq \lambda^W$.

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