Platform Traps*

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Abstract

A novel feature of platforms such as marketplaces and social networks is that non-participants may become worse off as others join. We show that in such settings, rational agents can be induced to join a platform despite being better off without it — a phenomenon we call a platform trap. We provide a theory of how platform traps emerge through one or more of the following additional features: (i) the ability of the platform to make dynamic price adjustments; (ii) the interplay between on-platform and off-platform negative externalities; (iii) favorable equilibrium selection in settings where participation decisions admit multiple equilibria.

1 Introduction

Many businesses increasingly rely on major platforms to reach customers, including third-party sellers on Amazon, hotels on Booking.com and Expedia, and restaurants on DoorDash and Uber Eats. As consumers shift to these platforms rather than shopping directly with sellers, concerns arise about sellers' growing dependence on, and exploitation by, platforms. This can involve higher or new types of fees, as well as an increasing need to advertise to be discovered on the platforms they depend on. But why would businesses join such platforms in the first place, knowing they will ultimately be held up this way?

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In this paper, we provide a theory of why rational agents join a platform even when they end up worse off as a result. We call this a platform trap. The key departure from standard models of platforms is that each agent's relevant outside option is not constant but deteriorates as each agent joins the platform. The theory applies to sellers participating on a marketplace that attracts consumers who previously searched directly for them, and to other settings: neighborhood shops opening outlets in suburban malls or brands supplying Walmart (which draws traffic away from their traditional channels); individuals joining an online social network rather than relying on direct connections (those remaining outside risk social exclusion); and news organizations signing deals with Facebook and Google (cannibalizing traffic on their own websites as users increasingly access news through these platforms).

Because each agent's participation imposes a negative externality on other agents' outside options, agents face a collective action problem: each finds it individually attractive to join but doesn't take into account the resulting degradation of other agents' outside option. The platform exploits this through higher prices, which can ultimately leave all agents worse off. This differs from the standard setting in which a monopoly platform extracts an increasing share of the value created as users join, due to positive network effects. There, the outside option is fixed, so the platform cannot leave users strictly worse off from joining. Moreover, in our setting the platform trap can arise absent network effects, including when on-platform externalities are negative.

Our theory has a finite number of strategic agents, such as sellers on a marketplace, who receive offers sequentially and decide whether to join, taking into account how their decision may affect later agents' decisions. Both the benefit of joining the platform and the outside option can depend on how many agents join. The benefit of the outside option is weakly decreasing in the number of agents participating on the platform and strictly decreasing at some point. The difference between the benefit on the platform and the outside option is weakly increasing as more agents join. We microfound these properties in a marketplace model. As sellers join, they draw buyers from the direct channel to the marketplace and worsen the position of all sellers that are not on the marketplace. We provide conditions under which such a marketplace attracts sellers even though all sellers end up worse off; in some cases, all buyers are also worse off and total welfare is lower than without the platform.

Sequential participation decisions raise the possibility that some agents are pivotal: each of their decisions affects whether later agents can be profitably induced to participate. In this case, the platform must first attract a critical mass of participants before it can offer each later agent more value than the outside option. Once the platform achieves this critical mass, each later agent knows that, regardless of its own decision, the platform can induce all remaining agents to join by adjusting prices, so it is willing to join even when charged a price based on all other agents joining.

In both the microfounded marketplace model and the more general setting that encompasses it, we derive equilibrium prices for the strategic agents and a sufficient condition for a platform trap. When no agents are pivotal, the platform extracts maximum surplus by setting a single price that leaves each agent with the same low surplus as its outside option when all other agents participate. At the other extreme, when all agents are pivotal and the platform provides enough value, it attracts all agents with increasing prices, leaving each agent with the outside option it would obtain when only earlier agents join the platform. In intermediate cases, the platform raises prices until the marginal joining agent is no longer pivotal, and then charges the same maximal price to all remaining agents as when no agents are pivotal.

In this baseline setting, whenever agents participate, all agents would be weakly better off without the platform, and at least some will be strictly better off. Agents would therefore be better off collectively committing to boycott the platform. Furthermore, since the platform may need to offer negative prices to attract the first pivotal agents, it may profitably attract all agents even when doing so is inefficient. In this case, both agent surplus and total surplus can be lower because the platform exists. Yet because the platform also attracts agents when doing so is efficient, the platform-trap problem is distinct from traditional concerns about monopoly platforms causing inefficiency, with neither implying the other.

The baseline logic suggests that dynamic price adjustments are essential for a platform trap. Surprisingly, even a platform restricted to a single price can generate a trap if each agent's platform benefit is single-peaked and peaks before all agents participate.[1] The price is based on this maximum benefit, and if agents' outside option falls fast enough with participation, all agents join and would be better off without the platform. Thus, negative externalities on and off the platform can generate a platform trap even without dynamic price adjustments.

We also examine factors such as simultaneous decisions, imperfect information, negotiated offers, heterogeneous agents and competing platforms, which can amplify or moderate the trap. Multiple equilibria can arise when agents move simultaneously at a constant price or do not observe others' offers and decisions. With equilibrium selection favorable to the platform, the trap arises in a wider set of cases than in the baseline because agents can no longer be pivotal when their decisions are unobserved.

[1] We show how this possibility can arise when a marketplace platform attracts competing sellers.

With unfavorable equilibrium selection, a platform trap can still arise, but only if on-platform externalities are negative. When the platform negotiates with each agent rather than making take-it-or-leave-it offers, the model can generate increasing price dynamics even without pivotal agents. Prices increase because each joining agent endogenously weakens the bargaining position of remaining agents. When one agent is larger than all others (a "superstar" agent), the platform must decide whether to attract that agent first or last. The optimal choice depends crucially on the curvature of the externality on the outside option. Finally, platform competition can mitigate but need not eliminate the trap.

2 Literature review

A platform trap is closely related to the collective traps introduced and examined by Bursztyn et al. (2025a). They provide experimental evidence that total consumer welfare from the availability of Instagram and TikTok is negative in their student sample, even though these individuals are willing to pay to use them. They propose a mechanism in which users impose a negative externality on non-users, for example through social exclusion or fear of missing out. As more people join, marginal users may participate to avoid worse non-participation outcomes, even if they experience negative overall utility from the platform. This mechanism is consistent with ours: the outside option weakens as platform participation increases. Bursztyn et al. offer a conceptual framework to interpret their results, but do not formalize how, or under what conditions, a collective trap can emerge through individual choices, nor how platforms might engineer such traps in a dynamic setting. Their focus is instead empirical: demonstrating and quantifying the collective trap rather than providing a theory of it.[2]

On the theory side, our paper relates to settings where a principal (the platform) contracts with multiple agents subject to cross-agent externalities. Segal (1999) provides the most general formulation, encompassing many applications. One application is Katz and Shapiro (1986), where a sponsored technology (the principal) licenses its technology and competes for users with an unsponsored technology (the outside option), with both technologies subject to network effects. Another application is Segal and Whinston (2000), where an incumbent signs exclusive contracts with agents, deny-

[2]Sullivan (2024) provides further empirical evidence consistent with a platform trap. In a counterfactual simulation, he finds that abolishing food delivery platforms in the United States would increase restaurants' profits.

ing an entrant the scale needed to enter and harming agents who do not contract with the incumbent.[3] These settings, including Segal's general framework, differ from ours in various modeling choices, notably their focus on simultaneous agent decisions rather than our sequential baseline. But more fundamentally, they differ in their research question. They analyze inefficiencies in contracts offered by the principal, whereas we ask whether agents would be better off without the principal. For example, Segal and Whinston ask whether the incumbent's exclusive contracts lead to inefficient outcomes, not whether agents would be better off without the incumbent. Indeed, agents would never be better off without the incumbent in their setting, since the entrant, assumed to have the same costs, would then become the monopolist. Thus, there is no platform trap in Segal and Whinston.

Our paper also connects to the literature on data externalities. When more users share personal data, a platform can infer information about users who do not share, or share less. This weakens the outside option of non-participation or limited data disclosure, allowing the platform to collect and monetize more data while providing less compensation for the resulting privacy nuisance (Choi et al., 2019). More broadly, this is an example of social data: individual data are informative about others, creating data externalities that affect the terms on which platforms acquire and use personal information (Bergemann et al., 2022).

A platform trap also differs fundamentally from inefficiencies in the classic literature on network externalities, such as Farrell and Saloner (1985) and Katz and Shapiro (1985). This literature shows that users can become locked into an inferior technology, as in the classic QWERTY example, or inefficiently adopt a new technology due to excess momentum (Farrell and Saloner, 1986; Katz and Shapiro, 1986). Even if only one technology is sponsored, analogous to the platform in our setting, these papers ask whether adoption is inefficient, not whether agents would be better off without the sponsored technology. As we show, whether a platform's existence creates an inefficiency is distinct from whether it makes agents worse off; our setting permits scenarios where either, both, or neither occur.

Within the platform literature, most papers do not study pricing dynamics. Jullien

[3] Their setting differs in several respects: there are no on-platform externalities, and the incumbent can extract surplus from agents who do not sign up if it signs up enough agents exclusively to deter entry by the rival platform. This structure explains why they cannot obtain our result that the platform can trap all agents even under perfectly coalition-proof Nash equilibria with a simultaneous constant price offer when on-platform externalities are negative, or our result that it can be efficient, from a total surplus perspective, for agents to sign up even though they are necessarily worse off. Nevertheless, the notion of pivotal agents in the sequential version of their model is the same as in ours.

et al. (2021) survey some that do. Among them, Biglaiser et al. (2022) is arguably closest to our approach given their agents also make sequential participation decisions. But their focus differs. They study agents moving from an incumbent to an entrant platform to explain incumbency advantage. Our key contribution to platform pricing dynamics is to show how increasing prices can emerge from pivotal agents or from the platform's endogenously improving bargaining position. These effects are absent in standard platform settings where agents' outside option remains constant. A further novelty relative to platform models with network effects is that the optimal order of attracting agents should depend on the externality on other agents' outside option.

Finally, a literature shows how platforms intensify competition between sellers relative to a direct channel when sellers set the same price across both channels (e.g., Baye and Morgan, 2001; Wang and Wright, 2020; and Gomes and Mantovani, 2025). In these settings, sellers may be collectively better off without the platform because it lowers their margins on both channels by intensifying competition. Our mechanism does not rely on same-price constraints across channels or on the commoditization of sellers that underpins seller harm in these settings. Moreover, these works rule out the dynamic decision-making that explains why sellers join despite ending up worse off, one of our key contributions.

3 Model

This section provides a fully worked out microfoundation of a platform trap arising for a two-sided marketplace platform. The agents that make strategic decisions are sellers. Buyers play a passive role, but help determine the payoffs for sellers. For this reason, the microfounded model below naturally maps to a more general version of the model that can also represent a one-sided platform (e.g., a social network), as Section 5 explains.

We assume there is a single platform, $N \geq 2$ independent sellers (local monopolists), and a continuum of measure one of buyers. Each buyer wants to buy from one independent seller, drawn uniformly across the N sellers. We will discuss the case with competing sellers below.

Buyers and sellers can participate on one of two channels: the platform or a non-strategic alternative, which we call the direct channel.[4] Outside these channels, buyers and sellers have a fixed outside option normalized to zero. Let π_j be a seller's profit per

[4] In an earlier version of the paper, we analyzed the case where sellers kept their direct channel when joining the platform, which made it easier for the platform to engineer a platform trap.

matched buyer on channel j ($j = D$ for the direct channel and $j = P$ for the platform) and let v_j be the buyer's indirect utility from purchasing from her matched seller. These primitives are consistent with many types of buyer-seller interactions on a given channel. For concreteness, we focus on a standard monopoly pricing interpretation with elastic demand. Suppose a buyer's willingness-to-pay for quantity q of her matched seller's product is $w_j(q)$ on channel j, implying a seller's demand from each matched buyer is $q_j(p)$ on channel j at the seller's price p. Then, if all sellers have marginal cost c and $p_j \equiv \arg\max_p \{(p - c) q_j(p)\}$, we have $\pi_j = (p_j - c) q_j(p_j)$ and $v_j = w_j(q_j(p_j)) - p_j q_j(p_j)$. We treat v_P, v_D, π_P and π_D as the model primitives in what follows.

Buyers differ in their cost of using each channel. This could be a one-time search, travel, or general mismatch cost. Once they incur this cost, they have access to all sellers present on that channel. Specifically, we assume each buyer draws a cost $s_j \in \left[\underline{s}_j, \overline{s}_j\right]$ of using channel $j \in \{P, D\}$. We allow $\underline{s}_j < 0$, so one or both channels may give some buyers fixed benefits rather than costs. The draws of s_D and s_P need not be independent. Rather, we only assume the difference in draws, $s = s_D - s_P$, has an induced cumulative distribution function G with support equal to a non-degenerate subinterval of $[\underline{s}_D - \overline{s}_P, \overline{s}_D - \underline{s}_P]$. We assume $v_j \geq \overline{s}_j - \underline{s}_k$ for $k \neq j$, so a buyer's maximum possible additional cost of using a channel is always less than their match value. We also assume all buyers prefer going to at least one of the channels rather than not going to either (and getting a zero payoff), regardless of how sellers are split among the two channels — a sufficient condition for this is provided below.

The game consists of a single period, so payoffs are only realized once. Within the period, there are many stages. In stage k of the first N stages, the platform offers seller $k \in \{1, \ldots, N\}$ a fixed price (or fee) P^k to join the platform, which seller k accepts or rejects. Since sellers are homogeneous, their order does not matter. Each seller, in sequence, observes the platform's previous offers and the previous sellers' channel decisions, and makes its decision upon receiving its offer.[5] After all sellers have made their channel decisions, buyers observe their draws of s_D and s_P, as well as the number of sellers on each channel (but don't observe which specific sellers are on each channel), and simultaneously decide which channel to use. As is the case on most marketplaces, we assume the platform does not charge buyers for access. Sellers set their prices simultaneously with buyers' channel choices. Finally, buyers observe all the sellers and their prices in their chosen channel, and make purchasing decisions. Payoffs are then realized. We assume throughout that if a seller is indifferent between

[5] As long as a seller can observe the previous sellers' decisions, the platforms' previous prices are irrelevant; we assume they are observed so that the game is one of perfect (and complete) information.

joining the platform and not joining, the seller joins the platform.

This timing implies that only sellers are strategic: each decides sequentially whether to join, anticipating later sellers' responses and buyers' eventual channel choices. Buyers, by contrast, are non-strategic decision makers, choosing their preferred channel after seller participation is determined. We look for a subgame perfect equilibrium of this game.

3.1 Discussion of model assumptions

The multi-stage single-period nature of our model can be interpreted as capturing sellers making irreversible participation decisions over time on the platform. The payoffs for buyers and sellers then represent the present discounted value of future payoffs, over many (possibly infinite) periods. The absence of discounting across the sellers' decisions reflects that joining decisions occur over a short horizon relative to the longer horizon over which payoffs arise. Supplementary Appendix A provides an infinite period version of the model with discounting to illustrate how a very similar platform trap can still arise, although the analysis becomes much more complicated.

In the baseline setting, the platform can set a different price (or fee) for each seller, including negative prices if necessary. Section 6.1 considers the other extreme in which the platform cannot price discriminate, showing that whether a platform trap arises depends on additional payoff assumptions. Our timing implies the realistic feature that when making an offer to a given seller, the platform cannot commit to the prices it will charge to subsequent sellers. This shows that the platform trap does not depend on commitment power. Indeed, committing to a fixed sequence of prices independent of sellers' participation decisions is often worse for the platform than retaining the flexibility to adjust its offers, although full commitment to participation-contingent pricing always maximizes the platform's payoffs. The role of commitment is analyzed in Supplementary Appendix B.

In the baseline setting, the strategic agents (the sellers) decide sequentially whether to join with full information, ensuring equilibrium participation and payoffs are unique. This rules out platform traps arising merely from "bad" equilibrium selection by sellers. If instead multiple sellers decide simultaneously whether to participate on the platform (either based on private offers, or a single price offered to them all), multiple equilibria can emerge in their decision stage. In that case, whether a platform trap arises depends on equilibrium selection and the nature of sellers' payoffs. Section 6.2 addresses this.

3.2 Preliminaries

Since buyers do not know whether their matched seller is on a particular channel until they choose that channel, they care about how many sellers they can reach on each channel. The more sellers that participate on the platform, the greater the payoff a buyer gets from going to the platform relative to the direct channel, since the buyer is more likely to be able to find her preferred seller there. Specifically, if n sellers participate on the platform, a buyer chooses it if their expected utility $\frac{n}{N}v_P - s_P$ exceeds their expected direct channel utility $\frac{N-n}{N}v_D - s_D$.[6]

Let $m(n)$ be the measure of buyers choosing the platform, so $1 - m(n)$ choose the direct channel:

$$m(n) = 1 - G\left(\frac{N-n}{N}v_D - \frac{n}{N}v_P\right). \tag{1}$$

Since $G(\cdot)$ is increasing over a subinterval of $[\underline{s}_D - \overline{s}_P, \overline{s}_D - \underline{s}_P]$, $m(n)$ is weakly increasing in n, with $m(0) = 0$ (since $v_D \geq \overline{s}_D - \underline{s}_P$) and $m(N) = 1$ (since $v_P \geq \overline{s}_P - \underline{s}_D$). We make the weak assumption that

$$m(N-1) > 0, \tag{2}$$

or equivalently, that $G\left(\frac{1}{N}v_D - \frac{N-1}{N}v_P\right) < 1$. This says that when all but one seller joins the platform, it attracts a positive measure of buyers. This holds for high enough N, reflecting that $G(-v_P) = 0$ given $v_P \geq \overline{s}_P - \underline{s}_D$.

Let $u(n) = \frac{1}{N}(1 - m(n))\pi_D$ be a seller's expected gross profit from selling on the direct channel and $b(n) = \frac{1}{N}m(n)\pi_P$ its expected gross profit from selling on the platform, when n sellers in total join the platform. These are gross profits since they are calculated before considering any platform prices. From above, $b(\cdot)$ is weakly increasing in n and strictly increasing for at least some n, capturing positive indirect network effects: sellers benefit more from joining when more sellers join because this attracts more buyers. By the same token, $u(n)$ is weakly decreasing in n and strictly decreasing for at least some n. Since $u(0) = \frac{1}{N}(1 - m(0))\pi_D = \frac{\pi_D}{N} > 0$, condition (2) implies $u(N-1) < u(0)$: a seller that stays on the direct channel is strictly worse off if all other sellers join the platform.

[6] A sufficient condition for our assumption that all buyers prefer one of the two channels to the zero outside option is

$$\min_{0 \leq n \leq N}\left\{\max\left\{\frac{n}{N}v_P - \overline{s}_P, \frac{N-n}{N}v_D - \overline{s}_D\right\}\right\} \geq 0.$$

This implies $v_D \geq \overline{s}_D$ (for $n = 0$) and $v_P \geq \overline{s}_P$ (for $n = N$).

Define the marginal surplus function

$$\Delta(n+1) \equiv b(n+1) - u(n),$$

which measures the incremental payoff to a seller from joining the platform rather than selling directly when n other sellers join. It determines a seller's willingness to pay for access and will be central to our analysis. Clearly, $\Delta(n+1) \geq \Delta(n)$ for all relevant n, with strict inequality for at least some n. This means the marginal seller's incentive to join the platform weakly increases with the number of participating sellers, implying (as we will show) that in equilibrium, either all sellers join the platform or none do.

We also assume $\Delta(N) > 0$, which, since $m(N) = 1$, is equivalent to

$$\pi_P > (1 - m(N-1))\, \pi_D.$$

Thus, profit per buyer on the direct channel cannot be so high relative to profit per buyer on the platform that a seller prefers to remain alone on the direct channel when all other sellers join the platform. This is the weakest assumption ensuring that joining the platform yields strictly greater gross profit in that situation. Otherwise, if $\Delta(N) \leq 0$, the platform can never profitably attract sellers. Indeed, we assume that the platform must earn positive profits to operate.

Finally, in an equilibrium in which all N sellers join at the platform prices $\left(P^1, \ldots, P^N\right)$, seller k would be weakly better off without the platform if and only if $b(N) - P^k \leq u(0)$.[7]

4 Analysis

A key step to characterize platform pricing and strategic agents' (i.e., sellers') decisions is to recognize that some sellers can be pivotal. Specifically, $k_0 \in \{0, \ldots, N-1\}$ sellers are pivotal in the original game if the non-participation of any of the first k_0 sellers prevents the platform from profitably inducing any of the N sellers to participate.[8]

To build intuition, consider two extreme cases: no seller is pivotal in the original game, or every seller is pivotal.[9] For example, with two sellers ($N = 2$), suppose the

[7]If the platform doesn't exist, all sellers use the direct channel, which attracts all buyers there given $v_D - \bar{s}_D > 0$. Thus, each seller obtains $u(0)$.

[8]In the proof of Proposition 1 below, we will show that either the first k_0 agents in the sequence are pivotal for some $1 \leq k_0 \leq N-1$, or no agents are pivotal.

[9]When we say "every seller is pivotal", we mean all sellers are pivotal except the last to decide,

platform needs both to join for it to offer each seller strictly higher expected gross profit than the direct channel, i.e., $\Delta(2) > 0 \geq \Delta(1)$. Then the first seller to receive an offer is pivotal: if (and only if) it rejects, the second seller cannot be profitably attracted. Thus, the maximum price the platform can charge the first seller is $b(2) - u(0)$, and the maximum profit it can extract in total is $2b(2) - u(0) - u(1)$. Now consider three sellers ($N = 3$) under the same assumptions, and suppose in addition $2b(2) > u(0) + u(1)$. The first seller is no longer pivotal, because even if it rejects, the platform can still profitably attract the remaining two. If the first seller accepts, the second seller is not pivotal because $\Delta(2) > 0$ implies the platform will attract the third seller regardless of the second seller's decision. However, if the first seller rejects, the situation reduces to the $N = 2$ case, making the second seller pivotal even though it is not pivotal in the original game.

For general N, if $\Delta(1) > 0$, no seller is ever pivotal because $\Delta(\cdot)$ weakly increasing implies $\Delta(n) > 0$ for all $n \geq 1$. The platform then creates higher expected gross profit for a seller than the direct channel, regardless of how many other sellers are expected to join. As a result, the platform can always profitably induce any seller to participate. Using the payoffs in Section 3.2, $\Delta(1) = \frac{1}{N}(m(1)\pi_P - \pi_D)$, which can be positive or negative in our setting.

When $\Delta(1) > 0$, the last seller can be induced to join even if no one else has joined. Since $\Delta(n)$ is weakly increasing, each seller expects all subsequent sellers to join regardless of its own decision. Thus, if $N' < N$ prior sellers have joined, the k-th seller will accept the platform's offer if and only if the price P satisfies

$$P \leq P(N') = \Delta(N' + N - k + 1).$$

A higher price causes rejection, eliminating positive revenue from that seller and reducing revenue from subsequent sellers. By setting $P = P(N')$ at each stage, the platform ensures that all N sellers join. Along the equilibrium path, $N' = k - 1$, so the common price is $P^* = \Delta(N)$.

This is the same platform price that would arise if all sellers decided simultaneously and coordinated on the equilibrium most favorable to the platform (i.e., where all sellers join whenever that is an equilibrium). In that case, each seller expects the other $N - 1$ sellers to join, yielding a payoff $b(N) - P$ if they also join, and $u(N-1)$ if they do not. Setting $P = \Delta(N)$ makes each seller indifferent between joining and not, when they expect all others to participate.

which by definition can't be pivotal. Likewise for agents later on.

For any given $N > 1$, $\Delta(1) > 0$ is a sufficient condition for no seller to be pivotal. More generally, no seller is pivotal if and only if the platform can still profitably attract all $N-1$ remaining sellers after the first seller rejects. Obtaining a simple closed-form condition for this in terms of the payoff primitives is difficult for general $N > 2$, as we discuss after Proposition 3 below.

Now consider the opposite extreme, where every seller is pivotal. Specifically, suppose

$$\Delta(N-1) = b(N-1) - u(N-2) \leq 0 < \Delta(N),$$

which, given that $\Delta(\cdot)$ is weakly increasing, implies $\Delta(k) \leq 0$ for all $k \leq N-1$. In this case, losing even one seller prevents the platform from profitably attracting any remaining seller: the last seller would prefer the direct channel at any positive price, so the platform has no incentive to attract it, and it cannot attract the second-to-last seller either, and so on. Thus, every seller is pivotal. To induce participation, the platform must offer each seller at least its expected direct-channel profit assuming no subsequent sellers join. Although all sellers join in equilibrium, the price increases along the sequence, from $P^1 = b(N) - u(0)$ for the first seller to $P^N = b(N) - u(N-1) = \Delta(N)$ for the last. This is profitable for the platform if $b(N) > \frac{1}{N}\sum_{k=1}^{N} u(k-1)$. In the marketplace setting, this condition is equivalent to $\pi_P > \sum_{k=1}^{N} \frac{1}{N}(1 - m(k-1))\pi_D$.

Between the two extremes, i.e. when $\Delta(1) \leq 0 < \Delta(N-1)$, some but not all sellers may be pivotal. Specifically, the proof of the next proposition shows that there is a threshold $k_0 \geq 1$ such that the first $k \leq k_0$ sellers are pivotal and are charged $P^k = b(N) - u(k-1)$. Once these sellers are on board, the remaining $N - k_0$ sellers are no longer pivotal and are each charged $P^k = b(N) - u(N-1) = \Delta(N)$. These prices are consistent with the extreme cases above. A scenario with $1 \leq k_0 \leq N-1$ can be interpreted as one in which the platform must first attract a critical mass k_0 of sellers before it can offer more expected profit than the direct channel to each subsequent seller irrespective of whether these sellers ultimately join. The proof considers a general reduced-form game of which the two-sided marketplace setting is a special case, and uses induction over games with increasing values of N. As with all proofs not covered in the text, it is relegated to the Appendix.

Proposition 1. *If no sellers are pivotal, the platform optimally attracts all N sellers with the constant price $P^k = \Delta(N) > 0$ for all $k \in \{1, \ldots N\}$. Otherwise, there exists a threshold seller $k_0 \in \{1, \ldots, N-1\}$ such that the profit-maximizing prices that induce* *participation on the platform are*

$$P^k = \begin{cases} b(N) - u(k-1) & \text{if} \quad 1 \leq k \leq k_0 \\ b(N) - u(N-1) & \text{if} \quad k_0 + 1 \leq k \leq N \end{cases}. \tag{3}$$

If $\sum_{k=1}^{N} P^k > 0$*, these are the platform's optimal prices and all* N *sellers join, with the first* k_0 *sellers pivotal. If instead* $\sum_{k=1}^{N} P^k \leq 0$*, the platform cannot profitably attract any sellers, so no one joins in equilibrium. In any equilibrium where sellers join, they would all be weakly better off without the platform, and at least some would be strictly better off. Buyers, however, may be better or worse off without the platform, depending on parameters. Finally, a sufficient condition for all sellers to join is*

$$b(N) > \frac{1}{N} \sum_{k=1}^{N} u(k-1). \tag{4}$$

A platform trap is a scenario in which all strategic agents (i.e., the sellers) join the platform despite all being weakly worse off, and some strictly worse off, than if the platform did not exist. Sellers end up worse off because each seller that joins lowers the payoff remaining sellers obtain by not joining, which lets the platform extract more from all of them. That is, the platform exploits a collective action problem among sellers, inducing them to collectively cannibalize their direct channel. In the end, under the sufficient condition (4), all sellers join despite being better off without the platform, i.e., if they collectively boycotted it. The sufficient condition (4) for a platform trap holds in our baseline marketplace setting if $v_P \geq v_D$ and $\pi_P \geq \pi_D$. For example, this could arise if the platform provides additional transactional benefits relative to the direct channel, so $q_P(p) \geq q_D(p)$ for all p.

Seller and buyer surplus

In an equilibrium where sellers join the platform, all sellers join, so seller k gets $\frac{1}{N} m(N) \pi_P - P^k$. For non-pivotal sellers, this equals $\frac{1}{N}(1 - m(N-1)) \pi_D$. Recalling that (2) requires $m(N-1) > 0$, we conclude that non-pivotal sellers obtain strictly less than the $\frac{1}{N}\pi_D$ they would get without the platform. For pivotal seller k, the equilibrium payoff is $\frac{1}{N}(1 - m(k-1)) \pi_D$, which is weakly lower than $\frac{1}{N}\pi_D$, given $m(k-1) \geq 0$. The last seller to sign, pivotal or not, is strictly worse off because $m(N-1) > 0$. The first seller to sign is indifferent if it is pivotal, given $m(0) = 0$. Therefore, in general, all sellers are weakly worse off, and at least one seller is strictly worse off, due to the platform's existence.[10]

To understand what the platform's existence implies for buyers' expected utility, note that if the platform doesn't exist (or sellers collectively boycott it), buyers each get $v_D - s_D$ from the direct channel, which under our assumptions is always non-negative. Similarly, when all sellers join the platform, buyers each get $v_P - s_P$ from the platform, which is always their preferred option given $v_P \geq \overline{s}_P - \min\{\underline{s}_D, 0\}$. Then under a platform trap, buyers for which $s > v_D - v_P$ are strictly better off with the platform, while those for which $s < v_D - v_P$ are strictly worse off due to the platform's existence (where recall $s = s_D - s_P$). The possibility that some buyers are worse off reflects buyers who prefer to buy from sellers directly but, after the platform trap, cannot access sellers directly. If $BS(n)$ represents aggregate buyer surplus when n sellers join the platform, we have $BS(N) = v_P - \mathbb{E}[s_P]$ and $BS(0) = v_D - \mathbb{E}[s_D]$. Thus, aggregate buyer surplus is lower under a platform trap if and only if $v_P - v_D + \mathbb{E}[s] < 0$.

The question remains whether all buyers can be worse off in equilibrium. This is certainly possible. For simplicity, suppose $v_P = v_D = v$ and $\pi_P = \pi_D = \pi$, so the two channels are equivalent for matched buyer-seller transactions. Assume $\overline{s}_D = \underline{s}_D = 0$ and $\overline{s}_P > \underline{s}_P = 0$, so buyers only face a cost of going to the platform (for instance, if it is costly for them to adopt the new channel). Then $s = -s_P$, which is distributed with full support on $[-\overline{s}_P, 0]$. In this case, (2) is equivalent to $N > 2$, which also implies (4) holds,[11] so the equilibrium in which the platform attracts all sellers still exists. Then all buyers are weakly worse off, and all buyers with $s_P > 0$ are strictly worse off, obtaining $v - s_P$ with the platform instead of v without it, while total surplus is lower.

Another way in which all buyers could be worse off in a platform trap is if the platform charges ad valorem fees to sellers, rather than fixed prices, and sellers pass these fees through into higher buyer prices. In Supplementary Appendix C, we show that when the platform charges ad valorem fees, all buyers and sellers can be worse off due to the platform's existence even when $s_P = 0$ and $s_D \geq 0$ (so buyers only incur costs of going to the direct channel) and $q_P(p) > q_D(p)$ (so the platform provides additional transactional benefits relative to the direct channel).

Total surplus

Having considered how a platform's existence affects buyers and sellers, it remains to

[10] If G is strictly increasing on the interval $[-v_P, v_D]$, then $b(n)$ is strictly increasing and $u(n)$ is strictly decreasing for all n, and so the platform trap would always involve all sellers being strictly worse off, with the exception of the first seller to join if it is pivotal.

[11] Indeed, it is equivalent to $N > \sum_{k=1}^{N} G\left(\left(\frac{N-(k-1)}{N} - \frac{k-1}{N}\right)v\right)$, which holds because $G\left(\left(\frac{N-(k-1)}{N} - \frac{k-1}{N}\right)v\right) \leq 1$ for all $k = 1, ..N$, with strict inequality for $k = N$.

consider whether total surplus, including the platform's profit, can also be lower under a platform trap. Since the fixed prices paid by sellers to the platform are transfers, total surplus when n sellers join the platform is

$$W(n) = nb(n) + (N - n)\, u(n) + BS(n)\,.$$

In any equilibrium, either no sellers join or all sellers join, and in the latter case there is a platform trap. Thus, to see whether total surplus is higher with the platform when it is viable, we only need to compare $W(N)$ with $W(0)$. Since $BS(N) - BS(0) = v_P - v_D + \mathbb{E}[s]$, total surplus is strictly lower with the platform if and only if

$$b(N) < u(0) - \frac{1}{N}\left(v_P - v_D + \mathbb{E}[s]\right). \tag{5}$$

For a one-sided platform (with just strategic agents), or for a marketplace where buyer surplus is ignored, the condition for the platform to decrease total surplus is simply $b(N) < u(0)$. The same condition applies for a marketplace where buyers view the two channels symmetrically, in the sense that $v_P = v_D$ and $\mathbb{E}[s] = 0$.

Comparing (5) with (4), the condition for lower total surplus differs sharply from the sufficient condition for a platform trap: the inequalities in $b(N)$ go in opposite directions. Total surplus can nevertheless be lower under a platform trap. A sufficient condition is:

$$u(0) - \frac{1}{N}\left(v_P - v_D + \mathbb{E}[s]\right) > b(N) > \frac{1}{N}\sum_{k=1}^{N} u(k-1)\,,$$

which is possible if $u(n)$ decreases fast enough in n. However, total surplus can also be higher despite a platform trap if $b(N)$ is high enough. Moreover, because $v_P - v_D + \mathbb{E}[s]$ can be positive or negative, a platform that would be efficient if all sellers joined need not generate a platform trap, because it may not be able to profitably attract sellers. We summarize these findings in the following proposition.

Proposition 2. *The existence of a platform that attracts all sellers decreases total surplus if and only if condition (5) holds. If buyers view the two channels symmetrically (or their surplus is ignored), the condition is simply $b(N) < u(0)$, and in this case, if the existence of the platform increases total surplus, a platform trap must arise. More generally, there is no necessary relationship between whether a platform that attracts all sellers increases total surplus and whether a platform trap arises, in the sense that all four combinations are possible.*

Proposition 2 illustrates that the problem of a platform trap is quite distinct from traditional concerns about a monopoly platform generating inefficient outcomes, with neither concern necessarily implying the other.

The conclusions of Propositions 1 and 2 are driven by the negative externalities that platform participation has on the direct channel (or, more generally, the alternative option). If the direct channel instead has a fixed value to sellers or does not exist, then the existence of a platform will always make sellers weakly better off: sellers can always choose not to participate and obtain this fixed value. This is the case in the classic analysis of two-sided platforms with positive cross-side network effects that set fees to attract users on each side (e.g., Rochet and Tirole, 2003 and Armstrong, 2006). In those models, the alternative to a monopoly platform is not a direct channel where the two sides can interact, perhaps less efficiently, but no platform at all (i.e., a fixed outside option). So there is no sense in which the platform is attracting buyers and sellers away from something which itself is affected by the existence of the platform. As a result, buyers and sellers can never be worse off due to the platform's existence, and total surplus is always higher. Applying this to our baseline model, if we remove the direct channel option so that the only alternative to joining the platform is the fixed outside option giving a zero payoff, the platform would charge each seller k joining in sequence the same price $b(N)$.[12] There would no longer be any possibility of pivotal sellers with the corresponding increasing prices P^k that we characterized in (3), or any kind of platform trap.

Seller competition

So far we have focused on a marketplace setting where sellers do not compete. In Supplementary Appendix D, we allow sellers to compete. Seller competition counteracts the positive effect of having more sellers on a channel (i.e., attracting more buyers). However, if the competition effect is not too strong, the buyer-expansion effect dominates, so platform profit $b(n)$ can still increase and direct-channel profit $u(n)$ can still decrease in n, and all the reduced-form payoff conditions needed for Proposition 1 still hold. We illustrate this with an example in Supplementary Appendix D. On the other hand, if the competition effect is sufficiently strong on the platform channel, and stronger than on the direct channel, $b(n)$ may eventually decrease even while $u(n)$ is everywhere decreasing in n. Supplementary Appendix D provides such an example, in which $b(n)$ is single-peaked despite all the other reduced-form payoff conditions needed for the platform-trap result continuing to hold. This motivates the generalization in

[12] To make this comparison meaningful, we must assume $b(\cdot)$ is weakly increasing, so that $\Delta(\cdot)$ can be weakly increasing despite $u(\cdot)$ being constant and equal to zero.

Section 5, which does not require $b(n)$ to be increasing and allows it, in particular, to be single-peaked.

5 Generalizations

We now turn to a more general reduced-form game with a single platform and N strategic agents, under the same timing and pricing assumptions as in Section 3. The N agents play the same role as sellers in the marketplace setting, while buyers' decisions are subsumed into the agents' payoffs $b(\cdot)$ and $u(\cdot)$. This generalization also captures one-sided settings such as social networks. We state the assumptions for this general setup with N agents.

(A1) *Fully flexible prices.* The platform can set a different price P^k for each agent $k \in \{1, \ldots, N\}$.

(A2) *Take-it-or-leave-it offers.* The platform makes take-it-or-leave-it offers.

(A3) *Full information and sequential moves.* Agents make sequential participation decisions and observe all prior decisions.

(A4) *Ex-ante homogeneous agents.* All agents face the same payoff functions $b(\cdot)$ and $u(\cdot)$.

(A5) *Monotonicity of payoff functions.* The payoff functions $b(\cdot)$ and $u(\cdot)$ satisfy the following conditions: (a) $u(\cdot)$ is non-negative, weakly decreasing in its argument, with $u(N-1) < u(0)$; and (b) $\Delta(n) = b(n) - u(n-1)$ is weakly increasing in n, with $\Delta(N) > 0$.

Each of Assumptions A1-A4 will be relaxed in later sections. Unlike the baseline model of Section 3, we do not assume $b(\cdot)$ is increasing. In a social network, benefits may initially increase as more users join, but later decrease due to congestion. Supplementary Appendix D shows that $b(\cdot)$ may be single-peaked when sellers compete and the competition effect is stronger on the platform than in the direct channel.

We show that the platform trap result in Proposition 1 applies to this more general setting.

Proposition 3. *Assume there are N agents, satisfying Assumptions A1-A5. Then Proposition 1 applies to these agents (replacing "sellers" with "agents"), and ignoring the implications for buyers, if any.*

For any given primitives (the number of agents N, and their payoff functions $b(\cdot)$ and $u(\cdot)$) one of two outcomes arises. Either the platform attracts all agents, yielding a unique equilibrium outcome in which all join and prices satisfy the properties in Proposition 1; or it attracts none, in which case there is a continuum of equilibria (in prices) with no participation. Furthermore, when the platform attracts all agents, there exists $k_0 \in \{0, \ldots, N-1\}$ such that the first k_0 agents are pivotal and each agent $k \leq k_0$ is charged $P^k = b(N) - u(k-1)$, while the remaining $N - k_0$ agents are non-pivotal and each agent $k \geq k_0 + 1$ is charged $P^k = b(N) - u(N-1) = \Delta(N) > 0$.[13] This is because at any stage, the subgame has only two possible participation outcomes: either all remaining agents join or none do (as shown in the proof of Proposition 1).

While Proposition 1 and its generalization in Proposition 3 provide a sufficient condition for a platform trap, stating a necessary and sufficient condition and fully characterizing the number k_0 of pivotal agents becomes increasingly complex as N grows. Let $\Pi(n|k)$ denote the platform's continuation profit after attracting $k \leq N$ agents and conditional on having to induce all $n \leq N$ remaining agents to participate. Suppose $\Pi(N|0) > 0 > \Pi(N-1|0)$, so the platform can profitably attract all N agents, but the first agent is pivotal: if she rejects, the platform cannot earn positive profit from the remaining $N-1$ agents. If the first agent accepts the offer, $\Pi(N|0) > 0$ implies $\Pi(N-1|1) > 0$. Is the second agent pivotal? If the second agent rejects, continuation profit is $\Pi(N-2|1)$: there are $N-2$ agents left, and only one of the first two agents has accepted. $\Pi(N-2|1)$ can be positive or negative depending on how strongly the first agent's decision weakens later agents' outside options. If the externality is weak, $\Pi(N-2|1)$ is close to $\Pi(N-2|0)$, so $\Pi(N-2|1) < 0$ as $\Pi(N-2|0) \leq \Pi(N-1|0) < 0$, and in this case, the second agent is pivotal. If the externality is strong enough that $\Pi(N-2|1) > 0$, the second agent is not pivotal. Thus, there may be one or multiple pivotal agents. The same reasoning applies to any agent k, conditional on all previous agents being pivotal.[14] Agents that are not pivotal in the original game may become pivotal off the equilibrium path. Thus, each subgame may have a different number of pivotal agents, making a full characterization of the equilibrium (in particular, k_0) unwieldy.

To illustrate this and give a sense of how prices relate to model primitives, Supplementary Appendix E characterizes the equilibrium for $N = 2$ and $N = 3$. It also shows that even with linear $b(\cdot)$ and $u(\cdot)$, additional restrictions on their slopes are

[13] The prices charged to pivotal agents may or may not be negative. For example, if $b(N-1) - u(N-2) \leq 0$, then all agents are pivotal, but it is still possible to have $b(N) - u(0) \geq 0$, so that all agents face non-negative prices. This can happen if b increases a lot when the last agent joins.

[14] We are grateful to an anonymous referee who provided this explanation.

needed for a closed form solution for general N.

We can, however, characterize the necessary and sufficient condition for a platform trap, and the associated number of pivotal agents, for the two extreme cases discussed earlier.

Corollary 1. *If $\Delta(1) > 0$ or $b(N-1) > \frac{1}{N-1}\sum_{k=1}^{N-1} u(k-1)$, there is a unique equilibrium outcome in which the platform attracts all N agents and charges $P^k = \Delta(N) > 0$ for all $k \in \{1, \ldots, N\}$. Here $k_0 = 0$ so there are no pivotal agents. All agents would be strictly better off without the platform.*

Corollary 2. *If $b(N-1) - u(N-2) \leq 0$, the platform attracts all N agents if and only if (4) holds. Under these conditions, there exists a unique equilibrium outcome in which the platform charges $P^k = b(N) - u(k-1)$ to agent $k \in \{1, \ldots, N\}$. Here $k_0 = N-1$ so the first $N-1$ agents are pivotal. All agents would be weakly better off without the platform, with some strictly better off.*

Corollary 1 provides two alternative sufficient conditions for when no agent is pivotal. When $N > 2$ and $b(\cdot)$ is weakly increasing, the condition $b(N-1) > \frac{1}{N-1}\sum_{k=1}^{N-1} u(k-1)$, which is weaker than $\Delta(1) > 0$, suffices to rule out any agent being pivotal. On the other hand, Corollary 2 gives a sufficient condition for the case all but the last agent are pivotal.

Having characterized the platform trap with a fixed number of agents, a natural question is how prices, agents' net payoffs and platform profits change as the total number of agents N increases? Using the general formulation of payoffs in which we assume $b(\cdot)$ and $u(\cdot)$ are invariant to N, we find:

Corollary 3. *The net surplus left to agents weakly decreases in N, while platform profit increases in N. If the platform is profitable in the game with N agents, the number of pivotal agents is weakly lower in the game with $N' > N$ agents than in the game with N agents.*

Proposition 1 shows that prices for non-pivotal agents weakly increase with N, while prices for pivotal agents may increase or decrease, depending on whether $b(\cdot)$ increases or decreases in n. However, net payoffs for all agents always weakly decline as N increases because the total negative externality on the outside option is larger. Furthermore, it is intuitive that as N increases, each agent becomes less critical, making it easier for the platform to attract participation and reducing the likelihood of pivotal agents. These results suggest that as a platform becomes more well-known (so more agents enter the market), the trap deepens. Over time, the platform is less likely to use negative prices, its prices rise, and agents' net payoffs decline.

6 Additional mechanisms

Our main platform trap result in Section 5 relied on Assumptions A1-A5. In this section, we show that relaxing Assumptions A1 and A3 sometimes limits and sometimes expands the scope for platform traps. It can also give rise to new platform trap mechanisms.

6.1 Constant pricing

One might think dynamic price adjustments are essential for the platform trap to arise. Perhaps surprisingly, we show this is not always the case by relaxing Assumption A1 in this section. Even when the platform must offer the same price to all agents (and agents still decide sequentially), a platform trap may still emerge. Since the platform must earn a profit, its constant price must be positive. Yet a platform trap can still occur when negative externalities dominate after enough agents join.

Proposition 4. *Suppose $b(\cdot)$ is single-peaked and the platform must charge the same price to all agents. There is a unique equilibrium outcome in which the platform attracts all N agents at the price $P = \max_{1\leq n\leq N}\{b(n)\} - u(0)$, provided it is positive. In this case, agents are indifferent about the platform's existence if $b(N) = \max_{1\leq n\leq N}\{b(n)\}$ and would be strictly better off without the platform if $b(N) < \max_{1\leq n\leq N}\{b(n)\}$. If $\max_{1\leq n\leq N}\{b(n)\} \leq u(0)$, the platform attracts no agents.*

The platform trap disappears when the platform must charge the same price to all agents and $b(\cdot)$ is weakly increasing. In this case, at the price stated in Proposition 4, each agent is effectively pivotal because the platform cannot adjust prices to later agents based on the decisions of earlier agents. Thus, the platform can profitably attract all agents if and only if $b(N) > u(0)$ and all agents are indifferent about the platform's existence (in the baseline setting only the first pivotal agent was indifferent).

However, when $b(\cdot)$ is decreasing, or first increasing and then decreasing, the platform trap reemerges despite the platform's inability to dynamically adjust prices. To see this, suppose $b(\cdot)$ peaks at $n^* < N$. Then the first $N - n^* \geq 1$ agents are not pivotal: using the argument for increasing $b(\cdot)$, the platform can attract the last n^*

agents even if none of the first $N - n^* \geq 1$ agents has joined. Thus, the first agents are easier to attract, while later agents keep joining as their outside option worsens, even though each additional participant reduces others' benefits. After the first n^* agents join, the trap is driven by both $b(\cdot)$ and $u(\cdot)$ declining, with $u(\cdot)$ falling faster.

This new mechanism implies some agents may be worse off under constant pricing than under price discrimination. In the baseline, if agent k was pivotal, it paid $b(N) - u(k-1)$. With a constant price, agent k pays $b(n^*) - u(0)$, where $n^* = \arg\max_n \{b(n)\} < N$. If $n^* < N$ and

$$b(n^*) - b(N) > u(0) - u(k-1),$$

then agent k pays more and receives a lower net payoff under constant pricing.[15] This requires that $b(n)$ declines sufficiently from its peak to $b(N)$. Only early agents that were pivotal in the baseline may be worse off when the platform cannot price discriminate. By contrast, non-pivotal agents, including the last, are always weakly better off. Indeed

$$b(n^*) - u(0) \leq \Delta(n^*) \leq \Delta(N) = b(N) - u(N-1).$$

Turning to welfare, Proposition 2 shows that a platform attracting all agents decreases total surplus if and only if $b(N) < u(0)$. Depending on the shape of $b(n)$, even if the platform can profitably operate when it must set a constant price, it may make agents weakly worse off and raise total surplus, or strictly worse off and lower total surplus. The latter case arises when $b(n)$ exceeds $u(0)$ for some n, but falls below $u(0)$ at N.

Proposition 4 implies that when $b(\cdot)$ is weakly increasing, the platform trap relies on the ability to adjust prices for future agents if an agent deviates and rejects its offer. However, it may not require much price flexibility to restore the trap. Supplementary Appendix F shows that if the platform can change prices once, the condition $b(N-1) > u(0)$ ensures a unique equilibrium outcome in which all agents join at the price $\Delta(N) > 0$ and would be strictly better off without the platform.

6.2 The role of equilibrium selection

In the baseline model, agents decide whether to join the platform sequentially with full information, which ensures that equilibrium participation and payoffs are unique. If agents observe only their own offers, then multiple participation equilibria can arise

[15] In particular, this always holds for the first agent ($k = 1$) if they were pivotal in the baseline, since $b(n^*) - u(0) > b(N) - u(0)$.

for given platform prices, and equilibrium selection matters. This holds whether agents still decide sequentially and the platform can set different prices (only Assumption A3 is relaxed), or the platform has to set a constant price and agents decide simultaneously (both Assumptions A1 and A3 are relaxed).

The following proposition characterizes the best and worst equilibria for the platform when it sets a single price in stage 1 and agents decide simultaneously in stage 2. Supplementary Appendix G shows the same results hold when the platform can set different prices and agents only observe their own offers (regardless of whether they decide simultaneously or sequentially), provided we restrict attention to strong perfect Bayesian equilibria with passive beliefs in the sequential case.

Proposition 5. *Suppose the platform charges a single price in stage 1 and all agents decide simultaneously in stage 2. There is a continuum of equilibria.*

1. *In the best equilibrium for the platform, each agent pays $P = \Delta(N)$, all N agents join, and platform profits are $N\Delta(N)$. All agents would be strictly better off without the platform.*
2. *If $\Delta(1) > 0$, in the worst equilibrium for the platform, each agent pays $P = \Delta(1)$, all N agents join, and platform profits are $N\Delta(1)$. All agents would be strictly worse off without the platform if $b(N) > b(1)$, indifferent if $b(N) = b(1)$, and strictly better off if $b(N) < b(1)$. If $\Delta(1) \leq 0$, in the worst equilibrium for the platform, no agents join and the platform profits are zero.*

Depending on equilibrium selection, the platform trap may arise more broadly than in the baseline (case 1 in the proposition) or be more limited (case 2 when $b(\cdot)$ is weakly increasing). More generally, the comparison depends on equilibrium selection, the number of pivotal agents in the baseline, and the shape of $b(\cdot)$.[16] The key reason is that agents are no longer pivotal: one agent's rejection cannot influence others. As a result, in the best equilibrium, previously pivotal agents are strictly worse off than in the baseline, while the platform does better and the trap applies for a wider set of payoff functions.

More surprisingly, the full platform trap can arise even in the worst equilibrium for the platform. This occurs when the aggregate externalities on the platform are negative, i.e., $b(N) < b(1)$. This shows that a platform trap can arise even without

[16] For example, suppose $b(\cdot)$ is decreasing and in the baseline all agents are pivotal. Then total payoffs to agents in the baseline are $\sum_{k=1}^{N} u(k-1)$, whereas here they are $N(u(0) - (b(1) - b(N)))$ in the worst equilibrium. Either one could be higher.

any dynamics or price discrimination. In this worst equilibrium, agents may still benefit from the platform, as they face a relatively low price ($P = \Delta(1)$) when on-platform network effects are positive. However, if $b(N) < b(1)$, all agents would be better off without the platform. The trap arises because agents join the platform even though full participation reduces their benefit. Each agent pays $\Delta(1) = b(1) - u(0)$ to join, knowing they will receive $b(N)$ rather than $b(1)$, because rejecting yields $u(N-1)$, which is worse.[17]

This contrasts with Proposition 2 in Segal and Whinston (2000). They find that when an incumbent makes simultaneous offers to buyers and cannot discriminate, exclusion cannot occur in any perfectly coalition-proof Nash equilibrium. In our setting, under the single price $P = \Delta(1) > 0$, joining is the only Nash equilibrium: agents prefer to join even if no others do, and even more so if others join, so the perfectly coalition-proof refinement has no bite.

7 Extensions

In this section, we relax in turn three key assumptions — take-it-or-leave-it offers, homogeneous agents, and monopoly platform — and show that platform traps persist in these richer settings, while also examining how platforms actively engineer them. For brevity, the proofs for this section are contained in Supplementary Appendix H.

7.1 Negotiated prices

The baseline assumed the platform makes take-it-or-leave-it offers to agents (Assumption A2). If the platform must negotiate, its ability to extract surplus, and thus the scope of the platform trap, is limited. To explore this and the implications for price dynamics, we now assume the platform engages in sequential Nash bargaining with each agent. We focus on the case with no pivotal agents, so $\Delta(1) > 0$, which ensures the platform can profitably attract all agents. This scenario yields the strongest form of the trap and serves as a benchmark: even here, negotiation can imply some agents are not made worse off by the platform's existence. The assumption also ensures closed form solutions despite the recursive structure that arises because each participation decision affects all subsequent negotiations. Finally, assuming $\Delta(1) > 0$ shows that prices can still increase as more agents join even without pivotal agents, because each additional agent weakens the bargaining position of those who follow.

[17] Indeed, $u(N-1) \leq b(N) - \Delta(1)$ since $\Delta(1) \leq \Delta(N)$.

Proposition 6. *Suppose $\Delta(1) > 0$ and the platform bargains sequentially with N agents, where each agent has bargaining power $0 \leq \alpha < 1$ and the platform has $1-\alpha$. There is a unique equilibrium in which all agents participate and the price charged to agent k is*

$$P^k = (1-\alpha)\left(\sum_{j=0}^{N-k} C_j^{N-k} (1-\alpha)^j \alpha^{N-k-j} \Delta(k+j)\right)$$

for all $k \in \{1, \ldots, N\}$, where C_j^{N-k} is the binomial coefficient. With linear externalities such that $\Delta(n) = \beta_0 + \beta_1 n$ and $\beta_1 > 0$, prices P^k increase in k when bargaining power is intermediate (i.e., $0 < \alpha < 1$).

Since $\sum_{j=0}^{N-k} C_j^{N-k} (1-\alpha)^j \alpha^{N-k-j} = 1$ for any k, the optimal price to agent k is $(1-\alpha)$ times a weighted average of $\Delta(n)$ for n ranging from k to N. If the externalities $u(.)$ and $b(.)$ are linear and produce $\Delta(n) = \beta_0 + \beta_1 n$ with $\beta_1 > 0$, prices are

$$P^k = (1-\alpha)(\beta_0 + \beta_1 (N - \alpha(N-k))). \tag{6}$$

Given that $0 < \alpha < 1$ and $\beta_1 > 0$, prices are increasing in k. If $b(N) < u(0)$ or α is sufficiently low, all agents would be strictly better off without the platform. If $b(N) > u(0)$ and α is sufficiently high, all agents are better off with the platform. Finally, if $b(N) > u(0)$, there is an intermediate range of α, such that agents receiving early offers would be strictly worse off without the platform, while those receiving later offers would be strictly better off without it.

7.2 One superstar agent

So far all agents have been homogeneous (Assumption A4). We now relax this by adding one "superstar" agent alongside N regular (symmetric) agents. This raises the question: which type of agent should the platform attract first to engineer a platform trap?

We assume the superstar is equivalent to a coalition of $S > 1$ agents, with payoffs $Sb(\cdot)$ and $Su(\cdot)$, and the same impact on all agents' payoffs as S regular agents.[18] Thus, when the superstar agent and $n \leq N$ regular agents join the platform, an individual agent's payoff on the platform is $b(S+n)$ and its outside option payoff is $u(S+n)$. If the superstar does not join, payoffs are the usual $b(n)$ and $u(n)$. We adjust the baseline

[18] Supplementary Appendix H.2 analyzes a slightly different version in which the superstar has the size and the payoffs of an individual agent, but an outsized impact on agents' payoffs. The results are similar.

assumption that every agent receives positive surplus from the platform when all others join by imposing $b(N+S) > u(N)$, which also implies $b(N+S) > u(N+S-1)$ since $S > 1$. To streamline the exposition, we also assume $u(\cdot)$ is strictly decreasing.

Focusing on the most relevant parameter ranges, we obtain the following proposition.

Proposition 7. *Suppose there are N regular agents and one superstar agent, of size S and equivalent in impact to $S > 1$ regular agents.*

1. *If $\min\{b(N), b(N+S-1)\} > u(0)$, the platform attracts all agents and the order of offers is irrelevant.*
2. *If $b(N+S-1) > u(0) > u(N-1) > b(N)$, the platform attracts all agents and optimally offers to the superstar last.*
3. *If $b(S+N-1) < u(S+N-2)$ and $u(\cdot)$ is convex, the platform offers to the superstar last and attracts all agents if and only if $b(N+S) > \frac{Su(N)+\sum_{k=0}^{N-1} u(k)}{N+S}$.*
4. *If $b(S+N-1) < u(S+N-2)$ and $u(\cdot)$ is concave, the platform offers to the superstar first and attracts all agents if and only if $b(N+S) > \frac{Su(0)+\sum_{k=0}^{N-1} u(S+k)}{N+S}$.*

In cases (1) and (2), all agents would be strictly better off without the platform. In cases (3) and (4), all agents except the first would be strictly better off without the platform, and the first is indifferent.

When no agents are pivotal (first case in Proposition 7), each knows the platform can still attract all others regardless of their decision. Thus, the order of offers does not matter and the platform attains maximum profit by leaving each agent with their lowest outside option.

The platform can obtain the same profit even if only the superstar is pivotal (second case in Proposition 7) by approaching it last, eliminating its pivotal role. By first contracting with the regular (non-pivotal) agents, who anticipate the superstar will join, the platform extracts their full surplus. Their participation undermines the superstar's outside option, allowing the platform to charge it the highest feasible price at the end.

Finally, when all agents are pivotal (third and fourth cases in Proposition 7), the platform can no longer achieve the same maximum profit. The optimal timing for approaching the superstar now depends on the curvature of $u(\cdot)$: approach it last if $u(\cdot)$ is convex, and first if it is concave. Approaching the superstar early requires offering it a lower price (its outside option is higher), but once secured, its participation allows the platform to charge higher prices to more subsequent agents (their outside options fall more). With convex $u(\cdot)$ the first effect dominates; with concave $u(\cdot)$ the second does. If $u(\cdot)$ is linear, the timing is irrelevant.

The general logic is that the platform should first attract non-pivotal agents to minimize the price discounts needed to attract pivotal agents. Afterward, it should attract those pivotal agents whose participation creates the greatest negative externality on others' outside options. An implication is that agents with a constant outside option should be attracted first. In reality, this might mean first attracting agents with very weak (or no) outside options, since their presence on the platform helps attract others, but not vice versa.

7.3 Platform competition

To what extent does platform competition weaken the platform trap? To explore this, suppose there are two platforms. The joining process remains sequential over N stages, with one agent selected at random in each stage. At each stage, both platforms simultaneously make offers to the selected agent.

Agents receive $b_1(n_1, n_2)$ or $b_2(n_1, n_2)$ from joining platform 1 or 2, respectively, when n_1 agents are on platform 1 and n_2 are on platform 2. We assume $b_1(n_1, n_2)$ is weakly increasing in n_1 and weakly decreasing in n_2, and vice versa for $b_2(n_1, n_2)$, so platforms create network effects. The outside option is $u(n_1, n_2)$, strictly decreasing in both arguments, reflecting that more participation on either platform reduces the value of staying out.

We also assume $b_1(N, 0) > u(N-1, 0)$ and $b_2(0, N) > u(0, N-1)$, so both platforms offer positive surplus when attracting all agents, and $b_1(m, n) > b_2(n, m)$ for any (m, n) with $1 \leq m + n \leq N$, so platform 1 offers higher benefits at any configuration. Finally, we adopt the following tie-breaking rule: whenever agents are indifferent between the two platforms, they join the one offering higher gross benefits (and platform 1 if equal), and if indifferent between a platform and the outside option, they join a platform.

We characterize the outcome for $N = 2$ under two additional assumptions:

$$b_1(2,0) > u(0,0) \tag{7}$$

$$u(0,1) \geq u(1,0). \tag{8}$$

Assumption (7) ensures that if only platform 1 exists, it is more efficient for both agents to join platform 1 than for both to stay out. Assumption (8) states that the outside option is (weakly) worse when one agent joins platform 1 than when one agent joins platform 2, reflecting that platform 1 is better and so an agent joining it has a greater negative effect on the outside option. Together with the assumptions on b_1 and b_2 above, these ensure that platform 1 attracts both agents.

The following proposition focuses on the most relevant cases.

Proposition 8. *Suppose $N = 2$ and conditions (7)-(8) hold. Platform 1 always attracts both agents and agent 1 is weakly better off than agent 2.*

1. *If $b_2(1,1) > u(0,0)$, both agents are better off with the platforms.*
2. *If $b_2(1,1) < u(0,0)$ and $b_1(1,1) > \max\{u(0,0), b_2(0,2)\}$, both agents are worse off with the platforms.*
3. *If $b_2(1,1) < u(0,0)$ and $b_2(0,2) > \max\{u(0,0), b_1(1,1)\}$, agent 1 is better off and agent 2 is worse off with the platforms.*

In case 1, platform 2 provides more value when agents split between platforms than the outside option does when no agents join. Thus, platform 2 exerts enough competitive pressure on platform 1 that both agents are better off with the platforms. In case 2, platform 1 offers more value when agents split than both the outside option and platform 2 when it attracts both agents. Here, neither platform 2 nor the outside option provides enough pressure, so both agents are worse off with the platforms. Finally, if platform 2 exerts enough competitive pressure on platform 1 for the first agent, but not after the first agent joins platform 1 or stays out, the first agent is better off with the platforms, while the second agent is worse off.

Similar results can be obtained for $N > 2$, but a full characterization becomes very messy. The intuition is the same: for at least one agent to be better off with competing platforms, the less preferred platform must be sufficiently competitive relative to the initial outside option. The results also hold when the agents' payoffs on each platform are independent of the number of agents who join the other platform, i.e. $b_1(n_1, n_2) = b_1(n_1)$ and $b_2(n_1, n_2) = b_2(n_2)$ for all (n_1, n_2).

At the symmetric boundary, relaxing the preceding strict ranking, suppose $b_2(n_1, n_2) = b_1(n_2, n_1)$ for every combination of $n_1 \in \{0, 1, 2\}$ and $n_2 \in \{0, 1, 2\}$, agent 1 is always weakly better off with the platforms. This is not surprising: competition for the first agent is most intense, so that agent benefits. However, agent 2 may still be worse off, since case 3 can still arise, namely we can still have $b_2(0,2) > u(0,0) > b_1(1,1) = b_2(1,1)$. Whether agent 2 benefits depends on how effective platform 2 is as a competitor once platform 1 has attracted agent 1. If the value platform 2 can offer agent

2 in that case is low, platform 1 can extract enough surplus to leave agent 2 worse off than without the platforms.

So far, agents can join only one platform. If agents can multihome and benefit functions are independent, so $b_1(n_1, n_2) = b_1(n_1)$ and $b_2(n_1, n_2) = b_2(n_2)$, then payoffs are $b_1(n_1) + b_2(n_2)$ when multihoming and $u(n_1, n_2) = u(n_1) + u(n_2)$ when staying out. We then return to the baseline analysis. There is no real competition between platforms in this case. For example, this arises when agents are sellers on two marketplaces serving completely distinct buyer segments, each of which decides whether to go to the outside option or the particular marketplace of interest.

8 Real-world examples and policy implications

In this section, we connect our model's predictions to real-world applications and discuss some implications for policy. The first prediction is that platform prices increase as more agents enter the market and join the platform. This can occur because early participants are pivotal and receive better terms (Proposition 1), or, even without pivotal agents, because each additional agent who joins weakens the bargaining position of those who follow (Proposition 6). These predictions are consistent with the widespread practice among real-world platforms of offering highly favorable terms early on (e.g., low fees, subsidies, exclusivity, preferential treatment), then raising fees and standardizing contracts as they scale.

For example, Amazon has not only raised fees in some categories, but also introduced new ones, that are difficult for newer sellers to avoid. Mitchell (2023), in an issue brief for the Institute for Local Self-Reliance, calculates that Amazon captured 45 percent of U.S. sellers' revenue in the first half of 2023 (up from 35 percent in 2020 and 19 percent in 2014) through referral, advertising, fulfillment, and other fees. Another example is Uber. When entering new markets (cities), it subsidized drivers with incentives such as hourly or monthly earnings guarantees, which were later removed (Hall and Krueger, 2018). Similarly, Spotify initially offered record labels equity stakes, minimum revenue guarantees, and favorable licensing terms to secure key music catalogs. Later participants joined without such concessions.

A second key prediction concerns the order in which platforms attract agents. The logic of pivotal agents would seem to imply that platforms should target large, high-impact participants first and offer them preferential terms. That would also be the general prediction based on classic models of platforms with network effects: first attract the participants that generate the largest network effect on the platform. Shopping malls illustrate this logic: developers typically first secure anchor tenants such as department stores or supermarkets with generous deals (below-market or even zero rents, long-term leases, and prime locations), then fill in with smaller retailers that pay higher rents and face co-tenancy clauses that tie their viability to the continued presence of anchor stores (Pashigian and Gould, 1998).

However, this ordering need not be optimal when participation also worsens agents' outside options (the direct channel). As shown in Propositions 7 and 14 (the latter is in the Supplementary Appendix), when only the superstar is pivotal or when all agents are pivotal and the outside option deteriorates rapidly at first and then more slowly, platforms may optimally reverse the order: attract "regular" (small) agents first to erode the outside option of "superstar" (large) ones, who are then compelled to join on less favorable terms.

The evolution of YouTube illustrates this logic. Early on (2005-2008), it focused on attracting individuals and incentivizing them to upload user-generated content. Only later (from around 2010) did it pivot to professional content and large media partners such as TV networks and movie studios. By contrast, Brightcove, one of YouTube's early competitors in Internet video, initially targeted major media companies (e.g., New York Times, Discovery Channel, MTV, CBS) with enterprise video infrastructure. Its aim was to build critical mass via professional content, before turning to user-generated content. This strategy backfired: large media partners imposed constraints that limited Brightcove's ability to scale a consumer platform, while YouTube faced no such restrictions. Ultimately, those same media companies were compelled to join YouTube, despite their initial reluctance.[19]

Third, from a policy perspective, the possible presence of a platform trap is not sufficient to warrant drastic interventions (e.g., banning platforms or imposing broad behavioral restrictions). As we showed, such traps can coexist with higher total surplus, especially in marketplace settings where buyer-side gains may compensate for seller-side surplus losses. However, we do think platform traps merit attention when evaluating the impact of platforms and related policies. Some platforms highlight that their suppliers enjoy higher revenues. For example, a report commissioned by DoorDash[20] cites a survey of individual restaurants, in which 85% of respondents said they would have lower overall revenue if they were to stop using DoorDash. But this does not imply that restaurants are collectively better off because of food delivery platforms. The key issue is that outside options are endogenous: they may be weak precisely because the

[19] See "Brightcove, Inc. (TN)," Harvard Business School teaching note 714-441, 2013.
[20] See https://doordash2024.publicfirst.co/

platform is widely adopted. As a result, comparing a restaurant's current payoff on the platform to its current outside option can be misleading, even accounting for diverted sales. A restaurant may be worse off leaving unilaterally because customers remain on the platform, whereas the outside option could be much stronger if many restaurants exited or if the platform were less prevalent.

Rather than banning platforms or imposing broad behavioral restrictions (e.g., limits on pricing), policymakers could mitigate platform traps by allowing affected agents to coordinate. When these agents are competing sellers, such coordination is usually treated as collusion under competition law. Our results suggest there may be value in carving out exceptions for sellers negotiating with dominant marketplaces or publishers negotiating with Google and Meta, provided coordination is limited to their role as platform customers. There are precedents. Australia's Treasury Laws Amendment (News Media and Digital Platforms Mandatory Bargaining Code) Act 2021 requires designated platforms (e.g., Facebook and Google) to negotiate with news publishers or face arbitration, and explicitly allows publishers to bargain collectively — conduct that would otherwise risk being deemed collusive.

Another policy intervention would be to restrict platform practices that deliberately weaken agents' outside options. For instance, some food delivery platforms (like DoorDash) withhold customer data (e.g., customer details, order histories, and detailed feedback) from participating restaurants. This makes it harder for restaurants to serve these customers well via direct channels, thereby pushing more consumers to use the platform. Another example is Apple's iMessage, which shows texts coming from Android devices as green bubbles — Bursztyn et al. (2025b) find that this stigmatizes Android users and reduces the perceived quality of Android phones.

9 Concluding remarks

Commentators have noted that far from being a boon for participants, some platforms may end up hurting them. In online marketplaces, for example, this could involve saddling sellers with new fees to reach the same customers they previously served directly. Our model demonstrates how a platform trap can emerge despite agents being rational and forward-looking. Furthermore, the model explains increasing price dynamics: prices may rise because early agents are pivotal to attracting later agents, or because each joining agent endogenously weakens the bargaining position of those who follow. We also determine the optimal order for a platform to attract different (heterogeneous) agents in order to best engineer a platform trap.

Other forms of hold-up may also complement our theory in multi-period settings. For example, agents could be locked in by irreversible platform-specific investments, or, in the case of sellers joining a marketplace, by the buyers they bring to the platform becoming loyal to it (Karle et al., 2026). Unlike our mechanism, where an agent's decision reduces the outside option for all agents, these hold-up mechanisms affect only the individual agent's outside option. Hence, they do not directly produce a platform trap, but it would nevertheless be interesting to integrate these hold-up mechanisms into our framework.

It would also be interesting to explore multi-period versions of our model; we considered one such extension, as discussed in Section 3.1. With multiple agents making participation decisions each period and externalities across them, equilibrium selection plays a role in determining the outcome. In such a setting, equilibrium selection may be influenced by participants' past choices, following the ideas in Halaburda and Yehezkel (2019) and Halaburda et al. (2020). In particular, equilibrium selection may be unfavorable for the platform at first, but if it can attract a critical number of agents to join, the platform may be able to enjoy favorable equilibrium selection after some point. This suggests a further mechanism by which a dynamic platform trap may arise.

10 Appendix

In this appendix we provide the proofs of propositions not proven in the text.

10.1 Proof of Propositions 1 and 3

Denote by $\Gamma(n, b(\cdot), u(\cdot))$ the generalized game from Section 5, with $N = n$ agents and payoff functions $b(\cdot)$ and $u(\cdot)$ satisfying Assumption A5. This formulation encompasses the microfounded version of the game from the baseline model, where agents are sellers on a marketplace. Thus, the proof that follows applies to both Proposition 1 and Proposition 3.

We start with two lemmas, which will be useful later.

Lemma 1: *If the platform can profitably attract all agents in the game $\Gamma(n, b(\cdot), u(\cdot))$, then it profitably attracts all agents in the game $\Gamma(n+1, b(\cdot), u(\cdot))$ with optimal prices $P^k = \Delta(n+1)$.*

Proof: The proof is by induction over games with an increasing number of agents. If the platform can profitably attract the sole agent in the game $\Gamma(1, b(\cdot), u(\cdot))$, then $b(1) > u(0)$, i.e. $\Delta(1) > 0$. Thus, in the game $\Gamma(2, b(\cdot), u(\cdot))$, if the first agent doesn't join, the platform can still attract the second one. And $\Delta(2) \geq \Delta(1) > 0$, so if the first agent does join, the platform also profitably attracts the second one. Thus, it profitably attracts both agents in $\Gamma(2, b(\cdot), u(\cdot))$ by charging each $\Delta(2) > 0$.

Suppose the statement in the lemma is true for $N = n - 1 \geq 1$ and *any* payoff functions $b(\cdot)$ and $u(\cdot)$ such that $u(\cdot)$ is weakly decreasing and $\Delta(k) = b(k) - u(k-1)$ is weakly increasing. We show it is also true for $N = n$. Namely, suppose the platform profitably attracts all agents in the game $\Gamma(n, b(\cdot), u(\cdot))$, so $\Delta(n) > 0$. We want to show the platform profitably attracts all agents in the game $\Gamma(n+1, b(\cdot), u(\cdot))$ at prices $P^k = \Delta(n+1)$ for $k = 1, \ldots, n+1$.

If the first of $n+1$ agents doesn't join, when facing the second agent, the platform is in the same position as at the start of $\Gamma(n, b(\cdot), u(\cdot))$, so it still attracts the last n out of $n+1$ agents. If the first agent joins and the second does not, then when facing agents $\{3, \ldots, n+1\}$, the platform is in the same position as when facing the second agent in $\Gamma(n, b(\cdot), u(\cdot))$ after the first agent has joined. By assumption, the platform attracts all agents in this situation, so the platform profitably attracts all agents $\{3, \ldots, n+1\}$ in $\Gamma(n+1, b(\cdot), u(\cdot))$ after the first agent joins and the second does not. This is equivalent to the platform profitably attracting all agents in $\Gamma\left(n-1, \widetilde{b}(\cdot), \widetilde{u}(\cdot)\right)$, where $\widetilde{b}(k) \equiv b(k+1)$ and $\widetilde{u}(k) \equiv u(k+1)$ (note $\widetilde{u}(\cdot)$ is weakly decreasing and $\widetilde{\Delta}(k) = \widetilde{b}(k) - \widetilde{u}(k-1) = b(k+1) - u(k)$ is weakly increasing). The induction hypothesis ($N = n-1$) then implies the platform profitably attracts all agents in $\Gamma\left(n, \widetilde{b}(\cdot), \widetilde{u}(\cdot)\right)$. And this is equivalent to saying that in $\Gamma(n+1, b(\cdot), u(\cdot))$, after the first agent joins, the platform profitably attracts all remaining n agents.

Thus, the platform profitably attracts the last n agents in $\Gamma(n+1, b(\cdot), u(\cdot))$ regardless of whether the first agent joins or not, so it can attract the first agent with $P^1 = \Delta(n+1) \geq \Delta(n) > 0$. Using a similar logic, the platform attracts all agents in $\Gamma(n+1, b(\cdot), u(\cdot))$ with the same price $\Delta(n+1)$, which maximizes profits. ■

Lemma 2: *In the game* $\Gamma(n, b(\cdot), u(\cdot))$, *either the platform optimally attracts all agents, or optimally attracts none of them.*

Proof: Suppose to the contrary, the platform finds it optimal to only attract $0 < n_0 < n$ agents in game $\Gamma(n, b(\cdot), u(\cdot))$. In this case, $\Delta(n_0) > 0$ because this is the highest price the platform can charge to participating agents. If the last agent that does not join is the last one overall, then the platform can attract it with a price $\Delta(n_0 + 1) \geq \Delta(n_0) > 0$, which is a contradiction. Suppose instead the last agent that does not join is the $(n - k_0)$-th agent, so the last $k_0 \geq 1$ agents are all among the n_0 agents that join. Consider any one of these final k_0 agents. If that agent rejects, at most $n_0 - 1$ agents can ultimately join, so the price that can be charged to such an agent can be no higher than

$$b(n_0) - u(n_0 - 1) = \Delta(n_0).$$

Consequently, the platform's continuation profit from the final k_0 participating agents is at most

$$k_0 \Delta(n_0).$$

The platform profitably attracts all agents in the game $\Gamma\left(k_0, \widetilde{b}(\cdot), \widetilde{u}(\cdot)\right)$, where $\widetilde{b}(k) = b(n_0 - k_0 + k)$ and $\widetilde{u}(k) = u(n_0 - k_0 + k)$. Applying Lemma 1, the platform can also profitably attract all agents in the game $\Gamma\left(k_0 + 1, \widetilde{b}(\cdot), \widetilde{u}(\cdot)\right)$ by charging each of them

$$\widetilde{\Delta}(k_0 + 1) = \Delta(n_0 + 1) > 0.$$

This alternative continuation therefore earns

$$(k_0 + 1)\Delta(n_0 + 1) \geq (k_0 + 1)\Delta(n_0) > k_0 \Delta(n_0).$$

Revenues collected before this continuation are unchanged. The platform can therefore strictly increase its total profit, contradicting the supposed optimality of attracting only n_0 agents. ■

The proof of Proposition 1 also proceeds by induction over games with an increasing number of agents. Start with $\Gamma(N = 2, b(\cdot), u(\cdot))$. If $\Delta(1) > 0$, then neither agent is pivotal, so the platform maximizes profits by attracting both agents with prices $P^1 = P^2 = \Delta(2) \geq \Delta(1) > 0$. If instead $\Delta(1) \leq 0$, then the first agent is pivotal, so the profit-maximizing prices that attract both agents are $P^1 = b(2) - u(0)$ and $P^2 = b(2) - u(1)$. From Lemma 2, if the platform finds it optimal to attract any agents, it must attract both, so these prices are optimal if $2b(2) - u(1) - u(0) > 0$.

Suppose the following induction hypothesis holds for $N = n \geq 2$ (we have just shown it holds for $N = 2$) and *any* payoff functions $b(\cdot)$ and $u(\cdot)$ such that $u(\cdot)$ is weakly decreasing and $\Delta(k) = b(k) - u(k-1)$ is weakly increasing in k:

- If the platform can profitably attract all agents in the game $\Gamma(n-1, b(\cdot), u(\cdot))$, then it profitably attracts all agents in the game $\Gamma(n, b(\cdot), u(\cdot))$ with optimal prices $P^k = \Delta(n)$.

- If the platform cannot profitably attract agents in the game $\Gamma(n-1, b(\cdot), u(\cdot))$, there exists $k_0 \in \{1, \ldots, n-1\}$ such that the platform's profit-maximizing prices that induce agents to join in the game $\Gamma(n, b(\cdot), u(\cdot))$ are

$$P^k = \begin{cases} b(n) - u(k-1) & \text{if} \quad 1 \leq k \leq k_0 \\ b(n) - u(n-1) & \text{if} \quad k_0 + 1 \leq k \leq n \end{cases}.$$

If $\sum_{k=1}^{n} P^k > 0$, then these are the platform's optimal prices in $\Gamma(n, b(\cdot), u(\cdot))$ and all agents join. If instead $\sum_{k=1}^{n} P^k \leq 0$, then it is optimal to not attract any agents.

We now show this also holds for $N = n+1$. If the platform can profitably attract all agents in the game $\Gamma(n, b(\cdot), u(\cdot))$, then Lemma 1 directly implies it profitably attracts all agents in the game $\Gamma(n+1, b(\cdot), u(\cdot))$ with optimal prices $P^k = \Delta(n+1)$. Suppose instead the platform cannot profitably attract any agents in the game $\Gamma(n, b(\cdot), u(\cdot))$. In this case, the highest price at which the platform can attract the first agent in $\Gamma(n+1, b(\cdot), u(\cdot))$ is $P^1 = b(n+1) - u(0)$. Once the first agent participates, then when facing the remaining n agents, the platform is in a position equivalent to that at the start of game $\Gamma\left(n, \widetilde{b}(\cdot), \widetilde{u}(\cdot)\right)$, where $\widetilde{b}(k) = b(k+1)$ and $\widetilde{u}(k) = u(k+1)$. We can then apply the induction hypothesis to conclude:

- If the platform can attract all agents in $\Gamma\left(n-1, \widetilde{b}(\cdot), \widetilde{u}(\cdot)\right)$, then it attracts the last n agents in $\Gamma(n+1, b(\cdot), u(\cdot))$ after the first agent has joined with optimal prices
$$P^k = \widetilde{b}(n) - \widetilde{u}(n-1) = \Delta(n+1) > 0$$
for all $k \in \{2, \ldots, n+1\}$. So we have
$$P^k = \begin{cases} b(n+1) - u(0) & \text{if} \quad k = 1 \\ b(n+1) - u(n) & \text{if} \quad 2 \leq k \leq n+1 \end{cases}.$$

- If the platform cannot profitably attract agents in $\Gamma\left(n-1, \widetilde{b}(\cdot), \widetilde{u}(\cdot)\right)$, there exists $k_0 \in \{2, \ldots, n\}$ such that in $\Gamma(n+1, b(\cdot), u(\cdot))$ after the first agent has joined, the platform's profit-maximizing prices for agents $k \in \{2, \ldots, n+1\}$ that induce them to join are
$$P^k = \begin{cases} \widetilde{b}(n) - \widetilde{u}(k-2) = b(n+1) - u(k-1) & \text{if} \quad 2 \leq k \leq k_0 \\ \widetilde{b}(n) - \widetilde{u}(n-1) = b(n+1) - u(n) & \text{if} \quad k_0 + 1 \leq k \leq n+1 \end{cases}.$$
If $\sum_{k=2}^{n+1} P^k > 0$, these prices are optimal for agents $k \in \{2, \ldots, n+1\}$. If instead $\sum_{k=2}^{n+1} P^k \leq 0$, the platform cannot profitably attract agents $k \in \{2, \ldots, n+1\}$

even after the first agent joins, so the platform sets sufficiently high nonnegative prices, or makes no offers, and attracts no agents. Thus, the platform's profit-maximizing prices in $\Gamma(n+1, b(\cdot), u(\cdot))$ that induce all agents to join (recall Lemma 2) are

$$P^k = \begin{cases} b(n+1) - u(k-1) & \text{if} \quad 1 \le k \le k_0 \\ b(n+1) - u(n) & \text{if} \quad k_0 + 1 \le k \le n+1 \end{cases}.$$

If $\sum_{k=1}^{n+1} P^k > 0$, these are the platform's optimal prices and all agents join (note $\sum_{k=1}^{n+1} P^k > 0$ implies $\sum_{k=2}^{n+1} P^k > 0$ because P^k is weakly increasing in k). If instead $\sum_{k=1}^{n+1} P^k \le 0$, the platform finds it optimal to attract no agents in $\Gamma(n+1, b(\cdot), u(\cdot))$.

Thus, the induction hypothesis holds for $N = n+1$. By induction, it holds for all $N \ge 2$. Moreover, if (4) holds, then the platform optimally attracts all agents in $\Gamma(N, b(\cdot), u(\cdot))$ even in the worst case when all of them are pivotal, so it can always profitably attract N agents. Finally, agent k's payoff in $\Gamma(N, b(\cdot), u(\cdot))$ when all join is $b(N) - P^k$, which is either $u(N-1)$ (if agent k is not pivotal) or $u(k-1)$ (if agent k is pivotal). These are both weakly lower than $u(0)$ (strictly for $k = N$), which is the payoff each agent would obtain without the platform.

10.2 Proof of Corollary 1

The case $\Delta(1) > 0$ is already explained in the text before Proposition 1. If $b(N-1) > \frac{1}{N-1}\sum_{k=1}^{N-1} u(k-1)$, Proposition 1 implies that the platform profitably attracts the last $N-1$ agents even if the first agent does not join. The platform can then attract the first agent at price $P^1 = b(N) - u(N-1)$. By a similar logic, it attracts all remaining agents $k = 2, \ldots, N$ at prices $P^k = b(N) - u(N-1)$.

10.3 Proof of Corollary 2

If $b(N-1) - u(N-2) \le 0$, then $b(k) - u(k-1) \le 0$ for all $k \in \{1, \ldots, N-1\}$, i.e., all agents are pivotal. The logic used in the proof of Proposition 1 implies the platform optimally sets $P^k = b(N) - u(k-1)$ to the k-th agent for $k = 1, \ldots, N$ provided (4) holds. Otherwise, the platform finds it optimal to attract no agents.

10.4 Proof of Corollary 3

From Proposition 1, agent k's net surplus in the game $\Gamma(N, b(\cdot), u(\cdot))$ is either $u(k-1)$ or $u(N-1)$, so is weakly decreasing in N. From Lemma 2 in the Proof of Proposition 1, there are two possibilities:

- If the platform finds it optimal to attract all agents in $\Gamma(N, b(\cdot), u(\cdot))$, its profits are at most $N\Delta(N)$ and repeated application of Lemma 1 implies that platform profits in $\Gamma(N', b(\cdot), u(\cdot))$ are $N'\Delta(N') > N\Delta(N)$.
- If the platform finds it optimal to attract no agents in $\Gamma(N, b(\cdot), u(\cdot))$, then its profits in $\Gamma(N', b(\cdot), u(\cdot))$ are weakly higher (strictly when it can profitably attract all agents in $\Gamma(N', b(\cdot), u(\cdot))$).

If the platform is profitable in $\Gamma(N, b(\cdot), u(\cdot))$, then Lemma 1 implies that for any $N' > N$, the platform attracts all agents with prices equal to $\Delta(N') > 0$ in $\Gamma(N', b(\cdot), u(\cdot))$, so no agent is pivotal. Thus, the number of pivotal agents in $\Gamma(N', b(\cdot), u(\cdot))$ is weakly lower than in $\Gamma(N, b(\cdot), u(\cdot))$ (strictly whenever there are any pivotal agents in $\Gamma(N, b(\cdot), u(\cdot))$).

10.5 Proof of Proposition 4

Here, we denote by $\Gamma(n, b(\cdot), u(\cdot), P)$ the platform adoption game that unfolds when there are $N = n$ total agents, payoff functions are $b(\cdot), u(\cdot)$, and the platform has set price P.

First, we prove that the only possible equilibria in $\Gamma(n, b(\cdot), u(\cdot), P)$ are all agents join or no agents join. Suppose instead $1 \leq m < N$ agents join in equilibrium, and the last agent does not join. This implies $P > \Delta(m+1)$. The last agent to join obtains $b(m) - P$ in equilibrium. If that agent deviates to not joining, then all subsequent agents will continue to not join (they are less likely to do so because fewer prior agents will have joined; this can be easily proven by induction), so the agent's deviation payoff is $u(m-1)$. Since $P > \Delta(m+1) \geq \Delta(m)$, the deviation must be profitable. Thus, in an equilibrium in which the last agent doesn't join, there can be no prior agent that joins, so no agents join.

Suppose instead the equilibrium with $1 \leq m < N$ agents joining is such that the last agent joins, so $P \leq \Delta(m)$. The last agent that does not join obtains $u(m)$. If that agent deviates to joining, then all subsequent agents will continue to join (they are more likely to do so because more prior agents will have joined), so the agent's deviation payoff is $b(m+1)-P$. Since $\Delta(m+1) \geq \Delta(m) \geq P$, the deviation is profitable. Thus, in an equilibrium in which the last agent joins, there can be no prior agent that does not join, so all agents join.

Next, we show by induction that when $b(\cdot)$ is weakly increasing, all agents join in $\Gamma(N, b(\cdot), u(\cdot), P)$ if $P \leq b(N)-u(0)$ and no agents join otherwise. In $\Gamma(1, b(\cdot), u(\cdot), P)$, the agent joins if and only if (iff) the platform sets $P \leq b(1)-u(0)$. Suppose the result holds for $N=n$ and any payoff functions $b(\cdot)$ and $u(\cdot)$ such that $b(\cdot)$ is weakly increasing and $u(\cdot)$ is weakly decreasing (so $\Delta(k)=b(k)-u(k-1)$ is weakly increasing in k). Consider $\Gamma(N=n+1, b(\cdot), u(\cdot), P)$. If the platform sets $P \leq b(n+1)-u(0)$, then after the first agent joins, the game is equivalent to $\Gamma\left(n, \widetilde{b}(\cdot), \widetilde{u}(\cdot), P\right)$, where $\widetilde{b}(n)=b(n+1)$, $\widetilde{u}(n)=u(n+1)$, and

$$P \leq b(n+1)-u(0) \leq b(n+1)-u(1)=\widetilde{b}(n)-\widetilde{u}(0).$$

The induction hypothesis implies all n agents join in this game. Thus, if the first agent joins in $\Gamma(n+1, b(\cdot), u(\cdot), P)$, its payoff will be $b(n+1)-P$, whereas if the agent doesn't join, its payoff will be at most $u(0)$. So the first agent joins because $b(n+1)-P \geq u(0)$, and so will all remaining n agents.

If instead the platform sets $P > b(n+1)-u(0)$ in $\Gamma(n+1, b(\cdot), u(\cdot), P)$, the first agent's payoff from joining will be less than $u(0)$. If the first agent doesn't join, then the remaining subgame is equivalent to $\Gamma(n, b(\cdot), u(\cdot), P)$ with

$$P > b(n+1)-u(0) \geq b(n)-u(0).$$

The induction hypothesis then implies no agent joins, so the first agent's payoff from not joining is $u(0)$, higher than its payoff from joining. The first agent doesn't join and neither will any of the remaining n agents. The result thus holds for $\Gamma(n+1, b(\cdot), u(\cdot), P)$.

By induction, the result holds for any $N \geq 2$. And we can then conclude that when $b(\cdot)$ is weakly increasing, the platform's optimal price is $P=b(N)-u(0)$ if $b(N)-u(0)>0$, which attracts all agents, and any $P>0$ if $b(N)-u(0) \leq 0$, which attracts no agents.

Finally, suppose $b(\cdot)$ is single-peaked and let m_0 be the lowest point at which $b(\cdot)$ reaches its maximum, i.e.

$$m_0=\min\left\{m \mid b(m)=\max_{1 \leq n \leq N} b(n)\right\}.$$

We prove that in the game $\Gamma(N, b(\cdot), u(\cdot), P)$, all agents join if $P \leq b(m_0)-u(0)$ and no agents otherwise. Suppose $P \leq b(m_0) - u(0)$ and no agents join in equilibrium. The subgame starting with agent $N - m_0 + 1$ is equivalent to $\Gamma(m_0, b(\cdot), u(\cdot), P)$ (no prior agents have joined), where $b(\cdot)$ is weakly increasing from 1 to m_0 and $P \leq b(m_0) - u(0)$. The result above implies that all m_0 agents should join in the equilibrium of this game, which is a contradiction. Thus, all agents join in the equilibrium of $\Gamma(N, b(\cdot), u(\cdot), P)$ if $P \leq b(m_0) - u(0)$. If instead $P > b(m_0) - u(0)$, the only possible equilibrium is that no agents join. Indeed, if the first $N - 1$ agents do not join, the last agent does not join either because it obtains $u(0)$ by not joining and at most $b(m_0) - P$ by joining. Similarly, if the first $N - 2$ agents do not join, then neither do the last two. And so on, until we conclude that if the first agent does not join, then neither will the remaining $N - 1$ agents. So the first agent's payoff from not joining is $u(0)$, compared to at most $b(m_0) - P$ from joining. Thus, no agent joins in equilibrium if $P > b(m_0) - u(0)$. So the platform's optimal price in this case is $P = b(m_0) - u(0)$ if this is positive, and all agents join. Otherwise, no agents join.

10.6 Proof of Proposition 5

In the best possible equilibrium for the platform, the platform prices at $P = \Delta(N) > 0$ to all agents. Each agent joins because they expect all other agents to join. The platform attains its maximum feasible profits and all agents receive net payoff $u(N-1)$, so they would be strictly better off without the platform.

Suppose $\Delta(1) > 0$. If the platform prices at $P = \Delta(1)$, each agent joins because they expect all other agents to join at this price, so they also join given they get $b(N) - P \geq b(N) - \Delta(1) \geq u(N-1) > 0$ because $\Delta(\cdot)$ is weakly increasing. Note that, at this price, or any lower price, it is no longer an equilibrium for no agents to join, because even if an agent expects no other agent to join, it will want to join: i.e., $b(1) - P \geq u(0)$. So this is the worst the platform can obtain. And this is an equilibrium because if the platform deviates to $P > \Delta(1)$, then each agent expects no other agents to join, which is an equilibrium since under these expectations they would get $b(1) - P < u(0)$ from joining. In the equilibrium, agents get $b(N) - P = u(0) + (b(N) - b(1))$, so agents are strictly better off without the platform if and only if $b(1) > b(N)$.

If $\Delta(1) \leq 0$, the platform cannot profitably attract any agents in the worst equilibrium defined above.

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Supplemental Appendix
Platform Traps

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A Infinite period model

In this section, we consider a multi-period setting to illustrate how our single-period model can be interpreted as capturing the full decision-making process across periods. We had claimed in the discussion of our model that the absence of discounting across the sellers' decisions within a period reflects the assumption that joining decisions are made over a short time frame, relative to the longer horizon (possibly infinite) over which payoffs arise. We illustrate this here by considering an infinite period model to show that a platform trap can still arise in such a setting.

Let's consider the general setting of Section 5, of which the marketplace baseline setting is a special case (i.e., the sellers' payoff functions can be mapped to a special case of the agents' payoff functions). The platform sets a single price in each period that applies to all participating agents in that period. In each of the first N periods, each agent is selected in sequence to decide whether to participate on the platform or not, so one agent decides in each period. If some agents have previously joined the platform, in any subsequent period they can leave, but are assumed to stay whenever staying is consistent with an equilibrium given the price the platform sets in that period. If indifferent between joining or not, an agent will join. Agents only get one chance to participate, and this only happens in the first N periods. This means if they decide not to participate (or if they later leave the platform), they remain forever outside. The discount factor is $0 < \delta < 1$.

We consider the two extreme cases considered in Section 5, corresponding to Corollary 1 and Corollary 2.

A.1 No pivotal agents

Let's first assume $\Delta(1) > 0$ which implies no agents are ever pivotal. We solve by backwards induction. Suppose at the start of period $N+1$, after which no more agents can join, $1 \leq n \leq N$ agents have joined. They obtain a benefit of being on the platform of $b(n)$ in each period, provided they all remain. The option for an individual agent at that point is to leave and obtain the alternative $u(n-1)$ in all subsequent periods, assuming the rest of the agents remain on the platform. Therefore the highest price the platform can set each period such that it is an equilibrium

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for all agents to remain (when they expect others to do so) is $P^{N+1}(n) = \Delta(n) = b(n) - u(n-1) > 0$. Since this logic applies in all subsequent periods, we also have $P^t(n) = \Delta(n)$ for all $t \geq N+1$. Agents will therefore get $u(n-1)$ in period $N+1$ and each subsequent period, implying each participating agent's present discounted value (PDV) payoff from the perspective of period $N+1$ is

$$V^{N+1}(n) = \frac{1}{1-\delta} u(n-1),$$

while the platform will obtain a PDV payoff of

$$\Pi^{N+1}(n) = \frac{1}{1-\delta} n\Delta(n) > 0.$$

Now consider the last period N in which agents can decide to join, which means agent N is deciding whether to join and some n satisfying $0 \leq n \leq N-1$ agents who have already joined the platform decide whether to stay. If agent N joins and the other n agents remain, each of the $n+1$ participating agents obtains a payoff equal to

$$b(n+1) - P + \delta V^{N+1}(n+1),$$

where P is the price charged by the platform. This is an equilibrium iff agent N indeed wants to join and none of the n agents that have previously joined want to leave in period N. The payoff to such a deviation is $u(n)$ in each subsequent period, so agent N joins and the existing n agents stay iff

$$b(n+1) - P + \delta V^{N+1}(n+1) \geq \frac{1}{1-\delta} u(n).$$

Thus, the maximum price the platform can set to induce participation of these $n+1$ agents is

$$P^N(n) = \Delta(n+1) > 0. \tag{9}$$

If the platform sets a price greater than $\Delta(n+1)$, then the only equilibrium is that no agents participate in period N. To see this, suppose $k \geq 1$ agents participate in equilibrium, which implies we must have

$$b(k) - \Delta(n+1) + \delta V^{N+1}(k) > \frac{1}{1-\delta} u(k-1),$$

i.e.,

$$b(k) + \delta V^{N+1}(k) - \frac{1}{1-\delta} u(k-1) > \Delta(n+1).$$

Recall $V^{N+1}(k) = \frac{1}{1-\delta} u(k-1)$, so the LHS is equal to $\Delta(k)$, which is increasing in k. Thus, since $k \leq n+1$, we must have

$$\Delta(n+1) \geq \Delta(k) > \Delta(n+1),$$

which is a contradiction. Thus, when the platform charges a price higher than $P^N(n) = \Delta(n+1)$, the only possible equilibrium is that no agents participate. This means the platform's optimal price in period N is indeed $P^N(n) = \Delta(n+1) > 0$.

The platform attracts agent N at the price (9) and each participating agent's present-discounted value (PDV) payoff in period N is

$$V^N(n) = \frac{u(n)}{1-\delta}.$$

The PDV of platform profit is

$$\begin{aligned} \Pi^N(n) &= (n+1) P^N(n) + \delta \Pi^{N+1}(n+1) \\ &= \frac{1}{1-\delta}(n+1)\Delta(n+1) > 0. \end{aligned}$$

so the platform will indeed attract agent N regardless of the value of n.

For all $0 \leq n < t \leq N$, define

$$V^t(n) = \sum_{j=0}^{N-t} \delta^j u(n+j) + \frac{\delta^{N-t+1}}{1-\delta} u(n+N-t).$$

Note that $V^N(n) = \frac{u(n)}{1-\delta}$ and for all $0 \leq n < t \leq N-1$,

$$V^t(n) = u(n) + \delta V^{t+1}(n+1). \tag{10}$$

The induction hypothesis for $T+1 \in \{2, \ldots, N\}$ is as follows. For any $t \in \{T+1, ..., N\}$ and $0 \leq n \leq t-1$, if n agents have already joined the platform at the beginning of period t, the platform induces all of them to remain and agent t to join by setting $P^t(n) = \Delta(n+1)$. All participating agents obtain PDV of payoffs equal to $V^t(n)$.

We now want to show that the same is true in period T. Suppose n agents have already joined, with $0 \leq n \leq T-1$. When the platform charges P in period T, if agent T joins and the other n agents remain on the platform this period, the induction hypothesis implies that each participating agent's payoff is

$$b(n+1) - P + \delta V^{T+1}(n+1),$$

given that in all subsequent periods, every new agent is attracted and all other agents remain on the platform. This is an equilibrium iff agent T indeed wants to join and none of the n agents that have previously joined want to leave in period T. The payoff to such a deviation is

$$\sum_{j=0}^{N-T} \delta^j u(n+j) + \frac{\delta^{N-T+1}}{1-\delta} u(n+N-T) = V^T(n),$$

because by the induction hypothesis, all other agents remain on the platform in all subsequent periods and every new agent is attracted and stays. Thus, in period T all existing agents stay and the new agent joins iff

$$b(n+1) - P + \delta V^{T+1}(n+1) \geq V^T(n).$$

Using (10), this inequality is equivalent to

$$P \leq b(n+1) - u(n) = \Delta(n+1).$$

If the platform sets a higher price in period T, i.e., $P > \Delta(n+1)$, then in equilibrium no agents participate. To see this, suppose $k \in \{1, \ldots, n+1\}$ agents participate, so we must have

$$b(k) - P + \delta V^{T+1}(k) \geq u(k-1) + \delta V^{T+1}(k).$$

Indeed, if one of the k agents deviates to not participating this period, it obtains $u(k-1)$ this period and $V^{T+1}(k)$ from the perspective of next period by the induction hypothesis. Thus, we must have

$$b(k) - u(k-1) \geq P,$$

which implies

$$\Delta(n+1) \geq \Delta(k) \geq P,$$

a contradiction. Thus, when $P > \Delta(n+1)$, the only possible equilibrium in period T is that no agents participate. This cannot be optimal because $\Delta(\cdot)$ is weakly increasing, so retaining the current participants yields greater revenue in the current period and in every subsequent period.

The platform's optimal price in period T is therefore

$$P^T(n) = \Delta(n+1) > 0.$$

At this price, each participating agent's PDV payoff is

$$b(n+1) - \Delta(n+1) + \delta V^{T+1}(n+1) = V^T(n).$$

Since this is profitable for the platform in every period, induction logic establishes that in every period $t \in \{1, \ldots, N\}$, if $n \in \{0, \ldots, t-1\}$ agents have already joined, the platform can induce the selected agent to join and all existing agents to remain by setting $P^t(n) = \Delta(n+1)$.

On the equilibrium path the number of existing agents is $n = t-1$ at the beginning of each period $t \in \{1, \ldots, N\}$, and $n = N$ for all $t \geq N+1$. On the equilibrium path, the platform charges $P^t = \Delta(t)$ for $1 \leq t \leq N$ and $P^t = \Delta(N)$ for $t \geq N+1$. All agents join the platform and remain on it along the equilibrium path.

The PDV of payoffs for agent k along the equilibrium path is

$$\begin{aligned} W(k) &= \sum_{t=1}^{k-1} \delta^{t-1} u(t) + \delta^{k-1} V^k (k-1) \\ &= \sum_{t=1}^{k-1} \delta^{t-1} u(t) + \sum_{t=k}^{N} \delta^{t-1} u(t-1) + \frac{\delta^N}{1-\delta} u(N-1). \end{aligned}$$

Since $u(n)$ is decreasing (strictly decreasing for at least one n within $0 \leq n \leq N-1$), $W(k)$ is strictly lower than each agent's payoff in the absence of the platform, which is simply $\frac{u(0)}{1-\delta}$. Moreover, it is easily verified that $W(k)$ is weakly decreasing in k, so agents that are selected earlier obtain weakly higher PDV payoffs, strictly so whenever $u(k-1) > u(k)$ for some k. Specifically, in this case, the difference in PDV payoff between agent k and agent $k+1$ is $\delta^{k-1}(u(k-1) - u(k)) > 0$. This is because an agent's value of not joining worsens as other agents join, so agents selected later are made indifferent against a weaker outside option.

Finally, note that if we interpret agents' payoffs as $\delta \to 1$, then the term that dominates payoffs is the eventual payoff (after the initial $N-1$ periods) which is $u(N-1)$ in all subsequent periods, coinciding exactly with the payoffs in our one-period model in the case without pivotal agents.

A.2 All pivotal agents

Next we suppose the first $N-1$ agents are pivotal by assuming $\Delta(N-1) < 0 < \Delta(N)$. We focus on the case in which

$$\Delta(N) > \delta(u(0) - u(N-1)),$$

so that, once all N agents have joined, the platform can profitably retain them in every subsequent period. In period $t = N+1$, if $n \in \{1, ..., N-1\}$ have previously joined, then any price that retains the agents can be no greater than $\Delta(n) < 0$, which is not profitable, so the platform attracts no agents from here on out, and $\Pi^{N+1}(n) = 0$ and $V^{N+1}(n) = \frac{u(0)}{1-\delta}$. If $n = N$ agents have joined, then the most the platform can charge and keep all agents staying is P such that

$$\frac{b(N) - P}{1-\delta} \geq u(N-1) + \frac{\delta u(0)}{1-\delta}$$

since if any agent leaves, the remaining $N-1$ agents cannot profitably be retained and therefore leave from the following period onward. Thus, under our maintained condition, the platform charges $P = \Delta(N) - \delta(u(0) - u(N-1))$ and retains all agents, leading to $\Pi^{N+1}(N) = \frac{N}{1-\delta}(\Delta(N) - \delta(u(0) - u(N-1)))$ and $V^{N+1}(N) = u(N-1) + \frac{\delta u(0)}{1-\delta}$.

Now consider period $t = N$. If $n < N-1$ agents have been previously attracted, then the platform will attract no agents from next period onwards and the maximum price it can hope to charge this period is $P = \Delta(n+1) < 0$. Thus, the platform attracts no one in this period and in any of the subsequent periods, leading to $\Pi^N(n) = 0$ and $V^N(n) = \frac{u(0)}{1-\delta}$.

If $n = N-1$ agents have been previously attracted, then the platform attracts all N agents iff

$$b(N) - P + \delta V^{N+1}(N) \geq u(N-1) + \frac{\delta u(0)}{1-\delta}.$$

Indeed, if one agent deviates to not participating this period, then next period and thereafter the platform will attract no agents. Thus, the platform attracts all N agents in period N iff

$$P \leq P^N = \Delta(N) - \frac{\delta u(0)}{1-\delta} + \delta V^{N+1}(N).$$

This is profitable iff

$$NP^N + \delta \Pi^{N+1}(N) > 0,$$

which is equivalent to

$$\Delta(N) > \delta(u(0) - u(N-1)).$$

This condition holds by assumption.

Furthermore, it cannot be profitable for the platform to set a higher price, i.e., $P > P^N$. If it were, then suppose the platform attracts $k \in \{1, \ldots, N-1\}$ agents to participate with $P > P^N$ (we already know it cannot attract N agents). Then we must have

$$b(k) - P + \frac{\delta u(0)}{1-\delta} \geq u(k-1) + \frac{\delta u(0)}{1-\delta},$$

which implies $P \leq \Delta(k) < 0$. This cannot be profitable because in this case the platform attracts no agents in any subsequent period (since $k < N$).

Thus, if $n = N-1$ agents have been previously attracted, the platform optimally attracts all agents in period N and every period thereafter, leading to the following PDV of profits and agent surplus in period N:

$$\begin{aligned} \Pi^N(N-1) &= \frac{N(\Delta(N) - \delta(u(0) - u(N-1)))}{1-\delta} \\ V^N(N-1) &= u(N-1) + \frac{\delta u(0)}{1-\delta}. \end{aligned}$$

Otherwise, the platform does not attract any agents in period N and all periods thereafter, so

$$\begin{aligned}\Pi^N(N-1) &= 0\\ V^N(N-1) &= \frac{u(0)}{1-\delta}.\end{aligned}$$

For all $t \in \{1, \ldots, N-1\}$, define

$$\pi(t) \equiv \sum_{k=0}^{N-1-t} \delta^k (k+t)(\Delta(k+t) - \delta(u(0) - u(k+t))) + \delta^{N-t} \frac{N(\Delta(N) - \delta(u(0) - u(N-1)))}{1-\delta}.$$

Suppose the following induction hypothesis holds for period $T+1 \leq N-1$. If at the start of period $T+1$ the platform has $n \leq T$ existing agents, then

$$\begin{aligned}\Pi^{T+1}(n) &= \begin{cases} \pi(T+1) & \text{if} \quad n = T \text{ and } \pi(T+1) > 0 \\ 0 & \text{if} \quad n < T \text{ or } \pi(T+1) \leq 0 \end{cases}\\ V^{T+1}(n) &= \begin{cases} u(T) + \frac{\delta u(0)}{1-\delta} & \text{if} \quad n = T \text{ and } \pi(T+1) > 0 \\ \frac{u(0)}{1-\delta} & \text{if} \quad n < T \text{ or } \pi(T+1) \leq 0 \end{cases},\end{aligned}$$

We now want to show that the same holds true in period T. Suppose n agents have already joined at the start of period T, with $0 \leq n \leq T-1$. If $n < T-1$, then we know from the induction hypothesis that the platform makes zero profits from next period onwards and in the current period the maximum price it can charge is $P = \Delta(n+1) < 0$, so the platform attracts no agents this period either and therefore

$$\begin{aligned}\Pi^T(n) &= 0\\ V^T(n) &= \frac{u(0)}{1-\delta}.\end{aligned}$$

Now suppose $n = T-1$. The platform attracts all T agents in this period iff

$$b(T) - P + \delta V^{T+1}(T) \geq u(T-1) + \frac{\delta u(0)}{1-\delta}.$$

If $\pi(T+1) \leq 0$, then the platform makes zero profits from next period onwards, and we must have $P \leq \Delta(T) \leq \Delta(N-1) < 0$, so this cannot be profitable. Thus, attracting all T agents this period can only be profitable if $\pi(T+1) > 0$, so $V^{T+1}(T) = u(T) + \frac{\delta u(0)}{1-\delta}$. Then the platform attracts all T agents in period T iff

$$P \leq P^T = \Delta(T) - \delta(u(0) - u(T)).$$

Furthermore, it cannot be profitable for the platform to set a higher price, i.e., $P > P^T$. If it were, then suppose the platform attracts $k \in \{1, \ldots, T-1\}$ agents to participate with $P > P^T$ (we already know it cannot attract all T agents). Then we must have

$$b(k) - P + \frac{\delta u(0)}{1-\delta} \geq u(k-1) + \frac{\delta u(0)}{1-\delta},$$

which implies $P \leq \Delta(k) < 0$. This cannot be profitable because in this case the platform attracts no agents in any subsequent period (since $k < T < N$).

Thus, the platform optimally sets $P = P^T$ in period T and attracts all agents. This is profitable iff

$$TP^T + \delta \Pi^{T+1}(T) > 0.$$

From the induction hypothesis, we have $\Pi^{T+1}(T) = \pi(T+1)$, so

$$\begin{aligned} TP^T + \delta \Pi^{T+1}(T) &= T(\Delta(T) - \delta(u(0) - u(T))) \\ &+ \sum_{k=0}^{N-2-T} \delta^{k+1}(k+T+1)(\Delta(k+T+1) - \delta(u(0) - u(k+T+1))) \\ &+ \delta^{N-T} \frac{N(\Delta(N) - \delta(u(0) - u(N-1)))}{1-\delta} \end{aligned}$$

$$\begin{aligned} &= \sum_{k=0}^{N-1-T} \delta^k (k+T)(\Delta(k+T) - \delta(u(0) - u(k+T))) + \delta^{N-T} \frac{N(\Delta(N) - \delta(u(0) - u(N-1)))}{1-\delta} \\ &= \pi(T). \end{aligned}$$

Thus, attracting all T agents in period T with $P = P^T$ is profitable iff $\pi(T) > 0$, otherwise the platform attracts no agents from period T onwards and makes zero profits. Note also that $P^T < 0$, so $\pi(T) > 0$ implies $\pi(T+1) > 0$. And if $\pi(T) > 0$, then

$$V^T(T-1) = u(T-1) + \frac{\delta u(0)}{1-\delta}.$$

Otherwise, $\Pi^T(T-1) = 0$ and $V^T(T-1) = \frac{u(0)}{1-\delta}$.

By induction, this is true for all $T \in \{1, \ldots, N-1\}$. Thus, if $\pi(1) \leq 0$, then the platform cannot profitably attract any agents in any period, so it makes zero profits and each agent's PDV of payoffs is $\frac{u(0)}{1-\delta}$. If on the other hand $\pi(1) > 0$, then along the equilibrium path the platform charges $P^t = \Delta(t) - \delta(u(0) - u(t))$ for $1 \leq t \leq N-1$, $P^t = \Delta(N) - \delta(u(0) - u(N-1))$ for $t \geq N$. All agents join the platform and remain on it along the equilibrium path. In this case the platform's PDV of profits is

$$\Pi = \pi(1) = \sum_{k=1}^{N-1} \delta^{k-1} k \left(\Delta(k) - \delta(u(0) - u(k))\right) + \delta^{N-1} \frac{N\left(\Delta(N) - \delta(u(0) - u(N-1))\right)}{1-\delta} > 0$$

and the PDV of payoffs for agent n along the equilibrium path is

$$W(n) = \sum_{k=1}^{n-1} \delta^{k-1} u(k) + \delta^{n-1} V^n(n-1) = \sum_{k=1}^{n-1} \delta^{k-1} u(k) + \delta^{n-1} u(n-1) + \frac{\delta^n u(0)}{1-\delta}.$$

Since $u(k)$ is decreasing (strictly decreasing for at least one k within $0 \leq k \leq N-1$), $W(n)$ is weakly lower than each agent's payoff in the absence of the platform, strictly lower at least for agent N. Moreover, $W(n)$ is weakly decreasing in n, so agents that are selected earlier obtain weakly higher PDV payoffs.

Note that as $\delta \to 1$, the condition for the platform to be able to profitably attract all agents ($\pi(1) > 0$) becomes simply $b(N) > u(0)$. This assumes, as we assumed throughout the paper, that the platform does not face any financial constraints, so it could set a negative price for any finite number of periods provided the PDV of prices is positive. If instead there were a financial constraint on the platform that it must turn positive profits within a certain number of periods, this would clearly reduce the parameter range in which the platform can attract all agents and thereby engineer a platform trap.

B Commitment to prices

Let's consider the general setting of Section 5 with $b(.)$ increasing, of which the marketplace baseline setting is a special case. We first show that committing to a fixed sequence of prices independent of agents' participation decisions is often worse for the platform than retaining the flexibility to adjust its offers. We then show full commitment to participation-contingent pricing — if possible — always maximizes the platform's profit.

Suppose the platform commits upfront to the N prices that will be charged to the N agents, who still arrive sequentially.

Proposition 9. *If $b(\cdot)$ is weakly increasing, then the platform's maximum profit with commitment is*

$$\max\left\{ N b(N) - \sum_{k=0}^{N-1} u(k), 0 \right\}.$$

If this is positive, then a profit-maximizing sequence of prices is $P^k = b(N) - u(k-1)$ for $k = 1, \ldots, N$ and all agents participate. Otherwise the platform attracts no agents.

Proof: For every $m \in \{1, \ldots, N\}$, define

$$R(m) \equiv mb(m) - [u(0) + u(1) + ... + u(m-1)] \, .$$

We first establish two claims.

Claim 1: Continuation participation is monotone

Fix any committed sequence of prices $\{P^1, ..., P^N\}$. Consider the continuation game beginning with agent $k \in \{1, ..., N\}$, and let $T(k,s)$ denote the total number of agents who ultimately participate, including the s agents who have already joined. Then we must have

$$T(k, s+1) \geq T(k,s) + 1.$$

Thus, one additional prior participant cannot reduce the number of subsequent participants.

The result follows by backward induction. It holds trivially after the last agent has decided. Suppose it holds for the continuation game beginning with agent $k+1$. Agent k joins when s agents have already joined if and only if

$$b(T(k+1, s+1)) - P^k \geq u(T(k+1, s)).$$

Meanwhile, when $s+1$ agents have already joined, agent k joins if and only if

$$b(T(k+1, s+2)) - P^k \geq u(T(k+1, s+1)).$$

By the induction hypothesis

$$\begin{aligned} T(k+1, s+2) &\geq T(k+1, s+1) + 1 \\ T(k+1, s+1) &\geq T(k+1, s) + 1. \end{aligned}$$

And since $b(.)$ is weakly increasing and $u(.)$ is weakly decreasing, we have

$$\begin{aligned} b(T(k+1, s+2)) - P^k &\geq b(T(k+1, s+1)) - P^k \\ u(T(k+1, s+1)) &\leq u(T(k+1, s)). \end{aligned}$$

Therefore, if agent k joins the platform with s prior participants, (s)he also accepts with $s+1$ prior participants. The desired monotonicity of $T(k,s)$ follows.

In particular, an agent who rejects on the equilibrium path continues to reject following the rejection of any earlier participating agent, because such a rejection weakly reduces subsequent participation.

Claim 2: Any committed price sequence that attracts exactly m agents generates profit no greater than R(m)

Consider a committed price sequence under which exactly m agents participate. By Claim 1, agents who reject on the equilibrium path also reject following any unilateral rejection by one of the participating agents. Consequently, these non-participating agents do not affect the continuation outcomes relevant to the participating agents' participation constraints. We can therefore remove the non-participating agents and consider the reduced game consisting of the m participating agents, in their original order and facing their original prices. All m agents join in this reduced game.

We next show that, for every pair of weakly increasing and weakly decreasing payoff functions $b(.)$ and $u(.)$, inducing all m agents to join requires leaving them aggregate net payoff of at least

$$u(0) + u(1) + ... + u(m-1).$$

The result follows by induction on m. It is immediate for $m = 1$, since the sole agent must receive at least $u(0)$.

Now suppose the result holds for $m-1$ agents. Given the committed sequence of prices $\{P^1, ..., P^m\}$ (we have re-indexed the prices to focus on the m participating agents), since $b(.)$ is weakly increasing and $u(.)$ is weakly decreasing, the last agent follows a threshold strategy. That is, there exists some $t \in \{0, \ldots, m-1\}$ such that the last agent joins if at least t previous agents have joined and rejects if fewer than t have joined. And

$$b(m) - P^m \geq b(t+1) - P^m \geq u(t).$$

From the perspective of the first $m-1$ agents, define the effective platform payoff and outside-option functions $\widetilde{b}(.)$ and $\widetilde{u}(.)$ by

$$\begin{aligned}
\widetilde{b}(k) &= \begin{cases} b(k) & \text{if } k < t \\ b(k+1) & \text{if } k \geq t \end{cases} \quad \text{for } k = 1, ..., m-1 \\
\widetilde{u}(k) &= \begin{cases} u(k) & \text{if } k < t \\ u(k+1) & \text{if } k \geq t \end{cases} \quad \text{for } k = 0, ..., m-2.
\end{aligned}$$

Indeed, if $k < t$ of the first $m-1$ agents participate, the last agent rejects, whereas if $k \geq t$, the last agent joins. Both $\widetilde{b}(\cdot)$ and $\widetilde{u}(\cdot)$ satisfy the required monotonicity conditions, and $\widetilde{b}(m-1) = b(m)$. The sequence $\widetilde{u}(0), \ldots, \widetilde{u}(m-2)$ is the original sequence $u(0), u(1), \ldots, u(m-1)$ with $u(t)$ removed.

Applying the induction hypothesis to the first $m-1$ agents with the payoff functions $\widetilde{b}(.)$ and

$\widetilde{u}(.)$ gives

$$(m-1)\,b\,(m)-\left(P^1+...+P^{m-1}\right)\geq u(0)+...+u(t-1)+u(t+1)+...+u(m-1).$$

Adding $b\,(m)-P^m\geq u\,(t)$ yields

$$mb\,(m)-\left(P^1+...+P^m\right)\geq u(0)+...+u(m-1).$$

The platform's profit when m agents participate is therefore bounded above by

$$P^1+...+P^m\leq mb\,(m)-[u(0)+...+u(m-1)]=R\,(m)\,.$$

This proves Claim 2.

We now show that the platform cannot optimally attract a strict subset of agents while earning positive profit. Suppose a price sequence attracting exactly $m<N$ agents yields positive profit. Claim 2 implies that $R(m)>0$. Therefore

$$b(m)>\frac{u(0)+u(1)+...+u(m-1)}{m}\geq u(m-1).$$

It follows that

$$\Delta(m)=b(m)-u(m-1)>0.$$

Furthermore,

$$R(m+1)-R(m)=(m+1)b(m+1)-mb(m)-u(m)=m\,(b(m+1)-b(m))+\Delta(m+1).$$

Because $b\,(.)$ and $\Delta\,(.)$ are weakly increasing and $\Delta(m)>0$, we have $R(m+1)-R(m)>0$. Repeated application of this argument gives

$$R(N)>R(m)>0.$$

The platform can attain $R(N)$ by committing to the sequence of prices $P^k=b(N)-u(k-1)$ for every $k\in\{1,\ldots,N\}$. Indeed, backward induction shows that these prices induce all agents to participate. Agent N, after the first $N-1$ agents have joined, obtains $b(N)-P^N=u(N-1)$ from accepting and $u(N-1)$ from rejecting, so it accepts. Now suppose that agents $k+1,\ldots,N$ accept whenever the first k agents have accepted. If the first $k-1$ agents have accepted, agent k's acceptance therefore leads all N agents to participate and gives agent k a payoff of $b(N)-P^k=u\,(k-1)$. Its payoff from rejecting is at most $u(k-1)$ because $k-1$ agents have already accepted and $u(\cdot)$ is weakly decreasing. Agent k therefore accepts. Backward induction implies that all N agents participate.

The platform's profit with this sequence of prices is

$$P^1 + P^2 + ... + P^N = Nb(N) - [u(0) + u(1) + ... + u(N-1)] = R(N).$$

Because $R(N) > R(m)$, a price sequence attracting only $m < N$ agents cannot be optimal. Thus, any strictly profitable optimum attracts all N agents.

Finally, if $R(N) > 0$, the displayed price sequence profitably attracts all agents and, by Claim 2, maximizes platform profit. If instead $R(N) \leq 0$, no price sequence can generate strictly positive profit. Otherwise, some price sequence would attract m agents with $R(m) > 0$, which, by the preceding argument, would imply $R(N) > R(m) > 0$, a contradiction. The platform therefore optimally attracts no agents. ■

Proposition 9 implies that commitment yields the same profit as the case without commitment when all agents are pivotal. Platform profit is therefore weakly lower than without commitment, and is strictly lower whenever some agent k who is non-pivotal without commitment satisfies $u(k-1) > u(N-1)$.

Now, suppose instead, the platform can commit to an optimal contingent price schedule. Then the platform can always achieve maximal payoffs even without agent-specific pricing. As shown in the proposition below, it can do so by using second-degree price discrimination; i.e., by committing to a contingent price function $P(j)$, where all participating agents pay the same price based on the total number j of agents that join.[3] However, this approach may require the platform to commit to selling at a loss off the equilibrium path; i.e., if only $n < N$ agents join and $\Delta(n) < 0$. Moreover, even if $\Delta(1) > 0$, implementing contingent pricing mechanisms is likely to be difficult in practice.

Proposition 10. *If the platform can commit to a single contingent pricing function offered to all agents, it maximizes profits by offering each agent the same contingent price $P(j) = \Delta(j)$ where $j \in \{1, \ldots, N\}$ is the number of agents that end up joining. This induces all N agents to join and yields a profit of $N\Delta(N)$. All agents would be strictly better off without the platform.*

Proof: Suppose the platform offers the pricing function $P(j) = \Delta(j)$ for all $j = 1, \ldots, N$. The last agent will pay $\Delta(N'+1)$ if they join and N' prior agents have joined, so they join (they are indifferent between joining and not). Knowing the last agent will join no matter what, the second-to-last agent will pay $\Delta(N'+2)$ if they join and N' prior agents have joined, which yields the

[3]Similarly, the platform could replicate the maximum platform trap from Proposition 10 if it can commit to run an auction in each stage for all unsigned agents, inducing them to compete for joining earlier. We showed this formally in an earlier version of the paper.

same payoff $u(N'+1)$ as not joining. So the second-to-last agent joins too. Thus, by backwards induction, all agents join and along the equilibrium path they are all charged $\Delta(N)$. This is the maximum price that the platform can charge any agent while getting all agents to join. ■

C Ad valorem fees

We modify the model so there is positive pass-through from the fee a platform charges a seller to the seller's price. This provides an additional mechanism by which buyers could ultimately end up being worse off because of the platform. To do so we assume the platform charges ad valorem fees (a percentage of a seller's revenue) instead of a fixed fee per seller. When deciding whether to search on the platform or in the direct channel, we continue to assume buyers observe how many sellers have listed on the platform, but they do not observe the fees the sellers are charged or the seller prices.

Let the ad valorem fee charged to a seller be denoted τ, where $0 \leq \tau \leq 1$. Facing such a fee, a seller's per-buyer profit from sales on the platform is $\max_p \{((1-\tau)p - c)\, q_P(p)\}$. Let $p(\tau) = \arg\max_p \{((1-\tau)p - c)\, q_P(p)\}$, which is increasing in τ. Then $\pi_P(\tau) = ((1-\tau)p(\tau) - c)\, q_P(p(\tau))$ is a seller's profit per buyer, which is decreasing in τ, and $b(n,\tau) \equiv \frac{1}{N} m(n)\, \pi_P(\tau)$ is a seller's expected profit from sales on the platform when n sellers join the platform. Meanwhile, a seller's expected sales from the direct channel is $u(n) = \frac{1}{N}(1 - m(n))\, \pi_D$, where π_D is a seller's profit per buyer in the direct channel.

Given a buyer's indirect utility function $v_P(p)$, a buyer's indirect utility from purchasing from her matched seller on the platform is $v_P(\tau) = v_P(p(\tau))$, which is also decreasing in τ. For prices where either quantity is positive, we assume $q_P(p) > q_D(p)$, so the platform is more efficient at making transactions, which implies $\pi_P(0) > \pi_D$, and consistent with this, we also assume $v_P(0) > v_D$. Let the buyers' draws of costs to go to each channel be such that $s_P = 0$ and $s = s_D$ is distributed according to $G(\cdot)$ with full support over $[0, \overline{s}]$, so buyers face costs of going to the direct channel but not to going to the platform. We assume $v_D > \overline{s}$.

Suppose buyers expect a single equilibrium fee τ^*. If n sellers participate on the platform, a buyer will go to the platform if their expected utility $\frac{n}{N} v_P(\tau^*)$ is greater than the expected utility of going to the direct channel, which is $\frac{N-n}{N} v_D - s$. Note since a buyer's expected utility from going to the platform is non-negative, they will never choose the outside option. Thus, the measure of buyers going to the platform is

$$m(n) = 1 - G\left(\frac{N-n}{N} v_D - \frac{n}{N} v_P(\tau^*)\right), \tag{11}$$

with $1 - m(n)$ going to the direct channel, where this is bounded between zero and one. It follows from (11) that $m(n)$ is weakly increasing in n, with $m(0) = 0$ and $m(N) = 1$. We assume τ^* is such that

$$\frac{1}{N}v_D - \frac{N-1}{N}v_P(\tau^*) < \overline{s}$$

so that $m(N-1) > 0$ and therefore $u(N-1) < u(0)$ holds.

To simplify the analysis, suppose

$$m(k)\pi_P(0) > \pi_D \tag{12}$$

for some $k \leq N-1$. This ensures no seller is pivotal: the platform can profitably attract k sellers even if no seller has previously joined and, once these sellers join, can profitably induce every remaining seller to join. Following the rejection of any seller, all other $N-1$ sellers participate.

Thus, when deciding whether to join, each seller expects all other sellers to join regardless of its own decision. A seller therefore accepts a fee τ if and only if

$$\pi_P(\tau) \geq (1 - m(N-1))\,\pi_D.$$

Because buyers do not observe the fee charged to an individual seller, $m(N-1)$ remains determined by their equilibrium expectation τ^* when the platform considers alternative fees.

The platform's commission revenue per platform buyer is $\tau p(\tau)q_P(p(\tau))$. Consequently, the equilibrium fee satisfies

$$\tau^* = \arg\max_{0\leq\tau\leq1}\{\tau p(\tau)q_P(p(\tau))\} \quad \text{subject to} \quad \pi_P(\tau) \geq (1 - m(N-1))\,\pi_D. \tag{13}$$

We focus on parameter values for which the solution is unique and positive. Since no seller is pivotal, the platform charges this fee to every seller on the equilibrium path and all N sellers join. The participation constraint in (13) may either bind or be slack.

In the equilibrium with the platform, sellers end up with expected payoffs

$$\frac{1}{N}\pi_P(\tau^*),$$

compared with $\frac{1}{N}\pi_D$ in the absence of the platform. Consequently, all sellers are strictly worse off because of the platform's existence whenever

$$\pi_P(\tau^*) < \pi_D.$$

Now consider buyers' expected utility. If the platform didn't exist, they would get $v_D - s$, with s drawn from distribution G on $[0, \overline{s}]$. With the platform they get $v_P(\tau^*)$. Thus, some buyers are strictly worse off with the platform provided $v_P(\tau^*) < v_D$, while all buyers are strictly worse off provided $v_P(\tau^*) < v_D - \overline{s}$. E.g., this last inequality together with the seller-payoff inequality both hold (so that all buyers and sellers are strictly worse off due to the existence of the platform) if we assume G is uniform, each buyer chooses q on channel $j \in \{P, D\}$ to maximize the quasi-linear net utility form $\alpha_j q - \frac{1}{2}q^2 - pq$ (so that $q_j(p) = \alpha_j - p$), and $N = 10$, $c = 4$, $\alpha_D = 7$, $\alpha_P = 8$ and $\overline{s} = \frac{2}{5}$. For these parameter values, the unconstrained maximizer of the platform's commission revenue is $\tau^* = 0.3106$. Moreover, $m(6) = 0.7807$ and $\pi_P(0) = 4$, so

$$m(6)\pi_P(0) = 3.1229 > 2.25 = \pi_D.$$

Thus, condition (12) holds with $k = 6$, and no seller is pivotal. Since $m(9) = 1$, a seller obtains zero if it rejects and a strictly positive payoff if it joins at τ^*, so its participation constraint is slack.

D Competing sellers

Our model with independent sellers can be extended to capture the possibility of some competition between sellers. This allows the possibility that $b(n)$ may be decreasing in n. To do so, we modify the previous setting so rather than having elastic demand for a matching seller's product, buyers only want to buy one unit of a product, which they value at v, i.e., they get this same value from any of the N sellers. Buyers are initially informed of a single seller, randomly drawn from the N sellers. When a buyer goes to channel $j \in \{P, D\}$, if there are $k \geq 1$ sellers there, with probability $f_j(k)$ a buyer becomes informed of all other sellers on the channel regardless of whether the seller the buyer is initially informed of is among these sellers,[4] and buys from the seller on the channel with the lowest price among the ones that the buyer is informed of (randomizing if there is more than one such seller). We assume f_j is an increasing function of k that is bounded between zero and one over $1 \leq k \leq N$. With probability $1 - f_j(k)$ the buyer does not discover the other sellers on the channel, and therefore can only buy from the seller it is initially informed of if that seller is on the channel. This is a simple way to introduce increased price competition as the number of sellers participating on a channel increases. We retain all other assumptions as in the baseline model: buyers do not know which sellers are on which channel (or their prices) until they go to a channel, but they do know how many sellers are on each channel; buyers have an outside option (in case they do not buy from any seller) normalized to zero; and each seller has marginal cost c.

[4] This means if the seller they are initially informed of is not on the channel, the buyer becomes informed of k other sellers; while if it is on the channel, they become informed of $k - 1$ other sellers.

If $k=1$ so there is only one seller on a channel, the seller never faces any competition and so it always prices at v, meaning buyers will get zero surplus on the channel. We assume $f_j(1)=0$ on both channels, so if there is only one seller on a channel, the probability buyers are informed of the seller if it is not the seller they are initially informed about is normalized to zero. Thus, such a seller will obtain a profit of $\frac{1}{N}(v-c)$ per buyer on that channel, where here $\frac{1}{N}$ represents the probability that any given buyer is initially informed of the seller.

If there are $k \geq 2$ sellers at a channel, then sellers will price according to a standard mixed strategy equilibrium, with a seller's profit per buyer being $(1-f_j(k))\frac{1}{N}(v-c)$, which is a seller's certain profit per buyer if it prices at v and only sells to a buyer who doesn't discover the other sellers (here we multiply by $\frac{1}{N}$, the probability the buyer is initially informed about it). This implies if n sellers join the platform, a seller's expected profit on the platform is $b(n) = (1-f_P(n))\frac{1}{N}(v-c)\,m(n)$ and on the direct channel is $u(n) = (1-f_D(N-n))\frac{1}{N}(v-c)(1-m(n))$, where $m(n)$ is the measure of buyers choosing the platform channel.

To work out $m(n)$, we need to work out a buyer's expected net utility on each channel (initially, ignoring the costs of going to the channel) when n sellers participate on the platform. Obviously, if a buyer goes to a channel where there are no sellers, then it will get zero expected net utility. The same is true if there is just one seller on the channel, since it prices at v. If, instead, there are $k \geq 2$ sellers on channel j, a buyer's expected net utility on channel j can be obtained by subtracting expected seller industry profit per buyer from expected total welfare per buyer. To calculate the latter, note that with probability $\left(1-\frac{k}{N}\right)(1-f_j(k))$ a consumer that comes to channel j ends up not transacting because the seller they know of initially is on the other channel and the consumer does not discover other sellers on channel j. Thus, expected total welfare per buyer in channel j is

$$\left(1-\left(1-\frac{k}{N}\right)(1-f_j(k))\right)(v-c),$$

and therefore expected net consumer surplus on channel j is

$$\begin{aligned} & \left(1-\left(1-\frac{k}{N}\right)(1-f_j(k))\right)(v-c) - k(1-f_j(k))\frac{1}{N}(v-c) \\ = \; & f_j(k)(v-c). \end{aligned}$$

Thus, if n sellers participate on the platform with $2 \leq n \leq N-2$, a buyer will go to the platform if their expected utility $f_P(n)(v-c)-s_P$ is greater than the expected utility of going to the direct channel, which is $f_D(N-n)(v-c)-s_D$.[5] As a result, the measure of buyers going

[5]We continue to assume the draws s_D and s_P are such that buyers always prefer to go to one of the channels over the outside option. We will note this holds for the distribution of the draws used below.

to the platform is

$$m(n) = 1 - G\left(\left(f_D(N-n) - f_P(n)\right)(v-c)\right),$$

for $2 \leq n \leq N-2$, which is clearly increasing in n.

We first need to specify $b(n)$ and $u(n)$ for the extreme cases where there is at most one seller on one of the channels. In such cases, we have

$$\begin{aligned} m(0) &= 1 - G\left(f_D(N)(v-c)\right) \\ m(1) &= 1 - G\left(f_D(N-1)(v-c)\right) \\ m(N-1) &= 1 - G\left(-f_P(N-1)(v-c)\right) \\ m(N) &= 1 - G\left(-f_P(N)(v-c)\right) \end{aligned}$$

Note $b(0)$ and $u(N)$ are undefined since a seller can't be on the platform when $n = 0$, and can't be in the direct channel if $n = N$. Then we have $b(1) = \frac{1}{N}(v-c)\,m(1)$ and $u(N-1) = \frac{1}{N}(v-c)(1 - m(N-1))$.

Given the functions G, f_D and f_P, we can explore the properties of $b(n)$, $u(n)$, and $\Delta(n)$. To illustrate what is possible, we consider the simple case where all three functions are linear. Specifically, we assume s_D is uniformly distributed on $\left[-\frac{\overline{s}}{2}, \frac{\overline{s}}{2}\right]$, and $s_P = -s_D$. Note this assumption is consistent with buyers having a standard Hotelling-type channel preference. This implies $s = s_D - s_P = 2s_D$ is uniformly distributed over $[-\overline{s}, \overline{s}]$, i.e., $G(s) = \max\left\{\min\left\{\frac{s+\overline{s}}{2\overline{s}}, 1\right\}, 0\right\}$. And we assume $f_j(k) = \theta_j \frac{k-1}{N}$, with $0 < \theta_D \leq \theta_P < 1$ to capture the idea that the platform enables greater discoverability of the sellers than the direct channel. With this formulation, buyers always prefer to go to one channel because ignoring the s_D and s_P terms, they get non-negative surplus from going to either channel, and given $s_P = -s_D$, one of s_D and s_P is always non-positive. Note $f_j(1) = 0$ as assumed above, with f_j strictly increasing in k.

We can illustrate that this model can lead to all the properties of the baseline model holding except that $b(n)$ is no longer always weakly increasing. We consider two numerical examples. First suppose:

$$N = 10, \qquad \theta_D = 0.1, \qquad \theta_P = 0.1, \qquad \overline{s} = 1, \qquad v - c = 2,$$

so the competition effect on the two channels is the same. Then, as Table 1 below shows, $m(n)$ is strictly increasing, $b(n)$ is strictly increasing, $u(n)$ is strictly decreasing and $\Delta(n)$ is strictly increasing, with $b(10) > \frac{1}{10}\sum_{k=1}^{10} u(k-1)$ so all the properties required for Proposition 1 to hold apply, and there is a platform trap.[6]

Now consider the following numerical example, where the competition effect on the platform

[6] In the tables below, $u(0) = 0.10738$ represents a seller's payoff when the platform is available but none join. Each seller's payoff in the absence of the platform is even higher, at 0.12376.

Table 1: Numerical example with $N = 10$, $\theta_D = 0.1$, $\theta_P = 0.1$, $\overline{s} = 1$, and $v - c = 2$

n	$m(n)$	$b(n)$	$u(n)$	$\Delta(n)$
0	0.41000	—	0.10738	—
1	0.42000	0.08400	0.10672	-0.02338
2	0.44000	0.08712	0.10416	-0.01960
3	0.46000	0.09016	0.10152	-0.01400
4	0.48000	0.09312	0.09880	-0.00840
5	0.50000	0.09600	0.09600	-0.00280
6	0.52000	0.09880	0.09312	0.00280
7	0.54000	0.10152	0.09016	0.00840
8	0.56000	0.10416	0.08712	0.01400
9	0.58000	0.10672	0.08400	0.01960
10	0.59000	0.10738	—	0.02338

is much stronger:

$$N = 10, \qquad \theta_D = 0.1, \qquad \theta_P = 0.5, \qquad \overline{s} = 1, \qquad v - c = 2.$$

Under these parameter values, the implied values of $m(n)$, $b(n)$, $u(n)$, and $\Delta(n)$ are given in Table 2.

Table 2: Numerical example with $N = 10$, $\theta_D = 0.1$, $\theta_P = 0.5$, $\overline{s} = 1$, and $v - c = 2$

n	$m(n)$	$b(n)$	$u(n)$	$\Delta(n)$
0	0.4100	—	0.10738	—
1	0.4200	0.0840	0.10672	-0.02338
2	0.4800	0.0912	0.09672	-0.01552
3	0.5400	0.0972	0.08648	0.00048
4	0.6000	0.1020	0.07600	0.01552
5	0.6600	0.1056	0.06528	0.02960
6	0.7200	0.1080	0.05432	0.04272
7	0.7800	0.1092	0.04312	0.05488
8	0.8400	0.1092	0.03168	0.06608
9	0.9000	0.1080	0.02000	0.07632
10	0.9500	0.1045	—	0.08450

In this second example, $b(n)$ is single-peaked, increasing up to $n = 7$, and decreasing for $n > 8$, with its peak at $b(7) = b(8)$. However, despite this, $b(N) > \frac{1}{10}\sum_{k=1}^{10} u(k-1)$ still holds, so that Proposition 1 again applies, meaning a platform trap again arises in this case.

E Full equilibrium characterization for special cases

In this section, we fully characterize the equilibrium of the game for several special cases:

- $N = 2$
- $N = 3$
- Any $N \geq 3$, $b(n) = b_0 + nb$ and $u(n) = u_0 - nu$, with $b_0 \geq 0$, $b > \frac{N-3}{2}u$, $u > 0$ and $u_0 > Nu$.
- $N = 4$, $b(n) = b_0 + nb$ and $u(n) = u_0 - nu$, with $b_0 \geq 0$ and $u_0 > Nu$.

Suppose first $N = 2$. If $\Delta(1) \leq 0$, then the first agent is pivotal, so the highest prices the platform can charge to attract both agents are $P^1 = b(2) - u(0)$ to the first agent and $P^2 = b(2) - u(1)$ to the second agent. This is profitable iff $2b(2) - u(0) - u(1) > 0$. If $\Delta(1) > 0$, then no agent is pivotal, so the platform optimally charges $P^1 = P^2 = \Delta(2) > 0$ to both agents and makes positive profits. To sum up, platform profits are

$$\begin{cases} 0 & \text{if} \quad \max\left\{b(2) - \frac{u(0)+u(1)}{2}, b(1) - u(0)\right\} \leq 0 \\ 2b(2) - u(1) - u(0) & \text{if} \quad b(1) - u(0) \leq 0 < b(2) - \frac{u(0)+u(1)}{2} \\ 2(b(2) - u(1)) & \text{if} \quad b(1) - u(0) > 0 \end{cases}.$$

Now suppose $N = 3$. If the first agent doesn't join, from above, the platform attracts the two remaining agents if

$$b(2) - \frac{u(0) + u(1)}{2} > 0$$

or

$$b(1) - u(0) > 0.$$

If the first agent does join, the platform attracts the two remaining agents if

$$b(3) - \frac{u(1) + u(2)}{2} > 0$$

or

$$b(2) - u(1) > 0.$$

It is easily seen that

$$b(2) - u(1) \geq \max\left\{b(1) - u(0), b(2) - \frac{u(0) + u(1)}{2}\right\}$$

because $\Delta(2) \geq \Delta(1)$ and $u(\cdot)$ is weakly decreasing. Thus, if the platform attracts the last two agents when the first agent does not join, it necessarily attracts the last two agents after the first agent joins. There are therefore three relevant cases:

1. $\max\left\{b(1) - u(0), b(2) - \frac{u(0)+u(1)}{2}\right\} > 0$, in which case the platform attracts the last two agents regardless of whether the first agent has joined or not
2. $\max\left\{b(1) - u(0), b(2) - \frac{u(0)+u(1)}{2}\right\} \leq 0 < \max\left\{b(2) - u(1), b(3) - \frac{u(1)+u(2)}{2}\right\}$, in which case the platform attracts the last two agents iff the first agent has joined
3. $\max\left\{b(2) - u(1), b(3) - \frac{u(1)+u(2)}{2}\right\} \leq 0$, in which case the platform cannot profitably attract any agent.

In the first case, the platform optimally charges $P^1 = P^2 = P^3 = \Delta(3) > 0$ and makes profits $3\Delta(3) > 0$.

In the second case, the first agent is pivotal so the platform optimally charges $P^1 = b(3) - u(0)$ to attract the first agent. Its maximum profits from the last two agents are then (applying the case $N = 2$ and taking into account the first agent has joined)

$$\begin{cases} 2b(3) - u(2) - u(1) & \text{if} \quad b(2) - u(1) \leq 0 < b(3) - \frac{u(1)+u(2)}{2} \\ 2(b(3) - u(2)) & \text{if} \quad b(2) - u(1) > 0 \end{cases},$$

so total profits are

$$\begin{cases} 3b(3) - u(2) - u(1) - u(0) & \text{if} \quad b(2) - u(1) \leq 0 < b(3) - \frac{u(1)+u(2)}{2} \\ 3b(3) - 2u(2) - u(0) & \text{if} \quad b(2) - u(1) > 0 \end{cases}.$$

And in the third case, the platform makes zero profits.

Combining all cases above, we have four final cases:

- if $\max\left\{b(1) - u(0), b(2) - \frac{u(0)+u(1)}{2}\right\} > 0$, the platform attracts all three agents, none are pivotal and the platform's profits are $3\Delta(3)$
- if $\max\left\{b(1) - u(0), b(2) - \frac{u(0)+u(1)}{2}\right\} \leq 0 < b(2) - u(1)$ and $b(3) > \frac{2u(2)+u(0)}{3}$, the platform attracts all three agents, only the first agent is pivotal, and the platform's profits are $3b(3) - 2u(2) - u(0) > 0$
- if $b(2) - u(1) \leq 0 < b(3) - \frac{u(2)+u(1)+u(0)}{3}$, the platform attracts all three agents, the first two agents are pivotal, and the platform's profits are $3b(3) - u(2) - u(1) - u(0) > 0$.
- otherwise the platform attracts no agents.

Next, we show that for general $N \geq 2$, the equilibrium (including the number of pivotal agents) can be fully characterized for linear increasing $b(\cdot)$ and linear decreasing $u(\cdot)$, provided we add additional restrictions on their respective slopes in relation to N.

Proposition 11. *Suppose $b(n) = b_0 + bn$ and $u(n) = u_0 - un$, where $b_0 \geq 0$, $b > \max\left\{\frac{(N-3)u}{2}, 0\right\}$ and $u_0 > Nu$, with $N \geq 2$. Then in the game with N agents the platform attracts all N agents and creates a platform trap iff*

$$u_0 < b_0 + Nb + \frac{N-1}{2}u$$

and the number of pivotal agents is

$$k_0(N) = \begin{cases} 0 & \text{if} \quad u_0 < b_0 + (N-1)\,b + \frac{N-2}{2}u \\ k \in \{1, \ldots, N-2\} & \text{if} \quad \begin{array}{c} b_0 + (N-1)\,b + \frac{N+k-3}{2}u \leq u_0 \\ < b_0 + (N-1)\,b + \frac{N+k-2}{2}u \end{array} \\ N-1 & \text{if} \quad \begin{array}{c} b_0 + (N-1)\,b + (N-2)\,u \leq u_0 \\ < b_0 + Nb + \frac{N-1}{2}u \end{array} \end{cases}$$

Proof: The proof is by induction over N. For $N = 2$, we know from the results above:

- if $u_0 < b_0 + b$, then $k_0 = 0$ and the platform attracts both agents
- if $b_0 + b \leq u_0 < b_0 + 2b + \frac{u}{2}$, then $k_0 = 1$ and the platform attracts both agents.
- if $u_0 \geq b_0 + 2b + \frac{u}{2}$, the platform cannot profitably attract any agents.

Thus, the result holds for $N = 2$.

Now suppose the result holds with $N - 1 \geq 2$ agents and any payoff functions satisfying the assumptions in the text of the proposition. We want to show it also holds with N agents.

First, the induction hypothesis implies that if

$$u_0 < b_0 + (N-1)\,b + \frac{N-2}{2}u,$$

then the platform can attract the last $N - 1$ agents even if the first agent does not join. This implies that on this range, no agent is pivotal, so $k_0(N) = 0$ and the platform can attract all agents with a price $P = b_0 + Nb + (N-1)\,u - u_0 > 0$. This proves the result for N on this parameter range.

Now suppose

$$u_0 \geq b_0 + (N-1)\,b + \frac{N-2}{2}u,$$

so the first agent is pivotal. The game that starts with the second agent after the first agent has joined is a game with $N - 1$ agents and payoff functions $\widetilde{b}(n) = \widetilde{b}_0 + bn$ and $\widetilde{u}(n) = \widetilde{u}_0 - un$, where $\widetilde{b}_0 = b_0 + b > b_0 \geq 0$ and $\widetilde{u}_0 = u_0 - u > (N-1)\,u$. Thus, the payoff functions $\widetilde{b}\,(n)$ and $\widetilde{u}\,(n)$ satisfy the assumptions in the text of the proposition with $N-1$ instead of N. Applying the induction hypothesis, the platform can profitably attract the last $N-1$ agents conditional on having attracted the first agent iff

$$\widetilde{u}_0 < \widetilde{b}_0 + (N-1)\,b + \frac{N-2}{2}u,$$

which can be rewritten as

$$u_0 < b_0 + Nb + \frac{N}{2}u.$$

And the number of pivotal agents in the game with $N-1$ agents that starts after the first agent has joined is

$$\widetilde{k}_0(N-1) = \begin{cases} 0 & \text{if} \quad \widetilde{u}_0 < \widetilde{b}_0 + (N-2)\,b + \frac{N-3}{2}u \\ k \in \{1, \dots, N-3\} & \text{if} \quad \begin{array}{c} \widetilde{b}_0 + (N-2)\,b + \frac{N+k-4}{2}u \leq \widetilde{u}_0 \\ < \widetilde{b}_0 + (N-2)\,b + \frac{N+k-3}{2}u \end{array} \\ N-2 & \text{if} \quad \begin{array}{c} \widetilde{b}_0 + (N-2)\,b + (N-3)\,u \leq \widetilde{u}_0 \\ < \widetilde{b}_0 + (N-1)\,b + \frac{N-2}{2}u \end{array} \end{cases}$$

Plugging in $\widetilde{b}_0 = b_0 + b$ and $\widetilde{u}_0 = u_0 - u$, this can be re-written

$$\widetilde{k}_0(N-1) = \begin{cases} 0 & \text{if} \quad u_0 < b_0 + (N-1)\,b + \frac{N-1}{2}u \\ k \in \{1, \dots, N-3\} & \text{if} \quad \begin{array}{c} b_0 + (N-1)\,b + \frac{N+k-2}{2}u \leq u_0 \\ < b_0 + (N-1)\,b + \frac{N+k-1}{2}u \end{array} \\ N-2 & \text{if} \quad \begin{array}{c} b_0 + (N-1)\,b + (N-2)\,u \leq u_0 \\ < b_0 + Nb + \frac{N}{2}u \end{array} \end{cases}.$$

Thus, the number of pivotal agents in the original game with N agents is $k_0\,(N) = 1 + \widetilde{k}_0(N-1)$, so we have

$$k_0\,(N) = \begin{cases} k \in \{1, \dots, N-2\} & \text{if} \quad \begin{array}{c} b_0 + (N-1)\,b + \frac{N+k-3}{2}u \leq u_0 \\ < b_0 + (N-1)\,b + \frac{N+k-2}{2}u \end{array} \\ N-1 & \text{if} \quad \begin{array}{c} b_0 + (N-1)\,b + (N-2)\,u \leq u_0 \\ < b_0 + Nb + \frac{N}{2}u \end{array} \end{cases}.$$

When $b_0 + (N-1)\,b + (N-2)\,u \leq u_0 < b_0 + Nb + \frac{N}{2}u$, all agents are pivotal, so the platform's profit is

$$Nb(N) - \sum_{k=1}^{N} u(k-1) = Nb_0 + N^2 b + \frac{N(N-1)}{2} u - Nu_0,$$

which is positive only if

$$u_0 < b_0 + Nb + \frac{N-1}{2} u.$$

And since $b > \frac{(N-3)}{2} u$, we have

$$b_0 + (N-1)b + (N-2)u < b_0 + Nb + \frac{N-1}{2} u < b_0 + Nb + \frac{N}{2} u,$$

and thus the platform can profitably attract N agents iff

$$u_0 < b_0 + Nb + \frac{N-1}{2} u.$$

Combined with the expression of $k_0(N)$ above, this shows the result holds for N agents. ■

In order to illustrate why the assumption $b > \frac{(N-3)}{2} u$ is crucial to obtain a full closed form characterization of the equilibrium, consider the case with linear payoff functions ($b(n) = b_0 + bn$, $u(n) = u_0 - un$, with $\min\{b, u\} > 0$) and $N = 4$. Indeed, for $N \leq 3$, the assumption $b > \frac{(N-3)}{2} u$ holds trivially as long as $b > 0$, which we assume.

We can apply the general analysis of the case $N = 3$ above to fully characterize the equilibrium when the payoff functions are linear. The platform can profitably attract all three agents iff

$$u_0 < b_0 + 3b + u.$$

The number of pivotal agents is

$$k_0(3) = \begin{cases} 0 & \text{if} \quad u_0 < b_0 + 2b + \frac{u}{2} \\ 1 & \text{if} \quad b_0 + 2b + \frac{u}{2} \leq u_0 < b_0 + 2b + u \\ 2 & \text{if} \quad b_0 + 2b + u \leq u_0 < b_0 + 3b + u \end{cases}.$$

Now suppose $N = 4$.

If $u_0 < b_0 + 3b + u$, then the platform can attract the last three agents regardless of the participation of the first agent, so $k_0(4) = 0$ and the platform profitably attracts all four agents.

Suppose $u_0 \geq b_0 + 3b + u$, so the first agent with $N = 4$ is pivotal. Once the first agent joins, we can use the characterization with $N = 2$ to conclude that the second agent is non-pivotal iff $b(3) - \frac{u(2)+u(1)}{2} > 0$, i.e., iff

$$u_0 < b_0 + 3b + \frac{3u}{2}.$$

Thus, $k_0(4) = 1$ iff

$$b_0 + 3b + u \leq u_0 < b_0 + 3b + \frac{3u}{2},$$

and in this region the platform's profits are

$$4b(4) - u(0) - 3u(3) = 4\left(b_0 + 4b + \frac{9}{4}u - u_0\right) > 0.$$

Next, suppose $u_0 \geq b_0 + 3b + \frac{3}{2}u$, so the first two agents are pivotal. Once the first two agents have joined, the third agent is non-pivotal iff $b(3) - u(2) > 0$, i.e., iff

$$u_0 < b_0 + 3b + 2u.$$

Thus, if

$$b_0 + 3b + \frac{3}{2}u \leq u_0 < b_0 + 3b + 2u,$$

then $k_0(4) = 2$ and the platform's profits are

$$4b(4) - u(0) - u(1) - 2u(3) = 4\left(b_0 + 4b + \frac{7u}{4} - u_0\right).$$

Thus, $k_0(4) = 2$ iff

$$b_0 + 3b + \frac{3u}{2} \leq u_0 < \min\left\{b_0 + 4b + \frac{7u}{4}, b_0 + 3b + 2u\right\}.$$

Finally, if $u_0 \geq b_0 + 3b + 2u$, then $k_0(4) = 3$ and platform profits are

$$4\left(b_0 + 4b + \frac{3u}{2} - u_0\right).$$

Thus, $k_0(4) = 3$ is possible only if $b > \frac{u}{2}$, in which case the region where this holds is

$$b_0 + 3b + 2u \leq u_0 < b_0 + 4b + \frac{3u}{2}.$$

Bottom line:

- if $b \leq \frac{u}{4}$, then the platform can profitably attract all four agents iff $u_0 < b_0 + 4b + \frac{7u}{4}$ and

$$k_0(4) = \begin{cases} 0 & \text{if} \quad u_0 < b_0 + 3b + u \\ 1 & \text{if} \quad b_0 + 3b + u \leq u_0 < b_0 + 3b + \frac{3u}{2} \\ 2 & \text{if} \quad b_0 + 3b + \frac{3u}{2} \leq u_0 < b_0 + 4b + \frac{7u}{4} \end{cases}$$

- if $\frac{u}{4} < b \leq \frac{u}{2}$, then the platform can profitably attract all four agents iff $u_0 < b_0 + 3b + 2u$ and

$$k_0(4) = \begin{cases} 0 & \text{if} \quad u_0 < b_0 + 3b + u \\ 1 & \text{if} \quad b_0 + 3b + u \leq u_0 < b_0 + 3b + \frac{3u}{2} \\ 2 & \text{if} \quad b_0 + 3b + \frac{3u}{2} \leq u_0 < b_0 + 3b + 2u \end{cases}$$

- if $b > \frac{u}{2}$, then the platform can profitably attract all four agents iff $u_0 < b_0 + 4b + \frac{3u}{2}$ and

$$k_0(4) = \begin{cases} 0 & \text{if} \quad u_0 < b_0 + 3b + u \\ 1 & \text{if} \quad b_0 + 3b + u \leq u_0 < b_0 + 3b + \frac{3u}{2} \\ 2 & \text{if} \quad b_0 + 3b + \frac{3u}{2} \leq u_0 < b_0 + 3b + 2u \\ 3 & \text{if} \quad b_0 + 3b + 2u \leq u_0 < b_0 + 4b + \frac{3u}{2} \end{cases}.$$

This illustrates how the number of possible cases and piecemeal segments of the platform's profit function increase very rapidly once N increases beyond 3, even assuming linear payoff functions. It also clarifies why the additional restriction $b > \frac{(N-3)u}{2}$ was imposed in Proposition 11 above: it ensured that for every $N' \leq N$, the number of pivotal agents $k_0(N')$ could take all values between 0 and $N' - 1$, so that the highest value of u_0 for which the platform could profitably attract N' agents always fell in the region where all agents are pivotal, i.e., $k_0(N') = N' - 1$.

F Limited price discrimination

Suppose the platform can change its price only once: it sets P_1 for the first N_1 agents, and then P_2 for the remaining $N_2 = N - N_1$ agents, where P_1 can be different from P_2 if the platform wants. As in the general model, the platform cannot commit to future prices, so P_2 is chosen only when offered. The platform can also choose N_1, i.e., the point at which the price changes (but does not commit to N_1 ex-ante).

With this set-up, if $b(N-1) > u(0)$, the platform can replicate the maximal platform trap with full price discrimination from Corollary 1, despite having significantly less price flexibility. The platform can attract the first agent with an offer of $P_1 = \Delta(N) > 0$. Indeed, from Proposition 4, the agent knows that rejecting this offer will lead the platform to set a uniform price $P_2 = b(N-1) - u(0)$ to all remaining agents, which ensures their participation, and therefore a payoff of $u(N-1)$ for the first agent. And if the first agent accepts, it becomes even easier to attract the others, so the first agent obtains $b(N) - P_1 = u(N-1)$. Thus, anticipating that either way the remaining $N-1$ agents will join, the first agent accepts. The same logic applies for all subsequent agents: each of them accepts $P_1 = \Delta(N)$, so ultimately all agents join, just as in Corollary 1. Thus, we have proven:

Proposition 12. *If the platform can change its price only once, but chooses when to do so, and if $b(N-1) > u(0)$ and $b(\cdot)$ is weakly increasing, there exists a unique equilibrium outcome in which all agents join at the uniform price $\Delta(N) > 0$. All agents would be strictly better off without the platform.*

Comparing this result with Proposition 4 highlights that one of the key ways a platform trap can be generated is a platform's ability to adjust its prices to attract future agents. This ability can arise from the platform having some discretion to adjust its prices (Proposition 12) given we assumed commitment was not possible.

G Private offers

Suppose agents only observe their own offers and nothing else (we call these private offers). Thus, agents do not know their position in the offer sequence or how many agents have previously accepted or rejected offers. As a result, multiple equilibria can arise depending on agents' off-equilibrium path beliefs, and the relevant equilibrium concept in this case is perfect Bayesian equilibrium (PBE).

In order to rule out PBE supported by unreasonable off-equilibrium path beliefs, we make the following refinement. We assume each agent believes the prices the platform will set to subsequent agents are unaffected by its decision to accept or reject its offer. These beliefs are known as passive beliefs in the literature on private contracting (e.g. Rey and Vergé, 2004). In our setting, such beliefs are reasonable. Even if an agent deviates from the proposed equilibrium and rejects an offer, there is no reason to expect the platform to change its subsequent offers since subsequent agents cannot observe this deviation and would therefore be expected to continue to accept the original equilibrium offers, which extract as much as possible from the agents.

Another reason for focusing on equilibrium selection using passive beliefs is that these are also the equilibrium outcomes that would be selected if we reorder the moves of the players so that all agents make their joining decisions (possibly simultaneously) after the platform has made its private offers to all agents. In that case, the platform cannot change its prices based on agents' decisions. Thus, the PBE outcomes we focus on do not depend on the exact ordering of the moves of players (provided, of course, the platform sets each agent's private offer before the agent decides).

The following proposition characterizes the best and the worst such equilibria for the platform, which implies the identical prices and outcomes to Proposition 5 in the main text.

Proposition 13. *Suppose agents observe only their own offers. There exists a continuum of perfect Bayesian equilibrium outcomes under passive beliefs. These are also the equilibrium outcomes of* *the game in which the platform makes its private offers to all agents in the first stage and agents all decide simultaneously whether to accept or not in the second stage.*

1. *In the best equilibrium for the platform, each agent k is charged $P^k = \Delta(N)$, all N agents join, and platform profits are $N\Delta(N)$. All agents would be strictly better off without the platform.*
2. *If $\Delta(1) > 0$, in the worst equilibrium for the platform, each agent k is charged $P^k = \Delta(1)$, all N agents join, and platform profits are $N\Delta(1)$. All agents would be strictly worse off without the platform if $b(N) > b(1)$, indifferent if $b(N) = b(1)$, and would be strictly better off without the platform if $b(N) < b(1)$. If $\Delta(1) \leq 0$, in the worst equilibrium for the platform, no agents join and the platform profits are zero.*

Proof: In the best possible equilibrium for the platform, the platform prices at $P = \Delta(N)$ to all agents, and each agent joins because they believe every other agent faces the same price and also joins. The platform attains its maximum feasible profits and all agents receive net payoff $u(N-1)$, so they would be strictly better off without the platform.

Suppose $\Delta(1) > 0$. The worst equilibrium for the platform is supported by the following strategies and beliefs:

- The platform charges $P^k = \Delta(1)$ to every agent k in the sequence and all agents choose to join
- Provided $P^k \leq \Delta(1)$, each agent believes every other agent is charged $P^k = \Delta(1)$ and they will join, so they also join (indeed, joining yields $b(N) - P \geq b(N) - \Delta(1) \geq u(N-1)$ because $\Delta(\cdot)$ is weakly increasing)
- If an agent is charged a price $P > \Delta(1)$, they believe every other agent is also charged the same P and does not join because they believe no other agent will join either.

Given the agents' strategy, the platform's pricing is optimal to induce participation whenever it is profitable. Conversely, each agent's strategy is optimal given their beliefs, which are fulfilled along the equilibrium path. The resulting platform profit, $N\Delta(1)$, is the minimum profit the platform can obtain given $\Delta(1) > 0$. To see this suppose there is an even worse equilibrium in which the platform sets the price $P^k < \Delta(1)$ to at least one agent. The platform can profitably increase its price P^k to $\Delta(1)$ for any such agent. Then regardless of the number n of other agents this agent expects to join after the price change, it will want to join. Indeed, joining yields payoff $b(n+1) - \Delta(1)$, not joining yields $u(n)$ (recall our PBE refinement implies n does not depend on the agent's own decision), and $b(n+1) - u(n) = \Delta(n+1) \geq \Delta(1)$. Similarly, the platform could change the price of any agent that doesn't join in the proposed equilibrium to $\Delta(1)$ and also profitably induce them to join. Thus, the platform can profitably deviate, so we can rule out any such worse equilibrium.

If $\Delta(1) \leq 0$, the platform cannot profitably attract any agents in the worst equilibrium defined above. Moreover, if it tries to charge a price above $\Delta(1)$ to any agent, then following the beliefs in the worst equilibrium above, such an agent will not join. ■

H Proofs of remaining propositions

This section contains the proofs of the three propositions in Section 7.

H.1 Negotiated prices

Here we prove the results in Section 7.1 (including Proposition 6).

We first show by backwards induction that the price charged to the k-th agent to decide is

$$P^k = (1-\alpha)\left(\sum_{j=0}^{N-k} C_j^{N-k} (1-\alpha)^j \alpha^{N-k-j} \Delta(N'+j+1)\right) \tag{14}$$

and all subsequent agents (including the k-th agent) will participate, where N' is the number of agents that have decided to participate before k.

Consider the last agent to decide whether to participate and suppose N' other agents have already decided to participate. When the last agent decides whether or not to participate, it expects to get $b(N'+1) - P^N$ if it participates and $u(N')$ if it doesn't. Meanwhile, the platform obtains an additional profit of P^N if the agent participates and no additional profit if it doesn't. Given each agent's bargaining power relative to the platform is measured by the parameter α, the Nash bargaining solution is the price P^N that maximizes

$$\left(\Delta(N'+1) - P^N\right)^{\alpha} \left(P^N\right)^{1-\alpha},$$

which implies

$$P^N(N') = (1-\alpha)\Delta(N'+1).$$

Since $\Delta(N'+1) > 0$, the platform will induce the last agent to participate because it can extract a positive price from doing so.

Now suppose the result holds in (14) for all $k \geq K$ for some $K \leq N$. Let us now show it also holds for $k = K-1$. To determine P^{K-1}, suppose N' other agents have already decided to participate. The $(K-1)$-th agent knows regardless of whether it decides to participate, the next $N-K+1$ agents will participate (by the induction hypothesis). Thus, the agent's increase in surplus from participating relative to not participating is

$$\Delta\left(N' + N - K + 2\right) - P^{K-1}.$$

If the agent participates, the platform's profit from this and the next $N-K+1$ agents participating is

$$\begin{aligned}
& P^{K-1} + \sum_{m=0}^{N-K} P^{K+m} \\
= \quad & P^{K-1} + (1-\alpha) \sum_{m=0}^{N-K} \left(\sum_{j=0}^{N-K-m} C_j^{N-K-m} (1-\alpha)^j \alpha^{N-K-m-j} \Delta\left(N' + m + j + 2\right) \right).
\end{aligned}$$

If the $(K-1)$-th agent does not participate, then the platform's profit from the subsequent agents is

$$(1-\alpha) \sum_{m=0}^{N-K} \left(\sum_{j=0}^{N-K-m} C_j^{N-K-m} (1-\alpha)^j \alpha^{N-K-m-j} \Delta\left(N' + m + 1 + j\right) \right).$$

Thus, the Nash bargaining solution is

$$\begin{aligned}
P^{K-1} \quad = \quad & (1-\alpha)\Delta\left(N' + N - K + 2\right) \\
& -\alpha(1-\alpha) \sum_{m=0}^{N-K} \left(\sum_{j=0}^{N-K-m} C_j^{N-K-m} (1-\alpha)^j \alpha^{N-K-m-j} \left(\begin{array}{c} \Delta\left(N' + m + j + 2\right) \\ -\Delta\left(N' + m + 1 + j\right) \end{array} \right) \right)
\end{aligned}$$

After some re-arrangement of the various terms, this can be re-written

$$P^{K-1} = (1-\alpha) \left(\begin{array}{c} (1-\alpha)^{N-K+1} \Delta\left(N' + N - K + 2\right) + \alpha^{N-K+1} \Delta\left(N' + 1\right) \\ + \sum_{s=1}^{N-K} \alpha^{N-K+1-s} \Delta\left(N' + s + 1\right) \left(\begin{array}{c} \sum_{m=0}^{s} C_{s-m}^{N-K-m} (1-\alpha)^{s-m} \\ - \sum_{m=0}^{s-1} C_{s-1-m}^{N-K-m} \alpha (1-\alpha)^{s-1-m} \end{array} \right) \end{array} \right)$$

We wish to show this can be rewritten as

$$P^{K-1} = (1-\alpha) \left(\sum_{s=0}^{N-K+1} C_s^{N-K+1} (1-\alpha)^s \alpha^{N-K+1-s} \Delta\left(N' + s + 1\right) \right),$$

so we need to prove

$$\begin{aligned}
& \sum_{m=0}^{s} \frac{(N-K-m)!}{(N-K-s)!\,(s-m)!} (1-\alpha)^{s-m} - \sum_{m=0}^{s-1} \frac{(N-K-m)!}{(N-K-s+1)!\,(s-1-m)!} \alpha (1-\alpha)^{s-1-m} \\
= \quad & C_s^{N-K+1} (1-\alpha)^s
\end{aligned}$$

for all $s \in \{1, \ldots, N-K\}$ and $\alpha \in [0,1]$, which can be rewritten

$$\sum_{m=0}^{s} \frac{(N-K-s+m)!}{(N-K-s)!m!} (1-\alpha)^m - \sum_{m=0}^{s-1} \frac{(N-K-s+1+m)!}{(N-K-s+1)!m!} \alpha (1-\alpha)^m = C_s^{N-K+1} (1-\alpha)^s .$$

With the change of variable $x = 1-\alpha$, this is equivalent to proving

$$\begin{aligned} & \frac{1}{(N-K-s)!} \sum_{m=0}^{s} (N-K-s+m) \times \ldots \times (1+m) \, x^m \\ & - \frac{1}{(N-K-s+1)!} \sum_{m=0}^{s-1} (N-K-s+1+m) \times \ldots \times (1+m) (1-x) \, x^m \\ = \; & \frac{(N-K+1)!}{(N-K+1-s)!s!} x^s \end{aligned}$$

for all $s \in \{1, \ldots, N-K\}$ and $x \in [0,1]$.

To do this, let

$$\begin{aligned} f_s(x) &= \frac{1}{(N-K-s)!} \sum_{m=0}^{s} (N-K-s+m) \times \ldots \times (1+m) \, x^m \\ & \quad - \frac{1}{(N-K-s+1)!} \sum_{m=0}^{s-1} (N-K-s+1+m) \times \ldots \times (1+m) \left(x^m - x^{m+1}\right) \\ g_s(x) &= \frac{(N-K+1)!}{(N-K+1-s)!s!} x^s . \end{aligned}$$

Note that both $f_s(x)$ and $g_s(x)$ are polynomials in x with the highest power of x being s. We have

$$f_s(0) = 0 = g_s(0) .$$

For all $k \in \{1, \ldots, s-1\}$ we have

$$\begin{aligned} \frac{d^k f_s}{dx^k}(x=0) &= \frac{(N-K-s+k)!}{(N-K-s)!} - \frac{((N-K-s+1+k)! - (N-K-s+k)!k)}{(N-K-s+1)!} \\ &= \frac{(N-K-s+k)!}{(N-K-s)!} - \frac{(N-K-s+k)!}{(N-K-s)!} \\ &= 0 = \frac{d^k g_s}{dx^k}(x=0) \end{aligned}$$

Finally,

$$\begin{aligned}\frac{d^s f_s}{dx^s}(x=0) &= \frac{(N-K)!}{(N-K-s)!}+\frac{(N-K)!s}{(N-K-s+1)!} \\ &= \frac{(N-K+1)!}{(N-K-s+1)!}=\frac{d^s g_s}{dx^s}(x=0).\end{aligned}$$

Thus, we can conclude $f_s(x)=g_s(x)$ for all $x\in[0,1]$ and any $s\in\{1,\dots,N-K\}$.

So by induction we have proven

$$P^k(N')=(1-\alpha)\left(\sum_{j=0}^{N-k}C_j^{N-k}(1-\alpha)^j\alpha^{N-k-j}\Delta(N'+j+1)\right)$$

for any $k\leq N$, when N' agents have decided to participate prior to agent k.

Since all agents participate along the equilibrium path, we can conclude that in equilibrium the price charged to agent k is

$$P^k=(1-\alpha)\left(\sum_{j=0}^{N-k}C_j^{N-k}(1-\alpha)^j\alpha^{N-k-j}\Delta(k+j)\right)$$

as stated in Proposition 6.

Note the price P^k is independent of k in the special case of $\alpha=0$, so the platform sets the same price to all agents in equilibrium. When $\alpha=0$, the platform has all the bargaining power, and we have $P^k=\Delta(N)$, so this is identical to Corollary 1 in which the platform makes a take-it-or-leave-it offer to each agent. Also, as $\alpha\to 1$, agents have all the bargaining power, so we have $P^k\to 0$ for all k. Whether agents would be better off without the platform in that case just depends on whether the platform is inefficient or not, i.e. whether $b(N)<u(0)$.

The more interesting case arises when $\alpha>0$. Since $\sum_{j=0}^{N-k}C_j^{N-k}(1-\alpha)^j\alpha^{N-k-j}=1$ for any k, the platform's optimal price to the k-th agent is $(1-\alpha)$ multiplied by a weighted average of the surplus obtained from joining the platform for each of the subsequent $N-k+1$ agents (including the k-th agent) deciding. That is, it is $(1-\alpha)$ multiplied by a weighted average of the $\Delta(n)$ terms, with n ranging from k to N. We wish to determine whether the platform's price is increasing across agents that sign up and whether agents are better or worse off due to the platform.

Since no agent is pivotal, by assumption, an agent expects all other agents to participate regardless of its decision. Thus, an agent's gross surplus $\Delta(N)$ from joining versus not doesn't change with k. Instead, any change in P^k over k happens due to changes in the additional payoff that the platform stands to gain on subsequent agents joining when agent k joins. Specifically, when agent k joins rather than doesn't, the additional profit that the platform stands to gain beyond the current price can be measured by the extent to which its negotiated price is expected to increase over subsequent agents. As k increases, with fewer potential agents left to join, the impact on the platform's profit from not signing an agent is felt over fewer subsequent agents signing up. This suggests the platform has less to lose from not attracting an agent as k increases, leading to P^k being increasing in k. But the additional profit that can be extracted from subsequent agents also depends on $\Delta(\cdot)$. Given $\Delta(\cdot)$ is an increasing function, as k increases, the average of the additional profit that can be extracted from the remaining agents increases, meaning the platform could actually have more to lose. The overall net effect on how much the platform stands to lose by not attracting an agent as k increases is ambiguous, and depends on the particular shape of $\Delta(\cdot)$.

To proceed, assume linear externalities so $b(n) = b_0 + b_1 n$ and $u(n) = u_0 - u_1 n$ with $u_1 > 0$, which implies $\Delta(n) = \beta_0 + \beta_1 n$, where $\beta_0 = b_0 - u_0 - u_1$ and $\beta_1 = b_1 + u_1$. We must assume $\beta_1 > 0$ so that $\Delta(\cdot)$ is increasing and $\beta_0 + \beta_1 > 0$ so that $\Delta(1) > 0$. Plugging this $\Delta(n)$ function into the last expression of P^k above implies

$$P^k = (1-\alpha)\left(\beta_0 + \beta_1 \left(N - \alpha(N-k)\right)\right),$$

for all $k \in \{1, \ldots, N\}$, as given in Proposition 6. Clearly, P^k is increasing in k. The monotonicity of P^k in k allows us to characterize when the platform trap arises. Specifically, there are three possible cases:

- If $b(N) - P^N - u(0) > 0$, which is equivalent to

  $$\alpha > \frac{u_1(N-1)}{\beta_0 + \beta_1 N},$$

  then all agents are better off with the platform.

- If $b(N) - P^1 - u(0) < 0$, then all agents are worse off with the platform. Note that $b(N) - P^1 - u(0) < 0$ is equivalent to

  $$b_0 + b_1 N - (1-\alpha)\left(\beta_0 + \beta_1 N - \alpha\beta_1(N-1)\right) - u_0 < 0,$$

  and it is easily verified that the LHS is increasing in α for $\alpha \leq 1$, equal to $-u_1(N-1)$ when $\alpha = 0$ and $b_0 + b_1 N - u_0$ when $\alpha = 1$.

- If $b(N) - P^1 - u(0) > 0 > b(N) - P^N - u(0)$, then there exists $\overline{k} \in \{1, \ldots, N-1\}$ such that the first $\overline{k}$ agents are better off and the last $N - \overline{k}$ agents are worse off with the platform.

From this, we can conclude:

- If $b_0 + b_1 N - u_0 < 0$, which means $b(N) < u(0)$, then all agents are worse off with the platform regardless of α.
- If $b_0 + b_1 N - u_0 > 0$, then there exist $\underline{\alpha}$ and $\overline{\alpha}$ such that $0 < \underline{\alpha} < \overline{\alpha} < 1$ and
  - if $\alpha < \underline{\alpha}$, then all agents are worse off with the platform
  - if $\alpha > \overline{\alpha}$, then all agents are better off with the platform
  - if $\underline{\alpha} < \alpha < \overline{\alpha}$, then there exists $\overline{k} \in \{1, \ldots, N-1\}$ such that the first $\overline{k}$ agents are better off and the last $N - \overline{k}$ are worse off with the platform.

H.2 Heterogeneous agents

H.2.1 Proof of Proposition 7

Suppose first $\min \{b(N), b(N+S-1)\} > u(0)$. We show that in this case the platform can profitably attract all agents and extract maximum profits regardless of the order of offers. Suppose the platform has attracted the first N agents. If the last agent is the superstar, the platform attracts it with a price of $S(b(N+S) - u(N))$ and if it is a regular agent the platform attracts it with a price of $b(N+S) - u(N+S-1)$, both of which are positive by assumption.

Now suppose the platform has attracted the first $N-1$ agents it has made offers to and is now facing the second-to-last agent. We know from the previous case that if this agent participates, then so will the last agent. Now suppose the second-to-last agent does not participate. Then, if the last agent is the superstar, the platform attracts it by charging $S(b(N+S-1) - u(N-1)) > 0$, and if it is a regular agent, the platform attracts it by charging $b(N) - u(N-1) > 0$ (if the second-to-last agent was the superstar) or $b(N+S-1) - u(N+S-2) > 0$ (if the second-to-last agent was not the superstar). Thus, the last agent will be attracted regardless of what the second-to-last agent does, so the platform can attract it by charging $S(b(N+S) - u(N))$ if it is the superstar, or $b(N+S) - u(N+S-1)$ if it is a regular agent.

We can use the same logic all the way back to the first agent, concluding that the platform profitably attracts all agents, charging $S(b(N+S) - u(N))$ to the superstar and $b(N+S) - u(N+S-1)$ to regular agents, regardless of the order of offers. The platform cannot do any better because it extracts the maximum surplus from each agent given the participation of the other agents.

Next suppose $b(N+S-1) > u(0) \geq u(N-1) > b(N)$, so the superstar is pivotal but none of the regular agents are. In this case, the only way for the platform to achieve maximum profits is by approaching all regular agents first and the superstar last. All regular agents are offered a price $b(N+S) - u(N+S-1)$ and the superstar is offered a price $S(b(N+S) - u(N))$, resulting in total profits

$$(N+S)\,b\,(N+S)-Su\,(N)-Nu\,(N+S-1)>0.$$

This can be easily verified as an equilibrium outcome given the same backwards induction logic as above. Once again, the platform attains its absolute maximum profit, except that here the order matters. If the platform approaches the superstar earlier than in the last slot, say in slot $k+1<N+1$, then it would only be able to charge it $S\,(b\,(N+S)-u\,(k))$ (if the superstar does not participate, the platform will not be able to attract any of the remaining agents), which is strictly less than $S\,(b\,(N+S)-u\,(N))$.

Finally, suppose $u\,(N+S-2)>b\,(N+S-1)$, so all agents are pivotal. Note this implies $u\,(N-1)>b\,(N)$ because

$$b\,(N)-u\,(N-1)=\Delta\,(N)\leq\Delta\,(N+S-1)=b\,(N+S-1)-u\,(N+S-2)<0.$$

If the platform approaches the superstar in position $k_0\in\{1,...,N+1\}$, then it charges $P^k=b\,(N+S)-u\,(k-1)$ to regular agents approached in position $k<k_0$ and $P^k=b\,(N+S)-u\,(S+k-2)$ to regular agents approached in position $k\geq k_0+1$. The superstar is charged $P^{k_0}=S\,(b\,(N+S)-u\,(k_0-1))$. Thus, the platform's profits when it approaches the superstar in position k_0 are

$$\Pi\,(k_0)=(N+S)\,b\,(N+S)-\sum_{k=1}^{k_0-1}u\,(k-1)-Su\,(k_0-1)-\sum_{k=k_0+1}^{N+1}u\,(S+k-2)\,.$$

Now consider the difference

$$\Pi\,(k_0+1)-\Pi\,(k_0)=(S-1)\,u\,(k_0-1)-Su\,(k_0)+u\,(S+k_0-1)$$

for $k_0\in\{1,...,N\}$. Consequently, if $u\,(\cdot)$ is convex, then

$$\frac{(S-1)}{S}u\,(k_0-1)+\frac{1}{S}u\,(S+k_0-1)\geq u\left(\frac{(S-1)}{S}\,(k_0-1)+\frac{1}{S}\,(S+k_0-1)\right)=u\,(k_0)\,,$$

so $\Pi\,(k_0+1)-\Pi\,(k_0)\geq 0$ for all $1\leq k_0\leq N$, which implies $\Pi\,(k_0)$ is maximized by $k_0=N+1$.

And if $u\,(\cdot)$ is concave, then

$$\frac{(S-1)}{S}u\,(k_0-1)+\frac{1}{S}u\,(S+k_0-1)\leq u\,(k_0)\,,$$

so $\Pi\,(k_0+1)-\Pi\,(k_0)\leq 0$ for all $1\leq k_0\leq N$, which implies $\Pi\,(k_0)$ is maximized by $k_0=1$.

H.2.2 Superstar of the same size as regular agents

Now suppose the superstar has the size and therefore the payoffs of an individual agent. In this case, the sole defining characteristic of the superstar is its outsized impact (externality) on all other agents' payoffs — other than that, the superstar is identical to any other agent. In the following proposition, we characterize the outcome for the same three parameter ranges as in Proposition 7.

Proposition 14. *Suppose there are N regular agents and one superstar agent, of size one but equivalent in impact to $S > 1$ regular agents.*

1. *If $\min \{b(N), b(N+S-1)\} > u(0)$, the platform profitably attracts all agents, none are pivotal, and the order of the platform's offers is irrelevant.*
2. *If $b(N+S-1) > u(0) > u(N-1) > b(N)$, the platform profitably attracts all agents, only the superstar is pivotal, and the platform optimally offers to the superstar last.*
3. *If $b(S+N-1) < u(S+N-2)$ and $b(N+S) > \frac{u(0)+\sum_{k=0}^{N-1} u(S+k)}{N+1}$, the platform profitably attracts all agents, all are pivotal, and the platform optimally offers to the superstar first.*

In cases (1) and (2), all agents would be strictly better off without the platform. In case (3), the regular agents would be strictly better off without the platform and the superstar is indifferent.

Proof of Proposition 14:

The proof of the first two cases is identical to that in Proposition 7 above.

Now suppose $u(N+S-2) > b(N+S-1)$, so all agents are pivotal. Note this implies $u(N-1) > b(N)$ because

$$b(N) - u(N-1) = \Delta(N) \leq \Delta(N+S-1) = b(N+S-1) - u(N+S-2) < 0.$$

In this case, we show that it is optimal for the platform to approach the superstar agent first if it wants to attract any agents. Indeed, by doing so, it can charge $b(N+S) - u(0)$ to the superstar agent and $P^k = b(N+S) - u(S+k-1)$ to agent $k \in \{1, \ldots, N\}$ in the sequence of regular agents. Total profits are

$$(N+1)\, b(N+S) - u(0) - u(S) - u(S+1) - \ldots - u(S+N-1),$$

which is positive if

$$b(N+S) > \frac{u(0) + \sum_{k=0}^{N-1} u(S+k)}{N+1}.$$

Now suppose the platform approaches the superstar second (after one regular agent). Then it charges $b(N+S) - u(0)$ to the first regular agent, $b(N+S) - u(1)$ to the superstar, $b(N+S) -$

$u(S+1)$ to the second regular agent, and so on. Total profits are now

$$(N+1)\,b(N+S)-u(0)-u(1)-u(S+1)-\ldots-u(S+N-1)\,.$$

This is lower than the profit obtained by approaching the superstar first since $u(S)<u(1)$. And it is easily seen that the same will be true when the platform approaches the superstar in any but the first position.

H.3 Competing platforms

Proof of Proposition 8:

Ignoring knife-edge equality cases, which do not affect the three strict comparisons stated in the proposition, there are four cases to consider.

Case (i): $b_1(1,1)>\max\{u(0,0),b_2(0,2)\}$. In this case, platform 1 will profitably attract agent 2 regardless of what agent 1 does. Thus, if agent 1 joins platform 1, it obtains $b_1(2,0)-P_1^1$, if it joins platform 2 it obtains $b_2(1,1)-P_2^1$, and if it doesn't join either platform, it obtains $u(1,0)$. Platform 2 is not willing to offer agent 1 a negative price because it has no chance of attracting agent 2, therefore platform 1 will attract agent 1 because $b_1(2,0)>\max\{u(1,0),b_2(1,1)\}$. And platform 1's prices are

$$P_1^1=P_1^2=b_1(2,0)-\max\{u(1,0),b_2(1,1)\}>0,$$

meaning each agent's net payoff is

$$\max\{u(1,0),b_2(1,1)\}\,.$$

Thus, both agents are worse off with the platforms if $b_2(1,1)<u(0,0)$ and they are better off with the platforms if $b_2(1,1)>u(0,0)$.

Case (ii): $u(0,0)>b_1(1,1)\geq b_2(0,2)$. In this case, if agent 1 joins platform 1, then platform 1 will also profitably attract agent 2 because $b_1(2,0)\geq b_1(1,1)>b_2(1,1)$ and $b_1(2,0)>u(1,0)$. If agent 1 joins platform 2, then agent 2 will still join platform 1 because $b_1(1,1)\geq b_2(0,2)>u(0,1)$. Finally, if agent 1 does not join either platform, then agent 2 joins platform 1 if $b_1(1,0)>u(0,0)$, and it joins neither platform if $u(0,0)\geq b_1(1,0)$. Thus, platform 2 is unwilling to offer agent 1 a negative price because it has no chance of attracting agent 2. Thus, if agent 1 joins platform 1, its payoff will be $b_1(2,0)-P_1^1$, if it joins platform 2 its payoff is $b_2(1,1)-P_2^1$ and if it joins neither platform, its payoff will be $u(0,0)$ (if $u(0,0)\geq b_1(1,0)$) or $u(1,0)$ (if $b_1(1,0)>u(0,0)$). Since platform 2 is unwilling to offer a negative price, platform 1 will always attract agent 1 by charging

$$P_1^1 = \begin{cases} b_1(2,0) - u(0,0) & \text{if } \ u(0,0) \geq b_1(1,0) \\ b_1(2,0) - \max\{u(1,0), b_2(1,1)\} & \text{if } \ u(0,0) < b_1(1,0) \end{cases}$$

Then platform 1 can profitably attract agent 2 by charging

$$P_1^2 = b_1(2,0) - \max\{b_2(1,1), u(1,0)\} > 0.$$

Platform 1 profits are

$$2b_1(2,0) - u(0,0) - \max\{b_2(1,1), u(1,0)\} \geq 0$$

when $u(0,0) \geq b_1(1,0)$, or

$$2b_1(2,0) - 2\max\{b_2(1,1), u(1,0)\} \geq 0$$

when $u(0,0) < b_1(1,0)$. Note in both cases profits are positive because

$$b_1(2,0) > \max\{b_2(1,1), u(1,0)\}.$$

Agent 2's net payoff is $\max\{b_2(1,1), u(1,0)\}$. Agent 1's net payoff is $u(0,0)$ if $u(0,0) \geq b_1(1,0)$, or $\max\{b_2(1,1), u(1,0)\}$ if $u(0,0) < b_1(1,0)$. Since $b_2(1,1) \leq b_2(0,2) < u(0,0)$ in this case, agent 2 is strictly worse off with the platforms. Agent 1 is either indifferent or strictly worse off.

Case (iii): $b_2(0,2) > b_1(1,1) > u(0,0)$. In this case, if agent 1 joins platform 2, then platform 2 also profitably attracts agent 2. If agent 1 joins platform 1 or neither platform, then platform 1 profitably attracts agent 2 because

$$b_1(2,0) \geq b_1(1,1) > \max\{b_2(1,1), u(0,0)\} \geq \max\{b_2(1,1), u(1,0)\}$$

and $b_1(1,0) > b_2(0,1)$ and $b_1(1,0) \geq b_1(1,1) > u(0,0)$ imply

$$b_1(1,0) > \max\{b_2(0,1), b_1(1,1)\} \geq \max\{b_2(0,1), u(0,0)\}.$$

Thus, since $b_2(0,2) > u(0,0) \geq u(1,0)$, the binding constraint on platform 1 for attracting agent 1 is platform 2. Namely, platform 2 is willing to set its price for agent 1 as low as

$$P_2^1 = -(b_2(0,2) - \max\{b_1(1,1), u(0,1)\}) = -(b_2(0,2) - b_1(1,1)),$$

i.e., to incur a loss on agent 1 equal to the amount platform 2 would be able to extract from agent 2 were it able to attract agent 1. This means platform 1 must set

$$P_1^1 = b_1(2,0) - (2b_2(0,2) - b_1(1,1))$$

in order to attract agent 1, which then implies platform 1 attracts agent 2 by setting

$$P_1^2 = b_1(2,0) - \max\{b_2(1,1), u(1,0)\}.$$

Total profit for platform 1 is

$$2b_1(2,0) - 2b_2(0,2) + b_1(1,1) - \max\{b_2(1,1), u(1,0)\},$$

which is positive and confirms that platform 1 wins. Net payoffs are $2b_2(0,2) - b_1(1,1)$ for agent 1 and $\max\{b_2(1,1), u(1,0)\}$ for agent 2. Note

$$2b_2(0,2) - b_1(1,1) > u(0,0)$$

in this case, so agent 1 is strictly better off with the platforms. Agent 2 is better off if $b_2(1,1) > u(0,0)$, otherwise it is worse off.

Case (iv): $b_1(1,1) < \min\{b_2(0,2), u(0,0)\}$. In this case, if agent 1 joins either platform, then agent 2 will join the same platform because

$$\begin{aligned} b_2(0,2) &> \max\{b_1(1,1), u(0,1)\} \\ b_1(2,0) &> \max\{b_2(0,2), u(1,0)\} \geq \max\{b_2(1,1), u(1,0)\}. \end{aligned}$$

And if agent 1 joins neither platform, then agent 2 joins neither platform when $u(0,0) \geq b_1(1,0)$, or joins platform 1 when $u(0,0) < b_1(1,0)$. Thus, agent 1's payoffs are $b_1(2,0) - P_1^1$ from joining platform 1, $b_2(0,2) - P_2^1$ from joining platform 2, and $u(0,0)$ or $u(1,0)$ from joining neither platform. Platform 2 is willing to offer agent 1 a negative price by setting

$$P_2^1 = -(b_2(0,2) - \max\{b_1(1,1), u(0,1)\}),$$

so to win agent 1 platform 1 must set

$$P_1^1 = b_1(2,0) - \max\{u(0,0), 2b_2(0,2) - \max\{b_1(1,1), u(0,1)\}\}$$

if $u(0,0) \geq b_1(1,0)$ or

$$P_1^1 = b_1(2,0) - \max\{u(1,0), 2b_2(0,2) - \max\{b_1(1,1), u(0,1)\}\}$$

if $u(0,0) < b_1(1,0)$.

Platform 1 then attracts both agents and charges

$$P_1^2 = b_1(2,0) - \max\{b_2(1,1), u(1,0)\}$$

to agent 2. Total profits for platform 1 are

$$2b_1(2,0) - \max\{u(0,0), 2b_2(0,2) - \max\{b_1(1,1), u(0,1)\}\} - \max\{b_2(1,1), u(1,0)\}$$

if $u(0,0) \geq b_1(1,0)$ or

$$2b_1(2,0) - \max\{u(1,0), 2b_2(0,2) - \max\{b_1(1,1), u(0,1)\}\} - \max\{b_2(1,1), u(1,0)\}$$

if $u(0,0) < b_1(1,0)$. It is easily verified that under assumptions (7) and (8), these profits are always positive, so platform 1 does indeed profitably attract both agents. Agent 1's net payoffs are then

$$\max\{u(0,0), 2b_2(0,2) - \max\{b_1(1,1), u(0,1)\}\}$$

if $u(0,0) > b_1(1,0)$ or

$$\max\{u(1,0), 2b_2(0,2) - \max\{b_1(1,1), u(0,1)\}\}$$

if $u(0,0) \leq b_1(1,0)$. Agent 2's net payoffs are

$$\max\{b_2(1,1), u(1,0)\}.$$

So if $b_2(0,2) > u(0,0) > b_1(1,1)$, then agent 1 is strictly better off with the platforms and agent 2 is strictly worse off.

Based on the four cases laid out above, if $b_2(1,1) > u(0,0)$ (case 1 in the text of the proposition), then we are in cases (i) or (iii), and under this condition, in both cases both agents are better off with the platforms. If $b_1(1,1) > \max\{u(0,0), b_2(0,2)\}$ and $b_2(1,1) < u(0,0)$ (case 2 in the text of the proposition), then we are in case (i) and both agents are worse off with the platforms. Finally, if $b_2(0,2) > \max\{u(0,0), b_1(1,1)\}$ and $b_2(1,1) < u(0,0)$ (case 3 in the proposition), then we are in cases (iii) or (iv), and it is easily verified that agent 1 is better off with the platforms, while agent 2 is worse off.