Regulating platform fees*

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Abstract

We consider a platform that helps consumers more easily discover and transact with suppliers. Such platforms have come to dominate many sectors of the economy, raising issues about the high fees they sometimes charge, especially since they tend to commoditize the suppliers they aggregate. We show that the welfare-maximizing fee exceeds the platform's marginal cost by the extent to which suppliers obtain lower markups on the platform than in the direct channel. We examine the robustness of this simple principle, and explore factors that make a platform set its fee higher or lower than this level.

Keywords: platforms, marketplaces, aggregators, regulation

1 Introduction

Regulators are struggling with the right way to address market power concerns arising with large digital platforms that control third-party suppliers, app developers, online sellers, and other small businesses access to consumers; i.e. they act as gatekeepers. In Europe, the Digital Markets Act, which was recently introduced, seeks to do this primarily by prohibiting various types of platform restraints and behavior: e.g. self-preferencing, price-parity clauses and bundling/tying, while obliging

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platforms to make certain changes that are supposed to promote easier user choice and switching. It is unclear, however, the extent to which these changes will really limit platforms' ability to exercise market power via high prices to the businesses that use them to access consumers (either via a platform marketplace and/or via advertising over the platform). This motivates our interest in another, possibly complementary, solution, which is the regulation of the prices charged by platforms to the suppliers that use them to access consumers.

The issue of regulating platform prices is relevant beyond just the big-tech platforms that are the focus of recent regulatory efforts. Digital marketplaces have sprung up across almost every sector of the economy, from business software to dog walking. These marketplaces aggregate suppliers and in some verticals are dominated by one or two players. As digital marketplaces become the main place consumers discover and transact with suppliers, they get more market power, which may be exercised through high fees charged to suppliers.[1]

To address this concern, we develop a simple framework of a monopoly platform that connects consumers and suppliers, and charges a fee to suppliers for doing so. Our framework takes into account that suppliers may pass these fees back to consumers via higher prices. It also takes into account that the platform intensifies competition between suppliers, and that the platform has to attract consumers in the first place who can alternatively buy directly from suppliers.

Given the platform intensifies competition between suppliers and facilitates consumer choice of their preferred supplier, it will attract consumers to use it. However, intensified supplier competition comes at a cost to these suppliers, which consumers ignore. As a result too many consumers will use the platform if it was to only charge suppliers a fee equal to its marginal cost. Instead, we find the socially optimal fee is characterized by a very simple rule: it equals the platform's marginal cost plus the amount to which the platform decreases supplier margins (what we call, the markup differential). This ensures that consumers internalize the benefit that suppliers get when making their channel choice. We also show how the fee a monopoly platform sets can be higher or lower than this efficient level, and the factors driving any distortion.

[1]Some platforms where high supplier-side fees have been noted include: app stores (Apple's AppStore and Android's Playstore), e-commerce marketplaces (Amazon and eBay etc), food delivery apps (UberEats and Doordash), hotel booking sites (Booking.com and Expedia), and gaming consoles (Xbox and Playstation). See https://www.theverge.com/21445923/platform-fees-apps-games-business-marketplace-apple-google for the fees charged by some of these platforms, as well as others.

We then explore two extensions of our framework. The first is the case that there is showrooming, so consumers can switch to buy directly after discovering a supplier on the platform, if it is cheaper to so. We show how our same characterization of the socially optimal fee still applies, although the markup differential is lower due to showrooming, thus implying showrooming lowers the efficient fee. The second is we suppose that suppliers have to advertise if they want to be discoverable in the direct channel. In the limit as the number of suppliers becomes large, we find the characterization of the platform's profit maximizing fee is unchanged from the baseline setting but the efficient fee is lower. The latter result reflects that an efficiency benefit the platform creates and which consumers don't internalize is to reduce the extent of duplicated advertising which arises in the direct channel.

1.1 Related literature

There is surprisingly little prior research on the question of the right level at which to regulate prices set by digital platforms. One exception is for payment card platforms, where most of the focus was exactly on whether the interchange fee set by a card platform (and so the fee merchants pay for accepting card payments) was too high (Rochet and Tirole, 2002 and 2011, and Wright, 2004 and 2012), and if so, what was the right fee to set. Indeed, this paper is in part inspired by the work of Rochet and Tirole (2011) who propose a simple rule that could be used to regulate interchange fees (the so-called "Merchant Indifferent Test"), one that has been adopted by regulators in Europe, among other places. Their setting is different, however, for two main reasons (i) unlike the types of marketplace platforms we're focused on in this paper, card platforms don't help intensify competition between suppliers given they are not primarily used to discover merchants; (ii) a no-surcharge rule applies, so suppliers are not allowed to set a higher price to consumers who purchase using the card platform than those who pay with cash.

Gomes and Mantovani (2022) relax (i) but not (ii). In their setting, the platform expands the consideration set of consumers and in so doing also intensifies competition between suppliers. They show the platform's fee to suppliers is excessive under a price-parity clause which says suppliers cannot undercut the price they set on the platform in their direct channel.[2] However, their characterization of the

[2] Other papers also look at settings in which price parity clauses hold, and markups and surplus can differ between the platform channel and the direct channel (e.g. see Edelman and Wright, 2015 and Wang and Wright, 2020) but they differ in not exploring the regulation of the platform's fee.

socially optimal fee is different from ours. This reflects their focus on the case where price-parity binds. This means there is no role for consumers' channel choice to be influenced by fees, which is what drives our results. Rather, in their setting, it is the extensive margin between whether the platform operates or not given randomness in its fixed cost of operation that pins down the efficient fee. Specifically, their efficient fee is determined by the extent to which the platform expands consumers' consideration set as well as any convenience benefits it provides to suppliers. We will compare Gomes and Mantovani's characterization of the efficient fee with ours under the same model of supplier competition.

Our focus on the case without price parity clauses is motivated by the fact in many platforms, price-parity clauses have not been imposed, or in some cases, have been banned by regulators (see Baker and Scott Morton, 2018). More generally, we have in mind a setting in which restrictions like price-parity clauses have already been removed, for instance, because of regulations like those implied by the Digital Markets Act. And we ask the question of what fees platforms would set then, and if these are still too high, how to then regulate them.

Finally, our paper relates to the literature modelling price comparison websites. The seminal paper in this line is Baye and Morgan (2001), in which consumers can use the platform to find the lowest priced supplier (which are homogenous) or instead go to their local monopolist. They maintain price parity. Galeotti and Moraga-González (2011) extend their work to the case with differentiated firms, as well as allowing suppliers to set different prices across channels, like in our paper. A key difference in these papers is that they assume the platform can set a fixed fee to each side (both consumers and suppliers), and consumers all face the same fixed benefit of shopping via the platform vs shopping in the direct channel. Thus, they shut down the smooth channel choice that drives our results, and the efficient fees are just set so all consumers and suppliers participate on the platform. The models of price comparison websites by Ronayne (2021) and Ronayne and Taylor (2022) are closer to our setting, since they assume, more realistically, that such platforms charge firms a per-transaction fee and nothing to consumers directly. They also allow for differential prices across channels. However, in their setting the platform fee does not affect total welfare, and their interest lies rather in whether the existence of such platforms is good for consumers.

2 Baseline model

Suppose there are multiple suppliers (either a finite number or a continuum) producing horizontally differentiated products. For brevity, we will refer to suppliers as "firms", but the reader should keep in mind these can sometimes be individuals (e.g. a dog walker on Rover or a web designer on Fiverr). There is a unit mass of consumers, each with unit demand. There is an outside option, with surplus normalized to zero. The firms costs are normalized to zero.

Firms and consumers can trade directly. Let ϕ_D be the expected gross value of shopping directly, and $p_D = \mu_D$ be the symmetric equilibrium direct price, where μ_D represents firms' symmetric markup in the direct channel. We assume $\phi_D \geq \mu_D$ since otherwise consumers would never shop in the direct market.

A marketplace platform M can facilitate the trades between firms and consumers at a marginal cost $c \geq 0$, and for doing so it charges firms a per-transaction fee f, the most commonly used form of fee charged by such marketplaces.[3]

The expected gross surplus of joining M is ϕ_M. Assume the symmetric equilibrium price on M is

$$p_M = f + \mu_M,$$

where μ_M is firms' symmetric markup on M.

In commonly used search models, location models, and random utility models, $\phi_M \geq \phi_D$, $\mu_D \geq \mu_M$, $\phi_D \geq \mu_D$, and we will adopt those assumptions, but as will be obvious, for the most part, the framework does not rely on those assumptions.

Consumers's expected net utility of shopping directly is

$$\phi_D - p_D = \phi_D - \mu_D,$$

and their expected net utility of shopping on M is

$$\phi_M - p_M + b = \phi_M - (f + \mu_M) + b,$$

where b is an additive benefit (if positive) or cost (if negative) associated with shopping on M, which is distributed according to H on $[\underline{b}, \overline{b}]$. We assume a strictly positive density h and a weakly increasing hazard rate for H (which implies that

[3] As detailed in https://www.theverge.com/21445923/platform-fees-apps-games-business-marketplace-apple-google, marketplaces typically charge firms (sellers and developers) on a per transaction basis. Often such a fee is written as a percentage of the value of a transaction rather than a fixed amount per transaction. We adopt the latter type of fee for tractability. In the Online Appendix we show how our analysis can be modified to handle percentage fees.

demand for M as defined by $1 - H(\cdot)$ is weakly log-concave).

We assume for the relevant fees we consider (in particular, the higher of the socially optimal and privately optimal), all consumers who go to M always make a single transaction on M and get non-negative net surplus from doing so.[4] Thus, in this baseline setting we can also interpret b as the additional benefit consumers get from making a transaction on M.

Buyers observe the fee f but not the prices or firms' listing decisions.[5] Consumers choose one channel only, and in our baseline setting cannot change channels depending on the prices or listing decisions they discover. Finally, we allow firms to price discriminate across channels. This set of assumptions implies firms will always be willing to list on M to obtain incremental revenue.

Consumers who join the platform must have

$$\phi_M - (f + \mu_M) + b \geq \phi_D - \mu_D \;\Leftrightarrow\; b \geq f - (\phi_M - \phi_D) - (\mu_D - \mu_M).$$

Define $\Delta_s \equiv \phi_M - \phi_D$ as the surplus differential and $\Delta_m \equiv \mu_D - \mu_M$ as the markup differential. The condition above becomes

$$b \geq f - \Delta_s - \Delta_m \equiv \widehat{b}$$

which makes clear the only reason a consumer uses M is if the benefit of doing so plus the surplus and markup differential that M creates more than covers the fee it charges.

Some examples of micro-founded settings that fit this baseline model include the following:

- Sequential search model of a platform such as in Wang and Wright (2020). There are a continuum of firms with consumer's match value drawn iid from a distribution $G(\cdot)$. Firms are all available on either channel, and buyers choose one channel to search on. They search sequentially in their chosen channel, but search costs are lower on M. If x_j represents the equilibrium reservation utility for searching in channel j, we can show $\Delta_s = x_M - x_D > 0$ and

[4]For our specific applications later, we will add conditions on parameters to make sure this assumption is satisfied.

[5]The key assumption is that an individual firm cannot influence a buyers choice of channel by whether to list on M or what price to set. Yet it should be that M's choice of fee f, and so firms' equilibrium prices, will ultimately impact which channel consumers want to use. We could alternatively assume that consumers cannot observe f, but they can observe the modal price set by firms on M, which then works provided $n \geq 3$.

$\Delta_m = \frac{1-G(x_D)}{g(x_D)} - \frac{1-G(x_M)}{g(x_M)} > 0$. The positive surplus differentiation arises from a higher reservation utility on M (due to lower search costs on M), and lower search costs on M also explain the positive markup differential. More details of this model are given in Section 4.

- Salop's circular-city model with $n \geq 2$ firms located on a circle (Salop, 1979). Each good offers value v, and consumers face a standard linear mismatch cost parameter t. If consumers go direct, we assume they are randomly matched to one of the firms, whereas if they go on M they can choose from all listed firms. Consumers don't observe which particular firms are located on each channel, their prices, or product attributes until after they choose a channel. In the Online Appendix we characterize parameter restrictions on v (has to be large enough) and t (cannot be too high) so that the baseline model applies. In equilibrium, all firms list on M and we have: $\Delta_s = \frac{t}{4} - \frac{t}{4n} > 0$ and $\Delta_m = v - \frac{t}{2} - \frac{t}{n} > 0$ given $v > t$. The surplus differential is driven by lower mismatch costs on M, and the markup differential is driven by more firms competing on M. Two special cases of this setting are: (i) Hotelling model when $n = 2$; (ii) Bertrand competition if we take $t \to 0$, so $\Delta_s = 0$ and $\Delta_m = v$.

- Perloff and Salop (1985) model with $n \geq 3$ firms. This is similar to the previous Salop example, but here the utility a consumer can get from buying at firm i is $u^i = v - p^i + \beta\xi^i$, where ξ^i is iid from G across firms and consumers, and $\beta > 0$ is a parameter to measure the importance of the match value. Consumers randomly draw a set of $2 \leq n_D < n$ firms that they choose from in the direct market, whereas if they go on M they can choose from all listed firms. The difference between n and n_D then drives the surplus and markup differentials. For example, if G is a uniform distribution on $[0, 1]$, $\Delta_s = \beta\left(\frac{n}{n+1} - \frac{n_D}{n_D+1}\right) > 0$ and $\Delta_m = \beta\left(\frac{1}{n_D} - \frac{1}{n}\right) > 0$. The general expressions for Δ_s and Δ_m are given in (5)-(6) below, with the derivations of these provided in the Online Appendix.

3 Analysis of baseline model

As a profit-maximizing monopolist, M chooses f to maximize

$$(f - c)\left(1 - H(f - \Delta_s - \Delta_m)\right).$$

Note that $f^* = c$ only if the demand $1 - H(f - \Delta_s - \Delta_m) = 0$ for all $f > c$. Otherwise, M would be better off by setting some fee strictly above its marginal cost. The condition that $1 - H(f - \Delta_s - \Delta_m) = 0$, or equivalently, $H(f - \Delta_s - \Delta_m) = 1$, for all $f > c$ is equivalent to

$$\bar{b} \leq c - \Delta_s - \Delta_m \quad \Leftrightarrow \quad \bar{b} + \Delta_s + \Delta_m \leq c.$$

We rule out this uninteresting case by assuming

$$\bar{b} + \Delta_s + \Delta_m > c \tag{1}$$

throughout the paper.

With this assumption, we obtain the following characterization of M's optimal fee (as with other results not proven in the text, the proof is given in the Appendix):

Proposition 1. (The platform's profit maximizing fee)
M sets $f^* = \widehat{f}$, where $\widehat{f}$ is the unique solution to

$$\widehat{f} = c + \lambda\left(\widehat{f} - \Delta_s - \Delta_m\right), \tag{2}$$

and satisfies $c < \widehat{f} < \bar{b} + \Delta_s + \Delta_m$. Here $\lambda(x) = (1 - H(x))/h(x)$ is the inverse hazard rate, which is weakly decreasing.

We can illustrate this result when H takes the generalized Pareto distribution (GPD), which covers several well-known distributions such as uniform, exponential, normal, logistic, type I extreme value, and Weibull.

Example 1 (Generalized Pareto distribution)**.** *Given our assumption of a weakly increasing hazard rate, when H takes the generalized Pareto distribution (GPD) form, it can be written as*

$$\begin{aligned} H(b) &= 1 - \left(1 + \frac{\epsilon(b - \underline{b})}{\sigma}\right)^{-\frac{1}{\epsilon}} \quad \textit{if } \epsilon < 0 \\ &= 1 - e^{-\frac{b - \underline{b}}{\sigma}} \quad \textit{if } \epsilon = 0 \end{aligned}$$

over the support $\underline{b} \leq b \leq \underline{b} - \frac{\sigma}{\epsilon}$. Then (2) implies

$$\widehat{f} = \frac{c + \sigma - \epsilon(\underline{b} + \Delta_s + \Delta_m)}{1 - \epsilon} \tag{3}$$

so $\widehat{f} = \frac{c+\underline{b}+\sigma+\Delta_s+\Delta_m}{2}$ *when* $\epsilon = -1$ *(uniform distribution) and* $\widehat{f} = c + \sigma$ *when* $\epsilon = 0$ *(exponential distribution).*

Now let's determine the efficient fee. A consumer with $b \geq f - \Delta_s - \Delta_m$ joins M, and shops directly otherwise. So total welfare is

$$W = \int_{f-\Delta_s-\Delta_m}^{\overline{b}} (\phi_M + b - c) dH(b) + \int_{\underline{b}}^{f-\Delta_s-\Delta_m} \phi_D dH(b).$$

Differentiating W with respect to f gives the derivative

$$-(\phi_M - \phi_D + f - \Delta_s - \Delta_m - c) h (f - \Delta_s - \Delta_m).$$

Given second-order conditions clearly hold, setting the derivative above equal to zero implies the following result.[6]

Proposition 2. (The planner's welfare maximizing fee)
The planner which can only control the platform's fee and not firms' final prices maximizes total welfare by setting

$$f^e = c + \Delta_m. \tag{4}$$

Proposition 2 says from an efficiency perspective, the platform's fee should be set above its marginal cost by the extent to which the platform lowers firms margins. The result is simple yet surprising at first glance. Why should the efficient fee be anything other than the platform's marginal cost? Indeed, the only difference in price consumers should face across the two channels is the marginal cost that M faces to provide its intermediation service. However, given markups are lower on M (i.e., $\Delta_m > 0$), in equilibrium consumers will face a lower price differential than the marginal cost, which is why the efficient fee is higher than marginal cost in order to restore the correct price differential. Put differently, the margin difference makes consumers favor M, and as a result too many consumers join M. The social planner uses a fee above cost to correct for this distortion. Formally, if $f = c$, consumers will choose M if $\phi_M - c - \mu_M + b \geq \phi_D - \mu_D$, whereas in the efficient outcome,

[6] In case $c - \Delta_s < \underline{b}$ such that $h(f - \Delta_s - \Delta_m) = 0$, efficiency requires all consumers to use the platform. The fee proposed in (4) induces such an outcome, even though a continuum of fees can achieve the same outcome. A parallel argument holds when $c - \Delta_s > \overline{b}$ such that $h(f - \Delta_s - \Delta_m) = 0$, and anywhere in the paper where $h(\cdot) = 0$.

consumers should choose M if and only if $\phi_M - c + b \geq \phi_D$ (i.e. the difference in margins is removed in the efficient solution).

Another way to understand why the efficient fee is above M's marginal cost, is in terms of externalities. When consumers decide to use the platform, they do not take into account the negative effect on firms who earn lower margins on this channel. If they did, there would be no need for the fee to be set above M's marginal cost. By setting a higher fee, consumers pay a tax for using the platform that equals the loss in firms' margins that results from their choice, thereby getting them to internalize the full effects of their choice.

We can compare our characterization of the efficient fee to that in Gomes and Mantovani (2022). In their mature market setting in which there is no positive latent demand, and assuming competition is determined by the Perloff and Salop setting in which consumers get to see n_d firms in the direct market and n on the platform, they find the efficient fee just equals $\Delta_s + b_f$, where

$$\Delta_s = \beta \left(\int_{\underline{\xi}}^{\overline{\xi}} \xi dG(\xi)^n - \int_{\underline{\xi}}^{\overline{\xi}} \xi dG(\xi)^{n_D} \right) \tag{5}$$

and b_f is the convenience benefit they assume firms get from on-platform transactions. This compares to the efficient fee in our setting for the same competition model[7], which is Δ_m, where

$$\Delta_m = \beta \left(\frac{1}{n_D \int_{\underline{\xi}}^{\overline{\xi}} g(\xi) dG(\xi)^{n_D - 1}} - \frac{1}{n \int_{\underline{\xi}}^{\overline{\xi}} g(\xi) dG(\xi)^{n-1}} \right). \tag{6}$$

The parameters β, n_D and n have the same qualitative effects on the efficient fee across both settings, although for very different reasons. In Gomes and Mantovani, this is via the surplus differential, which provides a reason to make sure the platform can operate by setting a sufficiently high fee. In our setting, this is via the markup differential, which requires a sufficiently high fee to offset excessive use of the platform by consumers. Another difference is that allowing firms to enjoy a convenience benefit b_f would not change the efficient fee in (4) since such a benefit would be like a negative marginal cost for firms — it would lower their equilibrium prices on the platform since it is fully passed through by firms, which would induce consumers to correctly take it into account when deciding which channel to choose.

[7] We assume the platform has no marginal costs to make things as comparable to their setting as possible.

The next proposition compares the equilibrium fee in (2) and the efficient fee in (4).

Proposition 3. (Comparison of fees) *The profit-maximizing fee exceeds the efficient fee iff*

$$\lambda \left(c - \Delta_s \right) \geq \Delta_m. \tag{7}$$

An increase in the surplus differential Δ_s increases any concern that the platform's fee is too high (it increases the profit-maximizing fee but not the efficient fee). An increase in the markup differential Δ_m decreases any concern that the platform's fee is too high (it increases the profit-maximizing fee by less than the one-for-one increase in the efficient fee).

The result shows that Δ_s and Δ_m have opposite effects in determining whether the equilibrium fee is too high. Here a high Δ_s leads to a high equilibrium fee but has no effect on the efficient fee as consumers already take into account the surplus differential when making their choice of channel. On the other hand, a high Δ_m leads to a high efficient fee (one-for-one) but it doesn't get fully passed through into the equilibrium fee.

Proposition 3 does not rule out the possibility that the platform's monopoly fee is lower than the efficient fee. If $\Delta_m \leq 0$, by construction $f^* > f^e$, and the platform sets its fee too high. A necessary condition for $f^* \leq f^e$ is therefore that $\Delta_m > 0$; i.e. that the margins are strictly lower on M.

One case where M's fee is always too low is when M's added social value does not cover its marginal cost so $\Delta_s \leq c$ and it is always costly to join so $\bar{b} = 0$. The planner would prefer a higher fee to reduce the number of consumers going to M. Note that $1 - H(c - \Delta_s) = 0$ if $\bar{b} = 0$ and $c - \Delta_s \geq 0$. Then, given $\Delta_m > 0$, it must be that $f^* < f^e$.

A more extreme case arises when M's only value is in intensifying competition. This arises when we take $t \to 0$ in our Salop circular-city model so firms are Bertrand competitors on M. Assuming the value of each product is v, then $\mu_M = 0$, $\phi_M = v$, $\mu_D = v$, $\phi_D = v$, and so $\Delta_s = 0$ and $\Delta_m = v$. Then if $\bar{b} = 0$, the planner prefers no one uses M from an efficiency perspective which it can do by setting $f^e = v$. At this fee no consumers would incur a cost to participate on M. In contrast, M will always set a fee below the efficient level (but above c) to attract some consumers. In this example, M's only advantage over the direct market is the competition it creates for firms which firms can avoid if they can coordinate not to join, but since they can't coordinate, M is able to shift surplus from firms to consumers, and tax transactions in the process. This illustrates commodization — how a platform that destroys welfare can exist by shifting surplus from firms (who face a coordination failure) to itself and consumers.

Example 2 (Generalized Pareto distribution)**.** *Using the GPD for b to get λ in (7), M's monopoly fee exceeds the efficient fee iff*

$$\sigma - \epsilon\left(\underline{b} + \Delta_s - c\right) \geq \Delta_m. \tag{8}$$

For $\epsilon = -1$ (linear demand), the condition becomes $\overline{b} + \Delta_s - c \geq \Delta_m$, and for $\epsilon = 0$ (exponential distribution) it becomes $\sigma \geq \Delta_m$.

As can be seen from (8), the condition for the platform's fee to be too high depends on both Δ_m and Δ_s, as well as the parameters that define H. Since both Δ_m and Δ_s depend on the underlying primitives of competition on the platform and in the direct market, we can ask how changes in the primitives of competition affect the tendency for the platform to set its fee too high. We do this by defining the difference between the two sides of (8) as

$$L = \sigma - \epsilon\left(\underline{b} + \Delta_s - c\right) - \Delta_m,$$

and considering how L changes in changes in the primitives for each of our three competition applications from Section 2. For a change in some parameter x, we have

$$\frac{\partial L}{\partial x} = -\epsilon \frac{\partial \Delta_s}{\partial x} - \frac{\partial \Delta_m}{\partial x} \tag{9}$$

where recall $\epsilon \leq 0$ given our increasing hazard rate assumption. Applying to the three applications we get the following results:

Proposition 4. (Comparative statics) *Our measure of the tendency for the platform to set its fee too high (L) varies with the primitives of the respective competition models in the following way:*

- *Sequential search model: If the distribution G of match values drawn iid when searching is also GPD with parameters $\underline{b}_G$, σ_G, ϵ_G, then an increase in search* *costs in each channel (s_M and s_D) has the following effects*

$$\begin{aligned} \frac{\partial L}{\partial s_M} &> 0 \quad \Leftrightarrow \quad \epsilon > \epsilon_G \\ \frac{\partial L}{\partial s_D} &< 0 \quad \Leftrightarrow \quad \epsilon > \epsilon_G. \end{aligned}$$

- *Salop circular-city model: An increase in product differentiation between firms implies*

$$\frac{\partial L}{\partial t} > 0$$

  and an increase in the number of firms that can list on M implies

$$\frac{\partial L}{\partial n} < 0 \quad \Leftrightarrow \quad \epsilon > -4.$$

- *Perloff-Salop model: An increase in product differentiation between firms implies*

$$\frac{\partial L}{\partial \beta} < 0 \quad \Leftrightarrow \quad \epsilon > -\frac{\frac{1}{n_D \int_{\underline{\xi}}^{\bar{\xi}} g(\xi) dG(\xi)^{n_D - 1}} - \frac{1}{n \int_{\underline{\xi}}^{\bar{\xi}} g(\xi) dG(\xi)^{n-1}}}{\int_{\underline{\xi}}^{\bar{\xi}} \xi dG(\xi)^n - \int_{\underline{\xi}}^{\bar{\xi}} \xi dG(\xi)^{n_D}}, \tag{10}$$

  which implies ϵ must be sufficiently negative, and a change in the number of firms that consumers can evaluate on each channel implies

$$\frac{\partial L}{\partial n_D} > 0 \Leftrightarrow \quad \epsilon \geq \widehat{\epsilon}(n_D) \tag{11}$$

  and

$$\frac{\partial L}{\partial n} < 0 \Leftrightarrow \quad \epsilon \geq \widehat{\epsilon}(n), \tag{12}$$

  where $\widehat{\epsilon}(.) < 0$.

The results in Proposition 4 are not obvious because changes in competition primitives tend to affect the surplus differential and markup differential in the same direction. As can be seen from (9), since $\epsilon < 0$ this means they will have an ambiguous effect on the tendency for the platform to set its fee too high. For the search model, Proposition 4 shows that the direction of the effect from a change in search cost depends on whether the GPD for platform benefits H is more or less concave than the GPD for match values G. Interestingly, when both distributions have the same shape parameter (e.g. both are uniform or both are exponential), then a change in search cost (in either channel) has no effect on the distortion between the monopoly platform fee and the efficient fee.

Things are somewhat more clear cut for the two models with a finite number of firms. In the Perloff-Salop model, greater product differentiation increases both the markup differential (it has a larger positive impact on markups when there are fewer competitors) and surplus differential (it has a larger negative impact on gross surplus when there are fewer competitors). Provided H is not excessively concave, the positive effect on the markup differential dominates, which is why greater product differentiation decreases the tendency for the platform to set its fee too high. As an example, this is clearly true if H is uniform so that $\epsilon = -1$ and G is uniform on $[0, 1]$. Things work differently in the case of the Salop circular-city model in that greater product differentiation always increases the tendency for the platform to set its fee too high. Here greater product differentiation actually decreases the markup differential. This is because greater product differentiation reduces the amount firms in the direct market can extract in the direct market given they enjoy monopoly positions with respect to the consumers that come to them there.

Finally, in both the Salop circular-city model and Perloff-Salop model, increasing the number of firms in the direct market and/or decreasing the number of firms in total increases the tendency for the platform to set its fee too high provided H is not excessively concave. Unless H is excessively concave, reducing the markup differential matters more in driving excessive fees, which is why having less difference in the number of firms across the two channels leads to a greater incentive for the platform to set fees that are too high.

3.1 Incomplete pass-through

So far we have assumed that firms fully pass through M's fee into their prices. Here we consider how to adjust the formula for the welfare-maximizing fee when there is actually incomplete pass-through. If we write the equilibrium price on M as $p_M(f)$, total welfare is

$$W = \int_{p_M(f)-\mu_D-\Delta_s}^{\bar{b}} (\phi_M + b - c)\, dH(b) + \int_{\underline{b}}^{p_M(f)-\mu_D-\Delta_s} \phi_D dH(b).$$

Then FOC from welfare maximization implies

$$-(p_M(f) - \mu_D - c)\, p'_M(f)\, h(p_M(f) - \mu_D - \Delta_s) = 0.$$

So the planner would set f^w such that

$$p_M(f^w) = c + \mu_D.$$

Then by construction, if we define the markup at the corresponding price as

$$\mu_M(f^w) \equiv p_M(f^w) - f^w,$$

then by definition

$$\begin{aligned} f^w &= c + \mu_D - \mu_M(f^w) \\ &= c + \Delta_m(f^w). \end{aligned}$$

The result suggests the same formula for setting the welfare-maximizing fee in (4) applies. The problem for implementation of f^w is that the markup differential also now depends on f^w. Fortunately, using (4) with the observed markup is still a conservative way to proceed, and provided this exercise is updated over time, it would converge to the correct efficient fee level. To see this note that in measuring the markup $\mu_M(f^w)$ empirically, one would actually be calculating $f^{reg} = c + \Delta_m\left(\widehat{f}\right)$ since the observed markup from current data would correspond to $\Delta_m\left(\widehat{f}\right) = \mu_D - \left(p_M\left(\widehat{f}\right) - \widehat{f}\right)$. Given pass-through is assumed less than one, if $\widehat{f} > f^w$, then $p_M\left(\widehat{f}\right) - \widehat{f} < p_M(f^w) - f^w$, and $\Delta_m\left(\widehat{f}\right) > \Delta_m(f^w)$, which would imply using $f^{reg} = c + \Delta_m\left(\widehat{f}\right)$ would give an estimate of the efficient fee between f^w and $\widehat{f}$. Alternatively if $\widehat{f} < f^w$, by a parallel argument, $p_M\left(\widehat{f}\right) - \widehat{f} > p_M(f^w) - f^w$, and $\Delta_m\left(\widehat{f}\right) < \Delta_m(f^w)$, which would imply using $f^{reg} = c + \Delta_m\left(\widehat{f}\right)$ would give an estimate of the efficient fee between $\widehat{f}$ and f^w. Thus, using $f^{reg} = c + \Delta_m\left(\widehat{f}\right)$ would be a conservative treatment of regulating the fee (i.e. it would be closer to the platform's preferred fee than the true efficient fee). Moreover, if that regulation is updated over time, eventually it would converge to the efficient fee. As such imposing that the fee equals $f^e = c + \Delta_m$ where Δ_m is the empirically measured markup is a reasonably robust way to regulate the fee in the context of imperfect pass-through of fees.

3.2 Different welfare objectives

So far, when evaluating welfare we have used a total welfare standard. However, often policymakers care more about the users of a service (here both final consumers and the competing firms that want to reach them) than say that of a monopoly provider (i.e. the platform). This is particularly relevant in this setting, where the firms involved may be individuals or small businesses.

Consider the weighted average of the different surplus components making up total surplus, which can be written as

$$
\begin{aligned}
W^{ts} &= w_c \left(\int_{f-(\Delta_s+\Delta_m)}^{\bar{b}} (\phi_M + b - f - \mu_M) dH(b) + \int_{\underline{b}}^{f-(\Delta_s+\Delta_m)} (\phi_D - \mu_D)\, dH(b) \right) \\
&+ w_f \left(\int_{f-(\Delta_s+\Delta_m)}^{\bar{b}} \mu_M dH(b) + \int_{\underline{b}}^{f-(\Delta_s+\Delta_m)} \mu_D dH(b) \right) \\
&+ w_m \int_{f-(\Delta_s+\Delta_m)}^{\bar{b}} (f-c)\, dH(b),
\end{aligned}
$$

where the terms in the expression are consumer surplus (the first line), firms' total profit (the second line), and platform profit (the third line), and the respective weights satisfy $w_c + w_f + w_m = 1$. After simplifying, the derivative of W^{ts} with respect to f is

$$
\begin{aligned}
& w_f \Delta_m h\left(f - (\Delta_s + \Delta_m)\right) - w_c \left(1 - H\left(f - (\Delta_s + \Delta_m)\right)\right) \\
& + w_m \left(1 - H\left(f - (\Delta_s + \Delta_m)\right) - (f - c)\, h\left(f - (\Delta_s + \Delta_m)\right)\right).
\end{aligned}
$$

We consider several different special cases.

3.2.1 Consumer surplus only

Clearly if $w_f = 0$ and $w_m = 0$, so the planner is only interested in maximizing consumer surplus, then the planner should regulate f as low as is feasible to ensure the platform still operates. We take this to be equal to its marginal cost, although obviously if M has fixed costs to cover, then it should be based on average costs. The only reason to set a higher fee is to get consumers to internalize the profit of firms and/or the platform, which are absent here. Thus, we have:

Proposition 5. (Consumer surplus standard) *The fee that maximizes consumer surplus while ensuring the platform covers its cost is* $f^{cs} = c$.

3.2.2 Total user surplus

Next suppose $w_m = 0$, so we are only interested in total user surplus (or more generally, some weighted average of consumer plus firm surplus); i.e., we ignore the platform's profit altogether. In this case we find

Proposition 6. (Total user surplus standard) *The fee that maximizes a weighted average of consumer surplus and firms' profit while ensuring the platform covers its cost is either the lowest feasible fee $f^{us} = c$ (so that M can just cover its costs) or any fee such that $f^{us} \geq \overline{b} + \Delta_s + \Delta_m$ (so that no consumers will go to M). In case the standard is total user surplus ($w_c = w_f$), the planner prefers the high fee iff*

$$\int_{c-(\Delta_s+\Delta_m)}^{\overline{b}} \Delta_m dH(b) > \int_{c-(\Delta_s+\Delta_m)}^{\overline{b}} (\Delta_s + \Delta_m - c + b) dH(b) \tag{13}$$

or $\Delta_m > \Delta_s - c + \overline{b}$ in the case of linear H.

The proposition shows that any weighted average of consumer surplus and firm profit has an inverted U-shape, with the planner preferring either the lowest feasible fee, which just allows M to cover its costs, or any fee high enough that even the consumer with the highest possible b has no reason to go to M. The latter case is particularly likely when the markup differential is large and $\overline{b}$ is small (so consumers face mostly costs of joining).

How can total user surplus ever be increasing in f up to the point where consumers stop coming to M? Like in the welfare case, the consumer here does not internalize the additional margin that the firm gets when the consumer transacts directly. This means, from their joint perspective, too many consumers will transact via M, which suggests a positive transaction fee is needed. However, a positive transaction fee reduces the surplus of using M for consumers (with no offsetting benefit given we are no longer counting the platform's profit), so depending on the net effect, this either requires that f be set as low as possible, or an ever larger increase in f is desirable until consumers stop using M altogether.

3.2.3 Weighted average of total surplus

Suppose now $0 < w_m < w_c$, so that we consider the surplus of all parties including the platform. Also assume that H takes the exponential distribution as defined in our GPD example. Then second-order conditions always hold, and

$$f^{ts} = c + \frac{w_f}{w_m}\Delta_m - \sigma\left(\frac{w_c - w_m}{w_m}\right),$$

with $f^* > f^{ts}$ iff

$$w_c\sigma > w_f\Delta_m.$$

There are two cases of particular interest for this exponential case:

- Set $w_c = w_f = 1+\alpha > w_m = 1-\alpha$ so we weight the surplus of users (consumers and firms) more than the platform's profit. Then provided $\alpha < 1$ so $w_m > 0$,

$$f^{ts} = c + \Delta_m - \frac{2\alpha}{1-\alpha}(\sigma - \Delta_m).$$

- Set $w_c = 1+\alpha > w_m = w_f = 1-\alpha$ so weight consumer surplus more than profit (firms' and the platform's). Then provided $\alpha < 1$ so $w_m > 0$ and $w_f > 0$,

$$f^{ts} = c + \Delta_m - \frac{2\alpha}{1-\alpha}\sigma.$$

These results show that when consumer surplus gets more weight than profits (either of firms or the platform), the weighted-welfare maximizing fee is necessarily less than $c+\Delta_m$, while that is not true if the surplus of users (consumers and firms) gets more weight than the profit of the platform. And in either case, the welfare-maximizing level can still be above the equilibrium level if Δ_m is high relative to the standard deviation of b as measured by σ.

4 Search and showrooming

An application of the general setting is a standard sequential search model with a continuum of firms (see Wang and Wright, 2020). Here consumers' match values for one unit of firm i's product are drawn iid from the common distribution function G for each consumer and each firm. Consumers search to discover these match values as well as prices. In the absence of showrooming or price parity, firms' equilibrium prices are $p_M = f + \mu(x_M)$ on M and $p_D = \mu(x_D)$ in the direct market, where $\mu(x) = \frac{1-G(x)}{g(x)}$ is decreasing in x given that G is assumed to have an increasing hazard function, and x is the reservation value of the consumers in their search. Thus, $x_M > x_D$ across channels capturing that searching on M is less costly. Lower search costs on M also imply the gross surplus of searching on M is higher. Specifically, a consumer's net utility from searching (and buying) on M is $\phi(x_M)+b-f-\mu(x_M)$, and her utility from searching (and buying) directly is $\phi(x_D)-\mu(x_D)$, where ϕ is the expected match value which is an increasing function of x. Everything in our general model then applies where $\phi_M=\phi(x_M)$, $\phi_D=\phi(x_D)$, $\mu_M=\mu(x_M)$, $\mu_D=\mu(x_D)$, and note $\Delta_s=\phi(x_M)-\phi(x_D)$ and $\Delta_m=\mu(x_D)-\mu(x_M)$ are both positive. In addition, we need $x_M \geq p_M$ and $x_D \geq p_D$ in equilibrium such that all consumers who actively search will eventually buy.

Now consider the extension where an exogenous fraction of consumers α can, having searched on M, showroom and switch to buy directly from the firm they find on M. Firms cannot distinguish these consumers though, from those coming directly. Firms have an incentive to lower their direct price to induce more consumers who go to M to switch and buy directly, thereby saving f.

Proposition 7. *The welfare-maximizing fee f^w in the presence of showrooming is lower than that in the benchmark (i.e. $f^w < f^e$). An increase in α reduces f^w.*

The result is intuitive. Firms take into account that some fraction of consumers can be attracted to switch after searching from M. Since they are competing to attract these consumers and these consumers have more elastic demand, they will optimally lower their direct price to do so. This reduces the margin difference between the two channels, and so reduces the need to set a high fee to offset that margin difference.

If instead consumers do not expect to be able to showroom when deciding which channel to use, but upon choosing M, a fraction of consumers discover they can switch to buy directly, the analysis is very similar. The detailed analysis can be found in the online appendix.

5 Advertising

The main role of a marketplace platform is arguably to help consumers discover firms. As such, a platform can help firms avoid spending on costly advertising. In this section, we address how this affects the optimal choice of M's fee by considering a situation where consumers initially have limited information about potential trading partners and firms can use direct advertising or the platform to inform them.

There are n symmetric firms producing horizontally differentiated products. Their listing decision on M follows the baseline setting, and as a result all firms will list on M. Once consumers visit M, they can see all the prices and locations (and so their match values) of the n firms. They then select one firm to buy from (or choose the outside option which gives them zero utility).

Consumers can only know about firms in the direct channel when they receive ads from them. Consumers know the underlying configuration of firms but are unaware of their identity. Firms can advertise their identity (and so existence) to consumers. Formally, each firm $i = 1, 2, ..., n$ chooses an advertising intensity $a_i \in [0, 1]$ such that a consumer learns the identity of firm i with probability a_i. The advertising cost is given by $\frac{c_a (a_i)^2}{2}$ with $c_a > 0$. The ads only reveal the firms' identity but not the firm's price or location (and so the match value).[8] In the end, consumers are assumed to only pick one firm in the direct channel or M to go to. We assume advertising is sufficiently costly.

Assumption 1. *Assume*

$$c_a \geq \frac{\mu_D}{n}. \tag{14}$$

The timing of the model is given as follows.

- M sets its fee f.
- Firms decide whether to list on M, what prices to set on each channel, and their advertising intensity.[9]
- Consumers receive ads and observe the fee level M sets. They do not observe anything else.
- Consumers decide among the different firms they receive ads from and M, which one to visit.
- Upon learning the price and match value of the firm they visit, or all the prices and match values of the firms listed on M if they visit M, consumers decide whether to purchase or not.

[8] This type of advertising technology was used in the previous literature to study the competition for consumers' attention. For details, see Haan and Moraga-González (2011) for example.

[9] Note given each firm prices as a monopolist in the direct channel, it doesn't matter if each firm sets advertising choice first and then pricing or vice-versa, or it sets them together.

We first characterize the equilibrium, and then derive the efficient allocation and make the comparison. Suppose a consumer receives firm i's ads. The question is how many other firms' ads the consumer also receives. Let all other firms choose the symmetric advertising intensity $a \in (0,1)$. The probability that this consumer receives ads from one other firm is $C_{n-1}^1 a(1-a)^{n-2}$, and in this case the consumer visits firm i with probability $\frac{1}{2}$, and the probability that this consumer receives ads from two other firms is $C_{n-1}^2 a^2(1-a)^{n-3}$, and in this case the consumer visits firm i with probability $\frac{1}{3}$, and so on. So the total visits firm i receives is

$$\begin{aligned}
& a_i \left[C_{n-1}^1 a(1-a)^{n-2} \cdot \frac{1}{2} + C_{n-1}^2 a^2 (1-a)^{n-3} \frac{1}{3} + ... + C_{n-1}^{n-1} a^{n-1} \frac{1}{n} \right] \\
=& a_i \left[C_n^1 a^1 (1-a)^{n-1} \frac{1}{an} + C_n^2 a^2 (1-a)^{n-2} \frac{1}{an} + ... + C_n^n a^n \frac{1}{an} \right] \\
=& \frac{a_i}{an} \left[-C_n^0 (1-a)^n + C_n^0 (1-a)^n + C_n^1 a^1 (1-a)^{n-1} + C_n^2 a^2 (1-a)^{n-2} + ... + C_n^n a^n \right] \\
=& \frac{a_i}{an} \left[1 - (1-a)^n \right].
\end{aligned}$$

When deciding how much advertising to engage in, the only thing that matters for an individual firm is how that affects its profit from consumers that receive its ad. These are consumers who choose to purchase via the firm directly, which gives it additional profit of

$$\frac{a_i \left(1 - (1-a)^n\right)}{na} \mu_D H \left(f - \Delta_s - \Delta_m\right) - \frac{c_a a_i^2}{2},$$

after taking into account the cost of advertising. The profit it gets on M is unaffected by its choice of a_i. A firm's optimal a_i therefore solves

$$\frac{\left(1 - (1-a)^n\right)}{na} \mu_D H \left(f - \Delta_s - \Delta_m\right) = c_a a, \tag{15}$$

and we have the unique solution for a, denoted a^*.

Lemma 1. *Given that $H(f - \Delta_s - \Delta_m) \neq 0$, there exists a unique $a^* \in (0,1)$ such that (15) holds.*

An increase in a strictly decreases the left-hand side of (15), which then requires each individual firm decrease its advertising in response. The firms' advertising intensities are strategic substitutes. Moreover, since the left-hand side of (15) is strictly decreasing in a while the right-hand side is strictly increasing in a, an upward shift of the left-hand side must imply an increase in a, thus implying a^* increases in

μ_D. A higher f shifts demand onto the direct channel and leads to an increase in a^*. In particular, totally differentiating (15) we have

$$\frac{da^*}{df} = \frac{\left(1-(1-a^*)^n\right)\mu_D h\left(f-\Delta_s-\Delta_m\right)}{2c_a a^* n - n\left(1-a^*\right)^{n-1}\mu_D H\left(f-\Delta_s-\Delta_m\right)}. \tag{16}$$

The lemmas below characterize the equilibrium individual and aggregate advertising reach when $n \to \infty$.

Lemma 2. *The following equilibrium properties hold at the limit as $n \to \infty$: (i) $a^* \to 0$; (ii) $(1-a^*)^n \to 0$; (iii) $n(1-a^*)^{n-1} \to 0$; (iv) $na^* \to \infty$.*

Lemma 2 implies that in the limit as the number of firms gets large, (i) individual advertising intensity goes to zero; (ii) even though each firm's level of advertising goes to zero, the probability of being uninformed goes to zero, so the reach of advertising is complete: the probability a consumer receives at least one ad goes to one; (iii) moreover, conditional on receiving ads from one firm, the probability of not receiving ads from any other of the $n-1$ firms goes to zero; (iv) and finally, the average number of firms that a consumer can observe goes to infinity. In the proof of Lemma 2, we further show that $a^* \underset{n\to\infty}{\approx} \sqrt{\frac{1}{nx}}$, where $x \equiv \frac{c_a}{\mu_D H(f-\Delta_s-\Delta_m)}$.

Each consumer receives ads from at least one firm with probability $1-(1-a^*)^n$ and does not receive ads from any firm with probability $(1-a^*)^n$. In the former case, consumers choose to go to M if $b \geq f-\Delta_s-\Delta_m$. In the latter case, consumers choose to go to M if $b \geq f+\mu_M-\phi_M$. So the total transaction volume on M, conditional on receiving ads, is

$$T(f) \equiv \left(1-(1-a^*)^n\right)\left(1-H\left(f-\Delta_s-\Delta_m\right)\right) + (1-a^*)^n\left(1-H\left(f+\mu_M-\phi_M\right)\right).$$

It is clear $f-\Delta_s-\Delta_m \geq f+\mu_M-\phi_M$ given our assumption that $\phi_D \geq \mu_D$. This implies the total demand faced by M decreases as a^* increases.

M will choose f to maximize

$$f^* = \arg\max_f \left\{(f-c)T\left(f\right)\right\}.$$

Using Lemma 2, and provided second-order conditions hold, we can then prove the following limit result.

Proposition 8. (The platform's profit maximizing fee given firms advertise) *In the limit as $n \to \infty$, M's profit maximizing fee $f^* \to \widehat{f}$, where $\widehat{f}$ is the unique solution to (2).*

Thus, for a large number of firms, M's profit-maximizing choice of fee converges to the baseline without any advertising. This is not surprising given that with a large number of firms advertising, the probability consumers are informed of at least one firm goes to one, so M's problem is essentially the same as in the case where firms do not need to advertise.

Total welfare is given by

$$\begin{aligned} W &= (1-(1-a^*)^n)\int_{f-\Delta_s-\Delta_m}^{\bar{b}} (\phi_M + b - c)\, dH(b) \\ &\quad +(1-a^*)^n \int_{f+\mu_M-\phi_M}^{\bar{b}} (\phi_M + b - c)\, dH(b) \\ &\quad +(1-(1-a^*)^n)\phi_D H(f-\Delta_s-\Delta_m) - \frac{(a^*)^2}{2}. \end{aligned}$$

Suppose a planner maximizes the total welfare by choosing f. Provided second-order conditions hold, and the solution is interior, we get

Proposition 9. (The planner's welfare maximizing fee given firms advertise) *In the limit as $n \to \infty$, the welfare-maximizing fee satisfies*

$$f^w \to c + \Delta_m - \frac{\mu_D}{2} < f^e.$$

Proposition 9 implies that the welfare-maximizing fee is now lower, reflecting that there is an additional negative cost of setting a high platform fee that pushes consumers to buy directly: it encourages more duplicative advertising by firms which is socially costly in this setting.

Finally, a direct comparison of the equilibrium fee with the welfare-maximizing fee for large n implies:

Proposition 10. (Comparison of fees) *In the limit as $n \to \infty$, we have*

$$f^* \geq f^w \;\Leftrightarrow \lambda\left(c - \Delta_s - \frac{\mu_D}{2}\right) \;\geq \Delta_m - \frac{\mu_D}{2}. \tag{17}$$

A greater μ_M only decrease the right-hand side of (17), while having no impact on the left-hand side. Meanwhile, the left-hand side increases in μ_D, while the right-hand side, which is equal to $\frac{\mu_D}{2} - \mu_M$, also increases in μ_D. This suggests that an increase in μ_D has a more ambiguous impact on fee comparison compared to the baseline model. It encourages firms to advertise, increasing consumers' awareness of the direct channel and counterbalances the need for a higher fee to improve efficiency.

Above we have treated the cost of advertising as a real cost to society. However, the amount spent on advertising by firms may instead represent, at least partially, a transfer from the firms advertising to publishers and other advertising platforms (e.g. Facebook and Google). If we multiply the cost of advertising in the welfare expression by the discount factor $0 \leq \delta \leq 1$, then following the same steps, we get

$$f^w \to c + \Delta_m - \frac{\delta \mu_D}{2}.$$

This implies in the extreme case that advertising expenditure is a pure transfer (so $\delta = 0$), $f^w = f^e$ as in the baseline setting.

In the Online Appendix we explore the extension of the model in this section in which M also needs to advertise to attract consumers. Arguably, if M is still emerging (and so relies on advertising), then it is less likely to be a candidate for regulation. Nevertheless, we do see some well-established platforms advertising aggressively against firms' own ads (e.g. food delivery platforms and hotel booking platforms). To capture this, we assume M also has to advertise to be discovered, and has access to the same advertising technology. Unfortunately, we cannot get a closed form solution for the welfare-maximizing fee even for sufficiently large n, but we are able to show that as $n \to \infty$, the equilibrium fee characterization remains the same as in the baseline. Moreover, we show that if (17) holds, the equilibrium fee is also higher than the welfare-maximizing fee in the limit.

6 Conclusion

This paper proposes a simple yet flexible framework for studying the regulation of the fee a platform charges to suppliers when transactions can be done both directly between firms and consumers and indirectly via the platform. Taking into account that suppliers have lower markups on the platform than in the direct channel, we find the efficient fee exceeds the platform's marginal cost by the difference in markups across the two channels, which eliminates the otherwise excessive use of the platform by consumers. We also explore the conditions for a monopoly platform's fee to exceed this efficient benchmark, which loosely speaking require that the surplus differential created by the platform is high relative to the markup differential. We extend the framework to accommodate other important market elements such as consumer search and supplier advertising. When consumers need to search for products, the presence of showrooming lowers the efficient fee given excessive use of the platform is less of an issue. The need for suppliers to advertise to attract business in the direct channel is another reason the efficient fee may be lower, so as to drive more transactions to the platform and thereby reduce the extent of duplicative advertising done by suppliers.

There are several other factors beyond those that we explored that could drive a distortion between the platform's optimal fee and the socially optimal fee. Three important ones are (i) if there is a fixed cost for suppliers to participate on the platform, there can be excessive or insufficient entry onto the platform; (ii) elastic demand by consumers would push towards the platform's optimal fee being excessive, but the direction of any such distortion is no longer obvious if we also allow suppliers to make investment decisions that respond to their margins; and (iii) positive network effects between participating consumers and suppliers mean that both the socially optimal and privately optimal choice of fee would tend to be lower. None of these effects therefore necessarily lead to a distortion between the platform's and planner's optimal fee in one direction or the other, but they are interesting (and challenging) extensions to consider for future work on this topic.

Another important direction for future research is to consider how our analysis would be affected by platform competition, so that it can capture the case of two large marketplaces that dominate a particular vertical. To the extent consumers tend to singlehome and suppliers multihome, the competitive bottleneck logic should apply, and our analysis should flow through. Even allowing for one platform to intensify competition between suppliers more than the other, our simple markup rule for regulating platform pricing should still apply at the level of each platform based on the principle that for maximum efficiency, the relative prices consumers face for using different channels ought to only reflect differences in the platforms' costs.

Appendix.

Proof of Proposition 1. Note that the left-hand side (LHS) of (2) strictly increases from 0 to ∞ when f increases from 0 to ∞. The right-hand side (RHS) of (2) weakly decreases in f given λ is weakly decreasing (from the assumed weakly increasing hazard rate). Moreover, it decreases from a value greater than c to c when f increases from 0 to ∞. As a result there is a unique solution to (2) which satisfies the stated condition. ■

Proof of Proposition 3. Compare $\widehat{f}$ in (2) and f^e in (4). Notice that the term $\lambda(f-\Delta_s-\Delta_m)$ in (2) strictly decreases in f unless $H(b)$ is an exponential distribution in which case λ is a constant. Then we must have

$$\begin{aligned} f^e \leq \widehat{f} \quad &\Leftrightarrow \quad c+\lambda(f^e-\Delta_s-\Delta_m) \geq c+\lambda(\widehat{f}-\Delta_s-\Delta_m) = \widehat{f} \geq f^e = c+\Delta_m \\ &\Leftrightarrow \quad \lambda(c-\Delta_s) \geq \Delta_m \end{aligned}$$

Totally differentiate (2), and we get

$$\frac{d\widehat{f}}{d\Delta_m} = \frac{-\lambda'}{1-\lambda'}.$$

Since $\lambda' < 0$, we have $0 \leq \frac{d\widehat{f}}{d\Delta_m} < 1$. ■

Proof of Proposition 4. For the sequential search model, we have $\frac{\partial L}{\partial x_M} = -\epsilon + \frac{\partial\left(\frac{1-G(x_M)}{g(x_M)}\right)}{\partial x_M}$, which equals $-\epsilon + \epsilon_G$ for the GPD case. The result follows given the effect of an increase in search cost on a channel has the opposite effect (in terms of direction) as a change in the corresponding reservation value level x, and moreover, the effect of a change in x_D is the same (with opposite sign) to the effect of a change in x_M.

For the Salop circular-city model, see the Online Appendix for more details. Based on the resulting Δ_s and Δ_m, which are stated in the main text, we have $\frac{\partial L}{\partial t} = -\epsilon\left(\frac{1}{4}-\frac{1}{4n}\right)+\frac{1}{2}+\frac{1}{n}$, which is positive given $\epsilon \leq 0$. Moreover, $\frac{\partial L}{\partial n} = -\epsilon\left(\frac{t}{4n^2}\right) - \left(\frac{t}{n^2}\right) < 0$ iff $\epsilon > -4$.

For the Perloff-Salop model, see the Online Appendix for more details. Evaluating (9) using (5)-(6), we get

$$\frac{\partial L}{\partial \beta} < 0 \quad \Leftrightarrow \quad \epsilon > -\frac{\frac{1}{n_D\int_{\underline{\xi}}^{\bar{\xi}} g(\xi)dG(\xi)^{n_D-1}} - \frac{1}{n\int_{\underline{\xi}}^{\bar{\xi}} g(\xi)dG(\xi)^{n-1}}}{\int_{\underline{\xi}}^{\bar{\xi}} \xi dG(\xi)^n - \int_{\underline{\xi}}^{\bar{\xi}} \xi dG(\xi)^{n_D}}.$$

It is clear the RHS of the last inequality is negative given that $n_D \leq n$.

Moreover, we have

$$\frac{\partial \Delta_s}{\partial n_D} = -\beta \int_{\underline{\xi}}^{\overline{\xi}} \left(\xi g(\xi) G(\xi)^{n_D-1} + \xi n_D (n_D-1) G(\xi)^{n_D-2} g(\xi)^2 \right) d\xi < 0$$

$$\frac{\partial \Delta_m}{\partial n_D} = \frac{-\beta \left(\begin{array}{c} \int_{\underline{\xi}}^{\overline{\xi}} (n_D-1) g(\xi)^2 G(\xi)^{n_D-2} d\xi \\ + n_D \int_{\underline{\xi}}^{\overline{\xi}} g(\xi)^2 \left(G(\xi)^{n_D-2} + (n_D-1)(n_D-2) G(\xi)^{n_D-3} g(\xi) \right) d\xi \end{array} \right)}{\left(n_D \int_{\underline{\xi}}^{\overline{\xi}} (n_D-1) g(\xi)^2 G(\xi)^{n_D-2} d\xi \right)^2} < 0.$$

So using (9), we have

$$\frac{\partial L}{\partial n_D} \geq 0$$

$$\Leftrightarrow \quad \epsilon \geq \frac{-\left(\begin{array}{c} \int_{\underline{\xi}}^{\overline{\xi}} (n_D-1) g(\xi)^2 G(\xi)^{n_D-2} d\xi \\ + n_D \int_{\underline{\xi}}^{\overline{\xi}} g(\xi)^2 \left(G(\xi)^{n_D-2} + (n_D-1)(n_D-2) G(\xi)^{n_D-3} g(\xi) \right) d\xi \end{array} \right)}{\left(\begin{array}{c} \left(n_D \int_{\underline{\xi}}^{\overline{\xi}} (n_D-1) g(\xi)^2 G(\xi)^{n_D-2} d\xi \right)^2 \\ \left(\int_{\underline{\xi}}^{\overline{\xi}} \left(\xi g(\xi) G(\xi)^{n_D-1} + \xi n_D (n_D-1) G(\xi)^{n_D-2} g(\xi)^2 \right) d\xi \right) \end{array} \right)} \equiv \widehat{\epsilon}(n_D)$$

Notice that the $\widehat{\epsilon}(n_D)$ is always negative. Moreover, $\Delta_s(n_D, n) = -\Delta_s(n, n_D)$ and $\Delta_m(n_D, n) = -\Delta_m(n, n_D)$, which is why (12) is the same as (11) with the inequality reversed. ■

Proof of Proposition 6. Differentiating W^{us} with respect to f we get

$$\frac{dW^{us}}{df} = w_f \Delta_m h\left(f - (\Delta_s + \Delta_m)\right) - w_c \left(1 - H\left(f - (\Delta_s + \Delta_m)\right)\right).$$

Define f where

$$w_c \lambda \left(f - (\Delta_s + \Delta_m)\right) = w_f \Delta_m$$

as the unique value f^{us}. We have $\frac{dW^{us}}{df} = h\left(f - (\Delta_s + \Delta_m)\right)\left[w_f \Delta_m - w_c \lambda \left(f - (\Delta_s + \Delta_m)\right)\right]$. Recall λ is decreasing in f. Therefore, if $f < f^{us}$, $w_c \lambda \left(f - (\Delta_s + \Delta_m)\right) > w_f \Delta_m$ and $\frac{dW^{us}}{df} < 0$, and vice-versa when $f > f^{us}$. So f^{us} characterizes a minimum. The planner will either want to set f as low as possible (subject to M wanting to operate) or as high as possible so that no consumer would go to M. Comparing W^{us} across the two extreme values of f assuming $w_c = w_f$ we get (13).

Proof of Proposition 7. Since each firm is infinitesimal, they cannot affect the measure of consumers who choose to visit M. So they take the distribution of consumers over the two channels as constant. That is, each firm i maximizes

$$p_D^i\left(\alpha(1-H(\cdot))\frac{1-G(x_M-p_D+p_D^i)}{1-G(x_M)}+H(\cdot)\frac{1-G(x_D-p_D+p_D^i)}{1-G(x_D)}\right). \tag{18}$$

We implicitly assume that consumers who can showroom indeed want to switch in equilibrium. This requires $p_D \leq f^* + \mu_M$, which we need to verify when solving for f^*. This needs to be true for the planner's solution too as we assume α consumers switch to buy directly at the end.

Consumers, without knowing whether they can switch later (but taking into account the probability they will be able to), join M if and only if

$$\phi_M - (1-\alpha)(f+\mu_M) - \alpha p_D + b \geq \phi_D - p_D$$

or equivalent,

$$b \geq -\Delta_s + (1-\alpha)(f-\widetilde{\Delta}_m),$$

where $\widetilde{\Delta}_m = p_D - \mu_M$. Notice that p_D is a function of f as $H(\cdot) = H(-\Delta_s + (1-\alpha)(f-\widetilde{\Delta}_m))$.

The FOC of firm i, after imposing symmetry, gives

$$p_D = \frac{((1-\alpha)H(\cdot)+\alpha)\mu_D\mu_M}{(1-H(\cdot))\lambda\mu_D + H(\cdot)\mu_M}$$

Notice that p_D is only implicitly defined here as $H(\cdot)$ contains p_D. It is straightforward to verify that $\mu_M < p_D < \mu_D$.

Given that p_d is only implicitly defined, the determination of the equilibrium fee by M is complicated, and we don't have a nice characterization of it. Rather, we focus on characterizing the efficient fee. The social planner chooses f to maximize

$$\int_{-\Delta_s+(1-\alpha)(f-\widetilde{\Delta}_m)}^{\bar{b}} (\phi_M + b - (1-\alpha)c)\, dH(b) + \int_{\underline{b}}^{-\Delta_s+(1-\alpha)(f-\widetilde{\Delta}_m)} \phi_D dH(b).$$

The FOC gives the welfare-maximizing fee level

$$f^w = c + \widetilde{\Delta}_m.$$

Notice that f^w is implicitly defined as $\widetilde{\Delta}_m$ is affected by f, and also f^w is affected by α only through $\widetilde{\Delta}_m$. However, we know that $f^w < f^e$ as $\widetilde{\Delta}_M < \Delta_M$ for a given f.

Finally, we need to check that $p_D(f^w) < f^w + \mu_M = c + \widetilde{\Delta}_m + \mu_M = c + p_D(f^w)$, so that consumers indeed want to switch to buy directly. This is true by construction.

Notice that in (18), the weight of the first term in the bracket, $\alpha(1 - H(-\Delta_s + (1-\alpha)(f^w - \widetilde{\Delta}_m))) = \alpha(1 - H(-\Delta_s + (1-\alpha)c))$, increases in α, while the weight of the second term, $H(-\Delta_s + (1-\alpha)c)$, decreases in α. The maximization of the first term (multiplied by p_d^i) yields a solution equal to μ_M, while the maximization of the second term (multiplied by p_d^i) yields a solution equal to μ_D. Since $\mu_D \geq \mu_M$, we know that p_D decreases in α, and thus $f^w = c + \tilde{\Delta}_m = c + p_D - \mu_M$ decreases in α. ■

Proof of Lemma 1. (15) can be rewritten as

$$\frac{(1-(1-a)^n)}{(a)^2} = \frac{nc_a}{\mu_D H(f - \Delta_s - \Delta_m)}. \tag{19}$$

We first show that the LHS of (19) is strictly decreasing in a. The LHS can be written as

$$\exp[\ln[1-(1-a)^n] - \ln[a^2]].$$

Take the derivative of $\ln[1-(1-a)^n] - \ln[a^2]$ with respect to a and we get

$$\frac{(na + 2(1-a))(1-a)^{n-1} - 2}{a(1-(1-a)^n)}.$$

The sign of the derivative is the same as the sign of $L(a) \equiv na + 2(1-a))(1-a)^{n-1} - 2$ for all $a \in (0,1)$. We further have $\lim_{a\to 0} L(a) = 0$, $\lim_{a\to 1} L(a) = -2$, and $L'(a) = -((n-2)a+1)n(1-a)^{n-2} < 0$. So $\ln[1-(1-a)^n] - \ln[a^2]$ strictly decreases in a, and so does the LHS of (19). The LHS of (19) goes to ∞ when $a \to 0$ and 1 when $a \to 1$. By Assumption 1 and $0 \leq H(\cdot) \leq 1$, we know that the RHS of (19) is a constant greater than 1. We then can conclude there is always a unique $a \in (0,1)$ satisfying (19), provided $H(f - \Delta_s - \Delta_m) \neq 0$. ■

Proof of Lemma 2. Equation (15) can be re-written as

$$1 - (1-a)^n - xn(a)^2 = 0,$$

where $x \equiv \frac{c_a}{\mu_D H(f - \Delta_s - \Delta_m)}$. Let $a^*(n) \in [0,1]$ be the solution to this equation for a. We instead work with $y \equiv n(a^*(n))^2 \in [0,n]$ and

$$f(y,n) = 1 - \left(1 - \sqrt{\frac{y}{n}}\right)^n - xy.$$

We will also treat n as a continuous variable.

We have $f(0,n) = 0$ and $f(n,n) = 1 - xn$, with the latter being strictly negative given Assumption 1. Routine calculations show that $\frac{\partial^2 f}{\partial y^2} < 0$ for every $y \in (0,n)$ and $\lim_{y \to 0} \frac{\partial f}{\partial y} = \infty$. Hence, there exists a unique $y(n) \in (0,n)$ such that $f(y(n),n) = 0$. This is because, since f is strictly concave in y and $f(y,n)$ is strictly positive for y small, f must cross the horizontal axis from above. Moreover, $\left.\frac{\partial f}{\partial y}\right|_{y=y(n)} < 0$. Clearly, $a^*(n) = \sqrt{\frac{y(n)}{n}}$.

Assume for a contradiction that the function $y(n)$ is not bounded. Then, there exists a sequence $(n^k)_{k \geq 0}$ such that $y(n^k) \underset{k \to \infty}{\longrightarrow} \infty$. This gives the contradiction

$$0 = f(y(n^k), n^k) \leq 1 - xy(n^k) \underset{k \to \infty}{\longrightarrow} -\infty.$$

Hence, there exists $\overline{y} > 0$ such that $y(n) \leq \overline{y}$ for every n. It follows that $\lim_{n \to \infty} a^*(n) = 0$.

Routine calculations show that $\partial f / \partial n$ has the same sign as

$$-\frac{1}{2}\sqrt{\frac{y}{n}} - \left(1 - \sqrt{\frac{y}{n}}\right) \ln \left(1 - \sqrt{\frac{y}{n}}\right).$$

It follows that $\left.\frac{\partial f}{\partial n}\right|_{y=y(n)}$ has the same sign as

$$\begin{aligned} -\frac{1}{2}a^*(n) - (1 - a^*(n)) \ln(1 - a^*(n)) &= -\frac{1}{2}a^*(n) - (1 - a^*(n))(-a^*(n)) + o(a^*(n)) \\ &= \frac{1}{2}a^*(n) + o(a^*(n)) = \frac{1}{2}a^*(n) \left[1 + o(1)\right]. \end{aligned}$$

Therefore, $\left.\frac{\partial f}{\partial n}\right|_{y=y(n)}$ is strictly positive for n sufficiently high. It follows that

$$y'(n) = -\left.\frac{\partial f / \partial n}{\partial f / \partial y}\right|_{y=y(n)}$$

is strictly positive for n high enough. Therefore, there exists n^0 such that $y(\cdot)$ is strictly increasing on $[n^0, \infty)$. This implies that $\ell \equiv \lim_{n \to \infty} y(n)$ exists and is strictly positive. Moreover, since the function $y(\cdot)$ is bounded, ℓ is finite.

We have

$$\begin{aligned}
n\ln(1-a^*(n)) &= n(-a^*(n))\frac{\ln(1-a^*(n))}{-a^*(n)} \\
&= -\sqrt{y(n)n}\frac{\ln(1-a^*(n))}{-a^*(n)} \\
&= \underbrace{-\sqrt{\ell n}}_{\underset{n\to\infty}{\longrightarrow}-\infty}\underbrace{\sqrt{\frac{y(n)}{\ell}}}_{\underset{n\to\infty}{\longrightarrow}1}\underbrace{\frac{\ln(1-a^*(n))}{-a^*(n)}}_{\underset{n\to\infty}{\longrightarrow}1}\underset{n\to\infty}{\longrightarrow}-\infty.
\end{aligned}$$

It follows that

$$(1-a^*(n))^n = \exp\left[n\ln(1-a^*(n))\right]\underset{n\to\infty}{\longrightarrow}0.$$

Moreover,

$$n(1-a^*(n))^{n-1} = n\exp[n\ln(1-a^*(n)) - \ln(1-a^*(n))]\underset{n\to\infty}{\longrightarrow}0.$$

Note also that, for every n,

$$0 = f(y(n),n) = 1-(1-a^*(n))^n - xy(n)\underset{n\to\infty}{\longrightarrow}1-x\ell.$$

Hence, $\ell = \lim_{n\to\infty} y(n) = 1/x$. This implies that

$$a^*(n)\approx\sqrt{\frac{1}{nx}},$$

when n is sufficiently large. Then,

$$na^*(n)\underset{n\to\infty}{\longrightarrow}\infty.$$

We have proved the results in Lemma 2. ■

Proof of Proposition 8. Provided second-order conditions hold and the solution is interior, f^* is characterized by the first-order condition

$$\begin{aligned}
&(1-(1-a^*)^n)(1-H(f-\Delta_s-\Delta_m)) + (1-a^*)^n(1-H(f+\mu_M-\phi_M)) \\
&+(f-c)\bigg(n(1-a^*)^{n-1}(1-H(f-\Delta_s-\Delta_m))\frac{da^*}{df} - (1-(1-a^*)^n)h(f-\Delta_s-\Delta_m) \\
&-n(1-a^*)^{n-1}(1-H(f+\mu_M-\phi_M))\frac{da^*}{df} - (1-a^*)^n h(f+\mu_M-\phi_M)\bigg) = 0.
\end{aligned}$$

Applying the limits in Lemma 2 to the above, the FOC above implies $\lim_{n\to\infty} f^* \to \widehat{f}$. ■

Proof of Proposition 9. Given our assumptions, f^w is characterized by the first-order condition

$$\begin{aligned} & n(1-a^*)^{n-1}\frac{da^*}{df}\int_{f-\Delta_s-\Delta_m}^{\bar{b}}(\phi_M+b-c)dH(b) \\ & -(1-(1-a^*)^n)(\phi_M+f-c-\Delta_s-\Delta_m)h(f-\Delta_s-\Delta_m) \\ & -n(1-a^*)^{n-1}\frac{da^*}{df}\int_{f+\mu_M-\phi_M}^{\bar{b}}(\phi_M+b-c)dH(b) \\ & -(1-a^*)^n(f-c+\mu_M)h(f+\mu_M-\phi_M) \\ & +n(1-a^*)^{n-1}\frac{da^*}{df}\phi_D H(f-\Delta_s-\Delta_m) \\ & +(1-(1-a^*)^n)\phi_D h(f-\Delta_s-\Delta_m)-nc_a a^*\frac{da^*}{df} \\ = & \ 0. \end{aligned}$$

To evaluate this FOC in the limit as $n\to\infty$, we first prove the equilibrium a^* satisfies

$$\frac{da^*}{df}\to 0 \ \text{ and } \ na^*\frac{da^*}{df}\to\frac{\mu_D h(f-\Delta_s-\Delta_m)}{2c_a}$$

as $n\to\infty$. Based on equation (16) and the limit results in Lemma 2, when $n\to\infty$,

$$\frac{da^*}{df}=\frac{(1-(1-a^*)^n)\,\mu_D h\,(f-\Delta_s-\Delta_m)}{2c_a a^* n-n\,(1-a^*)^{n-1}\,\mu_D H\,(f-\Delta_s-\Delta_m)}\to 0, \tag{20}$$

as $1-(1-a^*)^n\to 1$, $a^*n\to\infty$, and $n(1-a^*)^{n-1}\to 0$. Similarly,

$$na^*\frac{da^*}{df}=\frac{(1-(1-a^*)^n)\,\mu_D h\,(f-\Delta_s-\Delta_m)}{2c_a-\frac{(1-a^*)^{n-1}}{a^*}\mu_D H\,(f-\Delta_s-\Delta_m)}\to\frac{\mu_D h(f-\Delta_s-\Delta_m)}{2c_a},$$

since $\frac{(1-a^*)^{n-1}}{a^*}\approx\sqrt{nx}\exp\left[-\frac{n-1}{\sqrt{nx}}\right]\to 0$, where $x\equiv\frac{c_a}{\mu_D H(f-\Delta_s-\Delta_m)}$ is a constant. Now applying these results together with Lemma 2, the first-order condition becomes

$$\left(-(\phi_M+f-c-\Delta_s-\Delta_m)+\phi_D-\frac{\mu_D}{2}\right)h(f-\Delta_s-\Delta_m)=0,$$

from which we obtain the result. ■

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Online Appendix: Regulating platform fees

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We present some extensions referred to in the main paper.

A Percentage fees

Suppose now firms face a marginal cost d, and the platform sets a percentage fee r, where $0 < r < 1$. In the case with transaction fees, a firm maximizes $(p_M^i - f - d)D_i^M(p_M^i, p_M)$ and the symmetric equilibrium price is $p_M = f + d - \frac{D_M^i(p_M, p_M)}{D_M^{i\prime}(p_M, p_M)}$. In the case of percentage fees, firm i maximizes $((1-r)p_M^i - d)D_M^i(p_M^i, p_M)$ and the symmetric equilibrium price is $p_M = \frac{d}{1-r} - \frac{D_M^i(p_M, p_M)}{D_M^{i\prime}(p_M, p_M)}$. Therefore, as long as $\mu_M = -\frac{D_M^i(p_M, p_M)}{D_M^{i\prime}(p_M, p_M)}$ is a constant that is independent of p_M, we can write $p_m = d + \frac{r}{1-r}d + \mu_M$ in the case of percentage fee. Note $p_d = d + \mu_D$ as before.

As in our baseline setting, consumers will join M iff

$$\phi_M - p_m + b \geq \phi_D - p_d$$

or equivalently

$$b \geq f - \Delta_s - \Delta_m,$$

where we have redefined $f \equiv \frac{r}{1-r}d$ given that f is one-to-one increasing in r. Note that with this definition, we have $p_m = d + f + \mu_M$, so that it takes the same form as our baseline model.

The welfare maximizing choice of f is then determined by the same consideration as before, so

$$f^e = c + \Delta_m.$$

This implies the efficient percentage fee is

$$r^e = \frac{c + \Delta_m}{c + d + \Delta_m}.$$

In contrast, the platform chooses r to maximize

$$(rp_m - c)\left(1 - H\left(\frac{r}{1-r}d - \Delta_s - \Delta_m\right)\right)$$

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or using the definition of f above, sets f to maximize

$$\Pi_M = \left(f - c + \frac{f\mu_M}{d+f} \right) \left(1 - H\left(f - \Delta_s - \Delta_m\right)\right). \tag{21}$$

Note if $\mu_M = 0$, which could for instance arise in certain microfoundations if $n \to \infty$, this is the same expression for profit maximization as in the baseline. In this case, percentage fees lead to equivalent results as the case with fixed per-unit fees.

The FOC corresponding to maximizing (21) is

$$\frac{f + \frac{f\mu_M}{d+f} - c}{1 + \frac{d\mu_M}{(d+f)^2}} = \frac{1 - H\left(f - \Delta_s - \Delta_m\right)}{h\left(f - \Delta_s - \Delta_m\right)}.$$

The LHS is strictly increasing in f. By the increasing hazard rate, the RHS is decreasing in f. The LHS goes to $\frac{-c}{1+(\mu_M/d)} < 0$ when $f \to 0$ and ∞ when $f \to \infty$. The RHS goes to a positive value when $f \to 0$ and 0 when $f \to \infty$. So there is a unique f^* solving the FOC. Then, we can conclude that if and only if the RHS is greater than the LHS at $f^e = c + \Delta_m$, then $f^* \geq f^e$, i.e.,

$$f^* \geq f^e \quad \Leftrightarrow \quad \frac{1 - H\left(c - \Delta_s\right)}{h\left(c - \Delta_s\right)} \geq \frac{\Delta_m + \frac{(c+\Delta_m)\mu_M}{d+c+\Delta_m}}{1 + \frac{d\mu_M}{(d+c+\Delta_m)^2}}. \tag{22}$$

We have $f^e < f^*$ otherwise. Note provided $\mu_M > 0$, the term on the RHS of the inequality is higher than in the fixed per-unit fee case, where it was just Δ_m, implying that the condition for the profit-maximizing fee to exceed the efficient fee is now harder to satisfy. Thus, even though the efficient fee is equivalent under percentage and fixed per-unit fees, the percentage fee that maximizes M's profit is lower than the corresponding one under fixed per-unit fees.

The comparative statics of the comparison on the RHS in (22) with respect to Δ_s are similar to before; a greater Δ_s will make it easier for $f^* \geq f^e$. After substituting in $\Delta_m = \mu_D - \mu_M$ into the above expression, it can be confirmed that the RHS is increasing in μ_D and the RHS is decreasing in μ_M given that $\mu_D \geq \mu_M$, so a higher μ_D shifts the tradeoff towards M's fee being too high and a higher μ_M shifts the tradeoff towards M's fee being too low. Thus, the direction of the effect on Δ_s and μ_M and μ_D are preserved from the case with a fixed per-unit fee.

B Salop circular-city specification

Consider the application of the baseline model to the Salop circular-city model of competition. Assume there are a finite number $n \geq 2$ of firms which are equally located around a circle according to the standard Salop circular city model, with measure one of consumers uniformly located around the same circle. Consumers are willing to pay v for one unit of the good, but face a mismatch cost parameter of t. We assume consumers' value for the product is large enough.

Assumption 2. *We assume $t < v$.*

We assume each consumer can only visit one channel, either one of the firms directly or M. After selecting a firm to visit directly, the consumer observes this firm's price and location (so match value), and then decides whether to buy. Alternatively, if the consumer chooses to visit M, after observing price and location information on all listed firms, she decides which one of them to buy from.

Consider the direct market. Expecting that locations are random and that prices are symmetric, consumers randomly choose a firm. This means a consumer's visiting choice does not signal any information about her location on the Salop circle relative to the firm, and therefore her location can be viewed as being uniformly distributed on the Salop circle from the firm's perspective. After choosing a firm, the consumer will eventually buy from the firm if and only if $v - p - tx \geq 0$, or equivalently $x \leq \frac{v-p}{t}$. In addition, given firms and measure one of consumers are located along a circle of circumference one, the shortest distance between a consumer and a firm cannot exceed $\frac{1}{2}$. So the firm behaves as if it is a monopolist in the direct market and sets price to solve the following maximization problem

$$\max_{p} \left\{ 2p \min \left\{ \frac{v-p}{t}, \frac{1}{2} \right\} \right\}.$$

The symmetric equilibrium price and so markup $\mu_D = v - \frac{t}{2}$ given our assumption that $v \geq t$, and $\phi_D = v - \frac{t}{4}$.

Next, consider firm competition on M. Ignoring the platform's fee f, the standard competitive price that the n firms set when they compete and the market is covered (i.e. $\frac{t}{n}$) will be no higher than the standard monopoly price each firm would set when it prices as a local monopolist (i.e. $\frac{v}{2}$), given $v \geq t$ and $n \geq 2$. As we will show, even in the case the platform endogenously sets its fee f, in the case b is distributed uniformly, the assumption $v \geq t$ rules out the case that each firm on the platform behaves as a local monopoly with some consumers who visit M not being served, thereby guaranteing that all these consumers will buy in the resulting equilibrium.

On M, the competitive price when the market is covered will be $f + \frac{t}{n}$, so $\mu_M = \frac{t}{n}$, and the gross surplus of a consumer will be $\phi_M = v - \frac{t}{4n}$. In the competitive equilibrium price range, we have $\Delta_s = \frac{t}{4} - \frac{t}{4n} > 0$ and $\Delta_m = v - \frac{t}{2} - \frac{t}{n} > 0$ given $v > t$. For this case to arise (as opposed to a kinked equilibrium or a monopoly equilibrium where the firms prices are pinned down by different constraints), we require $f \leq v - \frac{3t}{2n}$. This condition also ensures that all consumers will buy after visiting M. We then need to determine M's optimal fee to see if it satisfies this constraint.

Lemma 3. *If $f < v - \frac{3t}{2n}$, firms set the competitive price $p_M = f + \frac{t}{n}$ on M.*

Anticipating the competitive equilibrium, consumers join M iff $b \geq f - \Delta_s - \Delta_m = f + \frac{5t}{4n} + \frac{t}{4} - v$. For consumers receiving ads from at least one firm, the fraction of them going through M will be $1 - H(f - \Delta_s - \Delta_m)$ and the fraction going directly will be $H(f - \Delta_s - \Delta_m)$.

Proof of Lemma 3. If f exceeds f^*, then the alternatives are:

- Kinked equilibrium. If $v - \frac{3t}{2n} \leq f < v - \frac{t}{n}$, each firm charges $p_m = v - \frac{t}{2n}$ and all consumers visiting M end up buying. In particular, the consumers who are at the exact middle of any two firms are indifferent about buying.

- Monopoly equilibrium. If $f \geq v - \frac{t}{n}$, each firm charges $p_m = \frac{v+f}{2}$ and consumers with $x < \frac{v-f}{2t}$ buy after visiting M. The total demand by consumers conditional on visiting M is $\frac{n(v-f)}{t}$.

We show M cannot do better setting a fee that induces a kinked equilibrium or a monopoly equilibrium. Consider now M setting a higher fee. In the kinked equilibrium range, consumers join M iff $b + v - \frac{t}{4n} - \left(v - \frac{t}{2n}\right) \geq \frac{t}{4}$ or equivalently, $b \geq \frac{t}{4n} + \frac{t}{4} = \frac{(1+n)t}{4n}$. This compares to in the competitive equilibrium range where the condition is $b \geq f + \frac{5t}{4n} + \frac{t}{4} - v$. But since $f < v - \frac{t}{n}$ for the kinked equilibrium range, we know that more consumers join under the competitive equilibrium range than the kinked equilibrium range for any f that is in the competitive or kinked equilibrium range. This implies M must be better off setting its unconstrained optimal fee under the competitive equilibrium range.

Finally, in the monopoly equilibrium range, if M treats consumer participation as constant in f, it would maximize its profit by solving

$$\max_{f} \left\{(f-c)\left(\frac{n(v-f)}{t}\right)\right\} \text{ subject to } f \geq v - \frac{t}{n}.$$

The solution is $f = \frac{v}{2}$ if $\frac{t}{n} \geq \frac{v}{2}$ and $f = v - \frac{t}{n}$ otherwise. Since $v > t$, the solution must be $f = v - \frac{t}{n}$ which coincides with the one inducing the kinked equilibrium. Taking into account that consumer participation is decreasing in f (reflecting that the firms' prices are increasing in f in this range), just reinforces that the constraint $f \geq v - \frac{t}{n}$ must be binding. Thus, in this range M will want to set $f = v - \frac{t}{n}$, which corresponds to the solution with the kinked equilibrium with $f = v - \frac{t}{n}$. Since we already showed this involves lower profit than in the competitive equilibrium range, the monopoly equilibrium solution must be worse for M. ■

C Perloff-Salop specification

We consider how the discrete-choice framework in Perloff and Salop (1985) fits our setup. There are n ex-ante symmetric firms and a unit mass of consumers. Each consumer can only encounter some fixed number $n_d = 1, 2, ..., n$ firms in the direct market. After, but not before, a consumer visits the direct market, she can see prices and match values of the n_d firms. So a unilateral change in direct price does not change consumers' visiting decisions. The identity of the n_d firms are randomly distributed across consumers. The utility a consumer can get from buying at firm i is

$$u^i = v - p^i + \beta\xi^i,$$

where ξ^i is distributed according to a CDF G on $[\underline{\xi}, \overline{\xi}]$, which is iid across firms and consumers. The parameter β measures consumers' taste for product differentiation. The consumer's outside option is assumed to be 0. We assume v is great enough compared to $\max\{\widehat{f} + \mu_M, \mu_D\} - \beta\underline{\xi}$ such that the market is always fully covered.

In the direct market, the demand of a firm i when it charges p_D^i while all other firms charge p_D is

$$D^i(p_D^i, p_D) = \Pr\left[u^i \geq \max_{j \neq i} u^j\right] = \int_{\underline{\xi}}^{\overline{\xi}} \left(1 - G\left(\xi + \frac{p_D^i - p_D}{\beta}\right)\right) dG(\xi)^{n_D - 1}.$$

Firm i chooses p_D^i to maximize $p_D^i D^i(p_D^i, p_D)$. The FOC and symmetry imply

$$\int_{\underline{\xi}}^{\overline{\xi}} (1 - G(\xi))\, dG(\xi)^{n_D - 1} - \frac{p_D}{\beta} \int_{\underline{\xi}}^{\overline{\xi}} g(\xi) dG(\xi)^{n_D - 1} = 0.$$

In a symmetric equilibrium, the first term is equal to $1/n_D$.[3] The symmetric equilibrium price then is

$$p_D = \mu_D = \frac{\beta}{n_D \int_{\underline{\xi}}^{\overline{\xi}} g(\xi) dG(\xi)^{n_D - 1}}.$$

A consumer's expected gross surplus of visiting the direct market is

$$\phi_D = v + \beta \int_{\underline{\xi}}^{\overline{\xi}} \xi dG(\xi)^{n_D}.$$

Similarly, the symmetric equilibrium price on M is

$$p_M = f + \mu_M = f + \frac{\beta}{n \int_{\underline{\xi}}^{\overline{\xi}} g(\xi) dG(\xi)^{n-1}}$$

and a consumer's expected gross surplus of visiting M is

$$\phi_M = v + \beta \int_{\underline{\xi}}^{\overline{\xi}} \xi dG(\xi)^{n}.$$

Clearly, $\phi_M > \phi_D$ as $G(\xi)^n$ first-order stochastically dominates $G(\xi)^{n_D}$. Zhou (2017) shows that when $g(\xi)$ is log-concave, $\mu_M < \mu_D$ and the mark-up converges to zero when the number of firms goes to infinity. The expressions in (5)-(6) follow directly from the above results.

Our specification in the direct market corresponds to the case of symmetry consideration sets and zero latent demand in Gomes and Mantovani (2022). If the latent demand is not zero, consumers who attend the direct market will either know n_D firms or no firms.

[3]If there exist some consumers who do not have access to the direct market, then we will have a latent demand $D_0 \in (0, 1)$. We then replace $1/n$ by $(1 - D_0)/n$.

D Emerging platforms

Suppose M also needs to advertise to attract consumers. M uses the same advertising technology, so its cost of choosing advertising intensity a_M is $\frac{c_a(a_M)^2}{2}$ and consumers can visit M only if they have received an ad from M. The model otherwise works in the same way as that in Section 5. In terms of timing, M chooses its advertising level a_M at the same time as the firms also set their advertising levels. Moreover, only consumers who receive ads from M observe its fee level.

An individual firm's profit is determined by five parts. One is the case that consumers receive both M's ad and a firm's direct ad, and choose to purchase via the firm. If other firms have advertised at the level a, this is

$$\frac{a_i\left(1-(1-a)^n\right)}{an}a_M\mu_D H\left(f-\Delta_s-\Delta_m\right).$$

The second is that consumers do not receive an ad from M but receive an ad from a firm and choose to go to the firm. This is

$$\frac{a_i\left(1-(1-a)^n\right)}{an}\left(1-a_M\right)\mu_D.$$

The third is the case that consumers receive both M's ad and some direct ad and choose to purchase from the firm via M. This is

$$\frac{1}{n}\left(1-(1-a)^n\right)a_M\mu_M\left(1-H\left(f-\Delta_s-\Delta_m\right)\right).$$

The fourth is that consumers do not receive an ad from any direct firm but receive an ad from M, and purchase from the firm via M. This is

$$\frac{1}{n}\left(1-a\right)^n a_M\mu_M\left(1-H\left(f+\mu_M-\phi_M\right)\right).$$

Costs are

$$\frac{c_a a_i^2}{2}.$$

Putting everything together, a firm's profit when it advertises at the level a_i, all other firms advertise at the level a, and M advertises at the level a_M is

$$\begin{aligned}\pi_i &= \frac{a_i\left(1-(1-a)^n\right)}{an}\mu_D\left(1-a_M\left(1-H\left(f-\Delta_s-\Delta_m\right)\right)\right) \\ &\quad +\frac{a_M\mu_M}{n}\begin{pmatrix} (1-a)^n\left(1-H\left(f+\mu_m-\phi_m\right)\right) \\ +\left(1-(1-a)^n\right)\left(1-H\left(f-\Delta_s-\Delta_m\right)\right)\end{pmatrix} \\ &\quad -\frac{c_a a_i^2}{2}.\end{aligned}$$

A firm's optimal a_i is the level of a solving

$$\frac{\left(1-(1-a)^n\right)}{n}\mu_D\left(1-a_M\left(1-H\left(f-\Delta_s-\Delta_m\right)\right)\right)=c_a a^2,$$

where we denote the solution of a to be a^*. The limit properties of the equilibrium level a^* are the same as in Lemma 2 with the only change being that now $x\equiv\frac{c_a}{\mu_D((1-a_M(1-H(f-\Delta_s-\Delta_m))))}$. Since by construction $0\le a_M\le 1$, all the limit properties in Lemma 2 go through. However, the expression of $\frac{da^*}{df}$ in (20) need to be rederived since now both a_M^* and a^* are affected by f.

Consider M's problem. For each consumer who receives M's ads, they also receive ads from at least one firm with probability $1-(1-a)^n$ and do not receive ads from any firm with probability $(1-a)^n$. In the former case, consumers choose to go to M if $b\ge f-\Delta_s-\Delta_m$. In the latter case, consumers choose to go to M if $b\ge f+\mu_M-\phi_M$. So the total transaction volume on M, conditional on receiving M's ads, is

$$T(f)\equiv\left(1-(1-a)^n\right)\left(1-H\left(f-\Delta_s-\Delta_m\right)\right)+(1-a)^n\left(1-H\left(f+\mu_M-\phi_M\right)\right).$$

Given f is set in the first stage, M and firms simultaneously choose advertising intensity. M will choose

$$a_M(f)=\arg\max_{a_M}\left\{a_M\left(f-c\right)T\left(f\right)-\frac{c_a a_M^2}{2}\right\}$$

so

$$a_M(f)=\min\left\{\frac{(f-c)T\left(f\right)}{c_a},1\right\}.$$

Suppose $a_M(f)$ is an interior value in $(0,1)$, so that $(f-c)T\left(f\right)-c_a a_M(f)=0$.

Now consider M's choice of f. M chooses f to maximize its profit

$$a_M (f-c) T(f) - \frac{c_a a_M^2}{2}.$$

The effect via a_M will be zero due to the Envelope theorem provided $\frac{da_M(f)}{df}$ is finite. Thus, the FOC is simply

$$a_M(f)\left(T(f) + (f-c)\frac{dT(f)}{df}\right) = 0,$$

and provided $a_M(f) \neq 0$, this reduces to

$$T(f) + (f-c)\frac{dT(f)}{df} = 0. \tag{23}$$

In order to derive f^*, we need to know $\frac{da^*}{df}$ in $\frac{dT(f)}{df}$. The two FOCs with respect to a and a_M are

$$\begin{aligned}
(1-(1-a^*)^n)\mu_D(1-a_M^*(1-H(f-\Delta_s-\Delta_m))) - nc_a(a^*)^2 = 0\\
(f-c)((1-(1-a^*)^n)(1-H(f-\Delta_s-\Delta_m)) + (1-a^*)^n(1-H(f+\mu_M-\phi_M))) - c_a a_M^* = 0.
\end{aligned}$$

Totally differentiating the two FOCs with respect to f we get

$$\begin{aligned}
&(1-(1-a^*)^n)\mu_D\left(-\frac{da_M^*}{df}(1-H(f-\Delta_s-\Delta_m)) + a_M^* h(f-\Delta_s-\Delta_m)\right)\\
&+n(1-a^*)^{n-1}\mu_D(1-a_M^*(1-H(f-\Delta_s-\Delta_m)))\frac{da^*}{df} - 2nc_a a^*\frac{da^*}{df} = 0,\\
&(1-(1-a^*)^n)(1-H(f-\Delta_s-\Delta_m)) + (1-a^*)^n(1-H(f+\mu_M-\phi_M))\\
&+(f-c)\Bigg(n(1-a^*)^{n-1}(1-H(f-\Delta_s-\Delta_m))\frac{da^*}{df} - (1-(1-a^*)^n)h(f-\Delta_m-\Delta_s)\\
&-n(1-a^*)^{n-1}(1-H(f+\mu_M-\phi_M))\frac{da^*}{df} - (1-a^*)^n h(f+\mu_M-\phi_M)\Bigg) - c_a\frac{da_M^*}{df} = 0.
\end{aligned}$$

Note that, when $n\to\infty$, $a^*\to 0$, $(1-a^*)^n\to 0$, and $n(1-a^*)^{n-1}\to 0$ still hold as in Lemma 2. As long as $\frac{da^*}{df}$ is finite, the total derivative of the FOCs in the limit reduce to

$$\mu_D\left(-\frac{da_M^*}{df}(1-H(f-\Delta_s-\Delta_m))+a_M^*h(f-\Delta_s-\Delta_m)\right)-2c_a\sqrt{\frac{n}{x}}\frac{da^*}{df}=0 \tag{24}$$

$$1-H(f-\Delta_s-\Delta_m))-(f-c)h(f-\Delta_s-\Delta_m)-c_a\frac{da_M^*}{df}=0. \tag{25}$$

Note that $x\equiv\frac{c_a}{\mu_D\left(\left(1-a_M^*(1-H(f-\Delta_s-\Delta_m))\right)\right)}$ is finite.

We can now characterize the equilibrium at the limit. From (24), as long as $\frac{da_M^*}{df}$ is finite, something we will confirm below, it must be that $\frac{da^*}{df}\to 0$ as $n\to\infty$, and as a result from (23), when $n\to\infty$,

$$f^*\to\widehat{f}.$$

Given $f^*\to\widehat{f}$, the equilibrium a_M^* at the limit is

$$a_M^*\to\frac{(\widehat{f}-c)(1-H(\widehat{f}-\Delta_m-\Delta_s))}{c_a}. \tag{26}$$

Finally, from (25), we know that

$$\frac{da_M^*}{df}\to\frac{1-H(f-\Delta_s-\Delta_m))-(f-c)h(f-\Delta_s-\Delta_m)}{c_a} \tag{27}$$

which is clearly finite.

Welfare given the equilibrium levels of a and a_M is

$$\begin{aligned}W \;=\;& a_M^*(1-(1-a^*)^n)\int_{f-\Delta_s-\Delta_m}^{\bar{b}}(\phi_M+b-c)\,dH\,(b)\\ &+a_M^*(1-a^*)^n\int_{f+\mu_M-\phi_M}^{\bar{b}}(\phi_M+b-c)\,dH(b)\\ &+(1-(1-a^*)^n)\phi_D\,(1-a_M^*\,(1-H\,(f-\Delta_s-\Delta_m)))\\ &-\frac{nc_a\,(a^*)^2}{2}-\frac{c_a\,(a_M^*)^2}{2}.\end{aligned}$$

We maximize W with respect to f. The FOC is

$$\left(\frac{da_M^*}{df}(1-(1-a^*)^n)+a_M^*n(1-a^*)^{n-1}\frac{da^*}{df}\right)\int_{f-\Delta_s-\Delta_m}^{\bar{b}}(\phi_M+b-c)dH(b)$$
$$-a_M^*(1-(1-a^*)^n)(\phi_M+f-\Delta_s-\Delta_m-c)h(f-\Delta_s-\Delta_m)$$
$$+\left(\frac{da_M^*}{df}(1-a^*)^n-a_M^*n(1-a^*)^{n-1}\frac{da^*}{df}\right)\int_{f+\mu_M-\phi_M}^{\bar{b}}(\phi_M+b-c)dH(b)$$
$$-a_M^*(1-a^*)^n(\mu_M+f-c)h(f+\mu_M-\phi_M)+n(1-a^*)^{n-1}\frac{da^*}{df}\phi_D(1-a_M^*(1-H(f-\Delta_s-\Delta_m)))$$
$$+(1-(1-a^*)^n)\phi_D\left(-\frac{da_M^*}{df}(1-H(f-\Delta_s-\Delta_m))+a_M^*h(f-\Delta_s-\Delta_m)\right)$$
$$-nc_aa^*\frac{da^*}{df}-c_aa_M^*\frac{da_M^*}{df}=0.$$

Taking the limit as $n\to\infty$, the FOC becomes

$$\frac{da_M^*}{df}\left(\int_{f-\Delta_s-\Delta_m}^{\bar{b}}(\Delta_s+b-c)dH(b)-c_aa_M^*\right)$$
$$-a_M^*\left(f-\Delta_m-c\right)h\left(f-\Delta_s-\Delta_m\right)-nc_aa^*\frac{da^*}{df}=0.$$

Substitute in the expressions of $\frac{da_M^*}{df}$ given in (27), a_M^* given in (26) but for an arbitrary f, and using that in the limit

$$\begin{aligned} nc_aa^*\frac{da^*}{df} &\approx c_a\sqrt{\frac{n}{x}}\frac{da^*}{df}=\frac{\mu_D}{2}\left(-\frac{da_M^*}{df}(1-H(f-\Delta_s-\Delta_m))+a_M^*h(f-\Delta_s-\Delta_m)\right)\\ &= \frac{\mu_D}{2c_a}\left(\begin{array}{c}-(1-H(f-\Delta_s-\Delta_m))-(f-c)h(f-\Delta_s-\Delta_m))(1-H(f-\Delta_s-\Delta_m))\\ +(f-c)(1-H(f-\Delta_m-\Delta_s))h(f-\Delta_s-\Delta_m)\end{array}\right)\end{aligned}$$

from (24) and the expression for a^*, we get that the FOC reduces to

$$\begin{aligned} &\left(\frac{1-H(f-\Delta_s-\Delta_m))-(f-c)h(f-\Delta_s-\Delta_m)}{c_a}\right)\left(\int_{f-\Delta_s-\Delta_m}^{\bar{b}}(\Delta_s+b-c)dH(b)-c_aa_M^*\right)\\ &-\frac{(f-c)(1-H(f-\Delta_m-\Delta_s))}{c_a}\left(f-\Delta_m-c\right)h\left(f-\Delta_s-\Delta_m\right)\\ &-\frac{\mu_D}{2}\left(\begin{array}{c}-\left(\frac{1-H(f-\Delta_s-\Delta_m))-(f-c)h(f-\Delta_s-\Delta_m)}{c_a}\right)(1-H(f-\Delta_s-\Delta_m))\\ +\frac{(f-c)(1-H(f-\Delta_m-\Delta_s))}{c_a}h(f-\Delta_s-\Delta_m)\end{array}\right)\\ =\ & 0.\end{aligned}$$

Note when evaluated at $f = \widehat{f}$, the LHS of the FOC is simply

$$-\frac{1}{c_a}\left(\widehat{f} - c - \Delta_m + \frac{\mu_D}{2}\right)(1 - H(\widehat{f} - \Delta_s - \Delta_m))^2.$$

This is strictly positive if

$$\widehat{f} < \Delta_m + c - \frac{\mu_D}{2},$$

thus implying that if $f^* < f^e$ in the case of mature platform, then welfare would also be increased by decreasing f below $\widehat{f}$ here.