Regulating platform fees*

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Abstract

We consider a platform that helps consumers more easily discover and transact with suppliers. Such platforms have come to dominate many sectors of the economy, raising issues about the high fees they charge suppliers, especially since they tend to commoditize the suppliers they aggregate. We show that in a baseline setting, the welfare-maximizing fee exceeds the platform's marginal cost by the extent to which suppliers obtain lower markups on the platform than in the direct channel. We examine the robustness of this simple principle, and explore factors that make the efficient fee higher or lower than this level.

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1 Introduction

Regulators are struggling with the right way to address market power concerns arising from large digital platforms that act as gatekeepers by providing key access to

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consumers for third-party suppliers, app developers, online sellers, and other small businesses. In Europe, the Digital Markets Act, which was recently passed, seeks to do this primarily by prohibiting various types of platform behavior: e.g. self-preferencing, price-parity clauses and bundling/tying, while obliging platforms to make certain changes that are supposed to promote easier user choice and switching. It is unclear, however, the extent to which these changes will really limit platforms' ability to exercise market power via high prices to the businesses that use them to access consumers (either via a platform marketplace and/or via advertising over the platform). This motivates our interest in another, possibly complementary, solution, which is the regulation of the prices charged by platforms to the suppliers that use them to access consumers.

The issue of regulating platform prices is relevant beyond just the big-tech platforms that are the focus of recent regulatory efforts. Digital marketplaces have sprung up across almost every sector of the economy, from business software to dog walking. These marketplaces aggregate suppliers and in some verticals are dominated by one or two players. As digital marketplaces become the main place consumers discover and transact with suppliers, they get more market power, which may be exercised through high fees charged to suppliers.[1]

To address this concern, we develop a simple framework of a monopoly platform that connects consumers and suppliers, and charges a fee to suppliers for doing so. The framework captures three key features of many such marketplace platforms: (i) suppliers pass the platform's fees back to consumers via higher prices; (ii) the platform intensifies competition between suppliers; and (iii) the platform has to attract consumers in the first place who can alternatively buy directly from suppliers.

Given the platform intensifies competition between suppliers and facilitates consumer choice of their preferred supplier, it will attract consumers to use it. However, intensified supplier competition comes at a cost to these suppliers, which consumers ignore. As a result too many consumers will use the platform if it just charges suppliers a fee equal to its marginal cost. Instead, we find in our baseline model, the socially optimal fee is characterized by a very simple rule: it equals the platform's marginal cost plus the amount to which the platform decreases supplier margins

[1] Some platforms where high supplier-side fees have been noted include: e-commerce marketplaces (Amazon and eBay etc), food delivery apps (UberEats and Doordash), hotel booking sites (Booking.com and Expedia), app stores (Apple's AppStore and Android's Playstore) and gaming consoles (Xbox and Playstation). See https://www.theverge.com/21445923/platform-fees-apps-games-business-marketplace-apple-google for the fees charged by some of these platforms, as well as others.

(what we call, the markup differential). This ensures that, when making their channel choice, consumers internalize the profit margin that suppliers lose by selling on the platform. We also show how the fee a monopoly platform sets can be higher or lower than this efficient level, and the factors driving any distortion.

We then explore three extensions of our framework. The first extends our baseline model to the case with imperfect pass-through, showing how the same simple efficient fee rule can still work in this case, provided it is updated over time. The second extension is the case that there is showrooming, so consumers can switch to buy directly after discovering a supplier on the platform, if it is cheaper to do so. We show that the same simple efficient fee rule applies, although the markup differential is lower due to showrooming, thus implying showrooming lowers the efficient fee. The third extension models what happens if suppliers have to advertise if they want to be discoverable in the direct channel. In the limit as the number of suppliers becomes large, we find the characterization of the platform's profit maximizing fee is unchanged from the baseline setting but the efficient fee is lower. The latter result reflects that an efficiency benefit the platform creates and which consumers don't internalize is to reduce the extent of duplicated advertising which arises in the direct channel.

1.1 Related literature

There is surprisingly little prior research on the question of the right level at which to regulate prices set by digital platforms. One exception is for payment card platforms, where most of the focus was exactly on whether the interchange fee set by a card platform (and so the fee merchants pay for accepting card payments) was too high (Rochet and Tirole, 2002 and 2011, and Wright, 2004 and 2012), and if so, what was the right fee to set. Indeed, this paper is in part inspired by the work of Rochet and Tirole (2011) who propose a simple rule that could be used to regulate interchange fees (the so-called "Merchant Indifferent Test"), one that has been adopted by regulators in Europe, among other places. Their setting is different, however, for two main reasons (i) unlike the types of marketplace platforms we're focused on in this paper, card platforms don't help intensify competition between suppliers given they are not primarily used to discover merchants; (ii) a no-surcharge rule applies, so suppliers are not allowed to set a higher price to consumers who purchase using the card platform than those who pay with cash.

Gomes and Mantovani (2022) relax (i) but not (ii). In their setting, the platform expands the consideration set of consumers and in so doing also intensifies competition between suppliers. But like the payments literature, they keep the assumption that prices must be the same across the platform channel and the direct channel (i.e. price parity holds). Under price parity, they show the platform's unregulated fee to suppliers is excessive.[2] Given they focus on price parity holding, not surprisingly, their characterization of the socially optimal fee is different from ours. Under price parity there is no role for consumers' channel choice to be influenced by fees, which is what drives our results without price parity. This is why our framework is not well suited for an analysis under price parity, and a setting like theirs is more appropriate. In their setting, it is the extensive margin between whether the platform invests or not given randomness in its fixed cost of investment that pins down the efficient fee. Specifically, their efficient fee is determined by the extent to which the platform expands consumers' consideration set as well as any convenience benefits it provides to suppliers. We compare Gomes and Mantovani's characterization of the efficient fee with ours under the same model of supplier competition.

Our focus on the case without price parity clauses is motivated by the fact in many platforms, price-parity clauses have not been imposed, or in some cases, have been banned by regulators (see Baker and Scott Morton, 2018). More generally, we have in mind a setting in which restrictions like price-parity clauses have already been removed, for instance, because of regulations like those implied by the Digital Markets Act. And we ask the question of what fees platforms would set then, and how to optimally regulate them.

Two recent papers explore caps on the fees platforms charge suppliers, but in contrast to our paper, they take into account the possibility that the platforms also charge fees on the consumer side. Bisceglia and Tirole (2022) views the lack of a platform fee to consumers as an indication that the platform would like to set negative prices to consumers if such fees were feasible, and explores the consequences of this missing price (as well as a zero price bound for the supplier's own prices) for the efficient cap. Their setting is quite different from ours in that the platform operates a hybrid marketplace and there is no direct channel. Their main focus is on the interplay between the platform fee to suppliers and whether the platform steers consumers to its own apps or squeezes (or forecloses) third-party apps. Sullivan (2022) empirically studies commission caps on food delivery platforms, and after

[2] Other papers also look at settings in which price parity clauses hold (e.g. see Boik and Corts, 2016, Edelman and Wright, 2015, Johnson, 2017, Ronayne and Taylor, 2020, and Wang and Wright, 2020) but they differ in not exploring the regulation of the platform's fee.

taking into account that such caps increase the platforms' delivery fees to consumers, he finds they lower consumer surplus and total welfare.

Finally, our paper relates to the literature modelling price comparison websites. The seminal paper in this line is Baye and Morgan (2001), in which consumers can use the platform to find the lowest priced supplier (which are homogenous) or instead go to their local monopolist. They maintain price parity. Galeotti and Moraga-González (2011) extend their work to the case with differentiated firms, as well as allowing suppliers to set different prices across channels, like in our paper. A key difference in these papers is that they assume the platform can set a fixed fee to each side (both consumers and suppliers), and consumers all face the same fixed benefit of shopping via the platform relative to shopping in the direct channel. Thus, they shut down the smooth channel choice that drives our results, and the efficient fees are just set so all consumers and suppliers participate on the platform. The models of price comparison websites by Ronayne (2021) and Ronayne and Taylor (2022) are closer to our setting, since they assume, more realistically, that such platforms charge firms a per-transaction fee and nothing to consumers directly. They also allow for differential prices across channels. However, in their setting the platform fee does not affect total welfare, and their interest lies rather in whether the existence of such platforms is good for consumers.

2 Baseline model

Suppose there are multiple suppliers (either a finite number or a continuum) producing horizontally differentiated products. For brevity, we will refer to suppliers as "firms", but the reader should keep in mind these can sometimes be individuals (e.g. a dog walker on Rover or a web designer on Fiverr). There is a unit mass of consumers, each with unit demand. There is an outside option, with surplus normalized to zero. The firms costs are normalized to zero.

Firms and consumers can trade directly. Let ϕ_D be the expected gross value of shopping directly, and $p_D = \mu_D$ be the symmetric equilibrium direct price, where μ_D represents firms' symmetric markup in the direct channel. We assume $\phi_D \geq \mu_D$ since otherwise consumers would never shop in the direct market.

A marketplace platform M can facilitate the trades between firms and consumers at a marginal cost $c \geq 0$, and for doing so it charges firms a per-transaction fee f, the most commonly used form of fee charged by such marketplaces.[3]

[3] As detailed in https://www.theverge.com/21445923/platform-fees-apps-games-business-

The expected gross surplus of choosing M is ϕ_M. Assume the symmetric equilibrium price on M is

$$p_M = f + \mu_M,$$

where μ_M is firms' symmetric markup on M.

In commonly used search models, location models, and random utility models, $\phi_M \geq \phi_D$, $\mu_D \geq \mu_M$, $\phi_D \geq \mu_D$, and we will adopt those assumptions, but as will be obvious, for the most part, the framework does not rely on those assumptions.

Consumers' expected net utility of shopping directly is

$$\phi_D - p_D = \phi_D - \mu_D,$$

and their expected net utility of shopping on M is

$$\phi_M - p_M + b = \phi_M - (f + \mu_M) + b,$$

where b is an additive benefit (if positive) or cost (if negative) associated with shopping on M, which is distributed according to H on $[\underline{b}, \overline{b}]$. We assume a strictly positive density h and a weakly increasing hazard rate for H (which implies that demand for M as defined by $1 - H(\cdot)$ is weakly log-concave). Corresponding to this, we define $\lambda(x) = (1 - H(x))/h(x)$ as the inverse hazard rate, which is weakly decreasing.

Firms often obtain extra transaction benefits (or equivalently, face lower marginal costs) when selling through the platform. We do not explicitly allow for such benefits in our baseline model given they can be implicitly captured by the consumer-side benefits b. This reflects that any firm-side benefits would be fully passed on to consumers given our assumption of full pass-through above. In our extension with partial pass-through (Section 4.1), we will explicitly allow for such firm-side benefits.

We assume for the relevant fees we consider (in particular, the higher of the socially optimal and privately optimal), all consumers who go to M always make a single transaction on M and get non-negative net surplus from doing so.[4] Thus, in this baseline setting we can also interpret b as the additional benefit consumers get from making a transaction on M.

marketplace-apple-google, marketplaces typically charge firms (sellers and developers) on a per transaction basis. Often such a fee is written as a percentage of the value of a transaction rather than a fixed amount per transaction. We adopt the latter type of fee for tractability. In the Online Appendix we show how our analysis can be modified to handle percentage fees.

[4]For our specific applications later, we will add conditions on parameters to make sure this assumption is satisfied.

Buyers observe the fee f but not the prices or firms' listing decisions.[5] Consumers choose one channel only, and in our baseline setting cannot change channels depending on the prices or listing decisions they discover. Finally, we allow firms to price discriminate across channels. This set of assumptions implies firms will always be willing to list on M to obtain incremental revenue.

Consumers who choose the platform must have

$$\phi_M - (f + \mu_M) + b \geq \phi_D - \mu_D \;\Leftrightarrow\; b \geq f - (\phi_M - \phi_D) - (\mu_D - \mu_M).$$

Define $\Delta_s \equiv \phi_M - \phi_D$ as the surplus differential and $\Delta_m \equiv \mu_D - \mu_M$ as the markup differential. The condition above becomes

$$b \geq f - \Delta_s - \Delta_m,$$

which makes clear the only reason a consumer uses M is if the benefit of doing so plus the surplus and markup differential that M creates more than covers the fee it charges.

Some examples of micro-founded settings that fit this baseline model include the following:

- Sequential search model of a platform such as in Wang and Wright (2020). There is a continuum of firms with consumers' match values drawn iid from a distribution $G(\cdot)$. Firms are all available on either channel, and buyers choose one channel to search on. They search sequentially in their chosen channel, but search costs are lower on M. If x_j represents the equilibrium reservation utility for searching in channel j, then $\Delta_s = x_M - x_D > 0$ and $\Delta_m = \frac{1-G(x_D)}{g(x_D)} - \frac{1-G(x_M)}{g(x_M)} > 0$. The positive surplus differentiation arises from a higher reservation utility on M (due to lower search costs on M), and lower search costs on M also explain the positive markup differential. More details of this model are given in Section 4.2.

- Circular-city model with $n \geq 2$ firms located on a circle (Salop, 1979). Each good offers value v, and consumers face a standard linear mismatch cost parameter t. If consumers go direct, we assume they are randomly matched to

[5] The key assumption is that an individual firm cannot influence a buyers choice of channel by whether to list on M or what price to set. Yet it should be that M's choice of fee f, and so firms' equilibrium prices, will ultimately impact which channel consumers want to use. We could alternatively assume that consumers cannot observe f, but they can observe the modal price set by firms on M, which then works provided $n \geq 3$.

one of the firms, whereas if they go on M they can choose from all listed firms. Consumers don't observe which particular firms are located on each channel, their prices, or product attributes after they choose a channel. In the Online Appendix we characterize parameter restrictions on v (has to be large enough) and t (cannot be too high) so that the baseline model applies. In equilibrium, all firms list on M and we have: $\Delta_s = \frac{t}{4} - \frac{t}{4n} > 0$ and $\Delta_m = v - \frac{t}{2} - \frac{t}{n} > 0$ given $v > t$. The surplus differential is driven by lower mismatch costs on M, and the markup differential is driven by more firms competing on M. Two special cases of this setting are: (i) Hotelling model when $n = 2$; (ii) Bertrand competition if we take $t \to 0$, so $\Delta_s = 0$ and $\Delta_m = v$.

- Random-utility model with $n \geq 3$ firms (Perloff and Salop, 1985). The utility a consumer can get from buying at firm i is $u^i = v - p^i + \beta\xi^i$, where ξ^i is iid from G across firms and consumers, and $\beta > 0$ is a parameter to measure the importance of the match value. Consumers randomly draw a set of n_D firms ($2 \leq n_D < n$) that they choose from in the direct market, whereas if they go on M they can choose from all listed firms. The difference between n and n_D then drives the surplus and markup differentials. For example, if G is a uniform distribution on $[0, 1]$, $\Delta_s = \beta\left(\frac{n}{n+1} - \frac{n_D}{n_D+1}\right) > 0$ and $\Delta_m = \beta\left(\frac{1}{n_D} - \frac{1}{n}\right) > 0$, where note $\Delta_m > \Delta_s$. The general expressions for Δ_s and Δ_m are given in (5)-(6) below, with the derivations of these provided in the Online Appendix.

3 Analysis of baseline model

As a profit-maximizing monopolist, M chooses f to maximize

$$(f - c)\left(1 - H(f - \Delta_s - \Delta_m)\right).$$

Note that $f^* = c$ only if the demand $1 - H(f - \Delta_s - \Delta_m) = 0$ for all $f > c$. Otherwise, M would be better off by setting some fee strictly above its marginal cost. The condition that $1 - H(f - \Delta_s - \Delta_m) = 0$, or equivalently, $H(f - \Delta_s - \Delta_m) = 1$, for all $f > c$ is equivalent to

$$\bar{b} \leq c - \Delta_s - \Delta_m \quad \Leftrightarrow \quad \bar{b} + \Delta_s + \Delta_m \leq c.$$

We rule out this uninteresting case by assuming

$$\overline{b} + \Delta_s + \Delta_m > c \tag{1}$$

throughout the paper.

With this assumption, we obtain the following characterization of M's optimal fee (as with other results not proven in the text, the proof is given in the Appendix):

Proposition 1. (The platform's profit maximizing fee)
M sets $f^* = \widehat{f}$, where $\widehat{f}$ is the unique solution to

$$\widehat{f} = c + \lambda\left(\widehat{f} - \Delta_s - \Delta_m\right), \tag{2}$$

and satisfies $c < \widehat{f} < \overline{b} + \Delta_s + \Delta_m$.

We can illustrate this result when H takes the generalized Pareto distribution (GPD), which covers several well-known distributions such as uniform, exponential, normal, logistic, type I extreme value, and Weibull.

Example 1 (Generalized Pareto distribution)**.** *Given our assumption of a weakly increasing hazard rate, when H takes the generalized Pareto distribution (GPD) form, it can be written as*[6]

$$H(b) = \begin{cases} 1 - \left(1 - \frac{\epsilon(b - \underline{b})}{\sigma}\right)^{\frac{1}{\epsilon}} & \text{if } \epsilon > 0 \\ 1 - e^{-\frac{b - \underline{b}}{\sigma}} & \text{if } \epsilon = 0 \end{cases}$$

over the support $\underline{b} \le b \le \overline{b}$, where $\overline{b} = \underline{b} + \frac{\sigma}{\epsilon}$. Note $\lambda(b) = \sigma - \epsilon(b - \underline{b})$. Then (2) implies

$$\widehat{f} = \frac{c + \sigma + \epsilon\left(\underline{b} + \Delta_s + \Delta_m\right)}{1 + \epsilon} \tag{3}$$

so $\widehat{f} = \frac{c + \underline{b} + \sigma + \Delta_s + \Delta_m}{2}$ when $\epsilon = 1$ (uniform distribution) and $\widehat{f} = c + \sigma$ when $\epsilon = 0$ (exponential distribution).

Now let us determine the efficient fee. A consumer with $b \ge f - \Delta_s - \Delta_m$ chooses

[6] To make our results easier to interpret, we define the shape parameter ϵ so it is non-negative, with a higher value of ϵ representing a more concave distribution for H.

M, and shops directly otherwise. So total welfare is

$$W = \int_{f-\Delta_s-\Delta_m}^{\overline{b}} (\phi_M + b - c) dH(b) + \int_{\underline{b}}^{f-\Delta_s-\Delta_m} \phi_D dH(b).$$

Differentiating W with respect to f gives the derivative

$$-(\phi_M - \phi_D + f - \Delta_s - \Delta_m - c) h\left(f - \Delta_s - \Delta_m\right).$$

Given second-order conditions clearly hold, setting the derivative above equal to zero implies the following result.[7]

Proposition 2. (The planner's welfare maximizing fee)
The planner which can only control the platform's fee and not firms' final prices maximizes total welfare by setting

$$f^e = c + \Delta_m. \tag{4}$$

Proposition 2 says from an efficiency perspective, the platform's fee should be set above its marginal cost by the extent to which the platform lowers firms margins. The result is simple yet surprising at first glance. Why should the efficient fee be anything other than the platform's marginal cost? Indeed, the only difference in the price consumers should face across the two channels is the marginal cost that M faces to provide its intermediation service. However, given markups are lower on M (i.e., $\Delta_m > 0$), in equilibrium consumers will face a lower price differential than the marginal cost, which is why the efficient fee is higher than marginal cost in order to restore the correct price differential. Put differently, the margin difference makes consumers favor M, and as a result too many consumers choose M. The social planner uses a fee above cost to correct for this distortion. Formally, if $f = c$, consumers will choose M if $\phi_M - c - \mu_M + b \geq \phi_D - \mu_D$, whereas in the efficient outcome, consumers should choose M if and only if $\phi_M - c + b \geq \phi_D$ (i.e. the difference in margins is removed in the efficient solution).

Another way to understand why the efficient fee is above M 's marginal cost, is in terms of externalities. When consumers decide to use the platform, they do not take

[7] In case $c - \Delta_s < \underline{b}$ such that $h\left(f - \Delta_s - \Delta_m\right) = 0$, efficiency requires all consumers to use the platform. The fee proposed in (4) induces such an outcome, even though a continuum of fees can achieve the same outcome. A parallel argument holds when $c - \Delta_s > \overline{b}$ such that $h(f - \Delta_s - \Delta_m) = 0$, and anywhere in the paper where $h(\cdot) = 0$.

into account the negative effect on firms who earn lower margins on this channel. If they did, there would be no need for the fee to be set above M's marginal cost. By setting a higher fee, consumers pay a tax for using the platform that equals the loss in firms' margins that results from their choice, thereby getting them to internalize the full effects of their choice.

The efficient fee f^e that we identify is relatively easy to implement. Note that the formula in (4) can be re-written as

$$f^e = c + \Delta_m = c + p_D - (p_M - f^*) = c + f^* + p_D - p_M.$$

Each of p_D, p_M, and f^* are directly observable, while c may be approximated by zero for some digital platforms. Note, if in practice, consumers might sometimes not find any product they like in the direct market, then p_D may overstate the margin that firms earn in the direct market. For example, suppose consumers, if shopping directly, can only find a desired product category with probability $\gamma \in (0, 1)$; they subsequently obtain a utility $\phi_D - \mu_D$ in this case. With probability $1 - \gamma$, the desired category does not exist in the direct market and the consumer obtains a zero utility. So a consumer only gets expected utility $\gamma(\phi_D - \mu_D)$ on the direct channel. In this case we can redefine $\Delta_s = \phi_M - \gamma\phi_D$ and $\Delta_m = \gamma\mu_D - \mu_M$. The efficient fee is still $f^e = c + \Delta_m$, but to implement it based on the above approach we need to use that $f^e = c + \gamma\mu_D - \mu_M = c + f + \gamma p_D - p_M$.

We can compare our characterization of the efficient fee to that in Gomes and Mantovani (2022) by assuming $c = 0$. In their mature market setting in which there is no positive latent demand, and assuming competition is determined by the random-utility framework in which consumers get to see n_D firms (with i.i.d. match value ξ and cumulative distribution function $G(\xi)$) in the direct market and n such firms on the platform, they find the efficient fee just equals $\Delta_s + b_f$, where

$$\Delta_s = \beta \left(\int_{\underline{\xi}}^{\overline{\xi}} \xi dG(\xi)^n - \int_{\underline{\xi}}^{\overline{\xi}} \xi dG(\xi)^{n_D} \right) \tag{5}$$

and b_f is the convenience benefit they assume firms get from on-platform transactions. This compares to the efficient fee in our setting for the same competition model[8], which is Δ_m, where

$$\Delta_m = \beta \left(\frac{1}{n_D \int_{\underline{\xi}}^{\overline{\xi}} g(\xi) dG(\xi)^{n_D - 1}} - \frac{1}{n \int_{\underline{\xi}}^{\overline{\xi}} g(\xi) dG(\xi)^{n-1}} \right). \tag{6}$$

The parameters β, n_D and n have the same qualitative effects on the efficient fee across both settings, although for very different reasons. In Gomes and Mantovani, this is via the surplus differential, which provides a reason to make sure the platform can operate by setting a sufficiently high fee. In our setting, this is via the markup differential, which requires a sufficiently high fee to offset excessive use of the platform by consumers. Another difference is that allowing firms to enjoy a convenience benefit b_f would not change the efficient fee in (4) since such a benefit would be like a negative marginal cost for firms — it would lower their equilibrium prices on the platform since it is fully passed through by firms, which would induce consumers to correctly take it into account when deciding which channel to choose.

The next proposition compares the equilibrium fee in (2) and the efficient fee in (4).

Proposition 3. (Comparison of fees) *The profit-maximizing fee exceeds the efficient fee iff*

$$\lambda (c - \Delta_s) \geq \Delta_m. \tag{7}$$

An increase in the surplus differential Δ_s increases any concern that the platform's fee is too high (it increases the profit-maximizing fee but not the efficient fee). An increase in the markup differential Δ_m decreases any concern that the platform's fee is too high (it increases the profit-maximizing fee by less than the one-for-one increase in the efficient fee).

The result shows that Δ_s and Δ_m have opposite effects in determining whether the equilibrium fee is too high. Here a high Δ_s leads to a high equilibrium fee but has no effect on the efficient fee as consumers already take into account the surplus differential when making their choice of channel. On the other hand, a high Δ_m leads to a high efficient fee (one-for-one) but it doesn't get fully passed through into the equilibrium fee (given log-concave demand). This means a higher Δ_s shifts the

[8] We assume the platform has no marginal costs to make things as comparable to their setting as possible.

tradeoff towards the equilibrium fee being too high, while a higher Δ_m shifts the tradeoff towards the equilibrium fee being too low.

Proposition 3 does not rule out the possibility that the platform's monopoly fee is lower than the efficient fee. If $\Delta_m \leq 0$, by construction $f^* > f^e$, and the platform sets its fee too high. A necessary condition for $f^* \leq f^e$ is therefore that $\Delta_m > 0$; i.e. that the margins are strictly lower on M.

One case where M's fee is always too low is when M's added social value does not cover its marginal cost so $\Delta_s \leq c$ and it is always costly to choose so $\bar{b} = 0$. The planner would prefer a higher fee to reduce the number of consumers going to M. Note that $1 - H(c - \Delta_s) = 0$ if $\bar{b} = 0$ and $c - \Delta_s \geq 0$. Then, given $\Delta_m > 0$, it must be that $f^* < f^e$.

A more extreme case arises when M's only value is in intensifying competition. This arises when we take $t \to 0$ in our Salop circular-city model so firms are Bertrand competitors on M. Assuming the value of each product is v, then $\mu_M = 0$, $\phi_M = v$, $\mu_D = v$, $\phi_D = v$, and so $\Delta_s = 0$ and $\Delta_m = v$. Then if $\bar{b} = 0$, the planner prefers no one uses M from an efficiency perspective which it can do by setting $f^e = v$. At this fee no consumers would incur a cost to participate on M. In contrast, M will always set a fee below the efficient level (but above c) to attract some consumers. In this example, M's only advantage over the direct market is the competition it creates for firms which firms can avoid if they can coordinate not to choose, but since they can't coordinate, M is able to shift surplus from firms to consumers, and tax transactions in the process. This illustrates commodization — how a platform that destroys welfare can exist by shifting surplus from firms (who face a coordination failure) to itself and consumers. To emphasize this interesting possibility, which underlies part of what is going on in our setting more generally, we state it as a Corollary.

Corollary 1. *A platform that destroys welfare can profitably exist by shifting surplus from firms to consumers, thereby attracting consumers to make transactions on its platform and taking a tax on such transactions.*

Underlying this result is that individual firms cannot circumvent the platform by delisting and redirecting consumers to their direct channel. This reflects our assumption that each individual firm's delisting decisions are not be observable, which more generally captures the idea that each individual firm is small. If firms were large and established, a platform that destroys welfare would unlikely be able to exist because each such firm could redirect its consumers to its direct channel via delisting.

Example 2 (Generalized Pareto distribution). *Using the GPD for* b *to get* λ *in (7), M's monopoly fee exceeds the efficient fee iff*

$$\sigma + \epsilon \left(\underline{b} + \Delta_s - c\right) \geq \Delta_m. \tag{8}$$

For $\epsilon = 1$ *(linear demand), the condition becomes* $\overline{b} + \Delta_s - c \geq \Delta_m$*, and for* $\epsilon = 0$ *(exponential distribution) it becomes* $\sigma \geq \Delta_m$*.*

As can be seen from (8), the condition for the platform's fee to be too high depends on both Δ_m and Δ_s, as well as the parameters that define H. Since both Δ_m and Δ_s depend on the underlying primitives of competition on the platform and in the direct market, we can ask how changes in the primitives of competition affect the tendency for the platform to set its fee too high. We do this by defining the difference between the two sides of (8) as

$$L = \sigma + \epsilon \left(\underline{b} + \Delta_s - c\right) - \Delta_m,$$

and considering how L changes in the primitives for each of our three competition applications from Section 2. An increase in L means the tradeoff shifts towards the unregulated fee exceeding the efficient fee. For a change in some parameter x, we have

$$\frac{\partial L}{\partial x} = \epsilon \frac{\partial \Delta_s}{\partial x} - \frac{\partial \Delta_m}{\partial x} \tag{9}$$

where recall $\epsilon \geq 0$ given our increasing hazard rate assumption. This implies changes in competition primitives will have an ambiguous effect on the tendency for the platform to set its fee too high as changes in these primitives tend to affect the surplus differential and markup differential in the same direction.

In the Online Appendix we apply (9) to each of the three applications (sequential search model, circular city model and random utility model) and fully characterize the resulting comparative static results. One way to summarize the results is to focus on the case where the distribution H of b is GPD and its shape parameter ϵ is sufficiently small (i.e. it is not too concave). Then the tendency for the platform to set its fee too high increases in the following factors: (i) in the search model, it increases in the search cost on the platform and decreases in the search cost in the direct channel; (ii) in the circular-city model, it increases in the degree of product differentiation (i.e. in the transport cost parameter t) and it decreases in the total number of firms altogether (so the number of firms listed on M); (iii) in the random-utility model, it decreases in the degree of product differentiation between firms, it decreases in the total number of firms altogether (so the number of firms listed on M), and it increases in the number of firms that consumers draw in the direct channel.

3.1 Different welfare objectives

So far, when evaluating welfare we have used a total welfare standard. However, often policymakers will want to put more weight on the surplus of consumers than that of a monopoly firm selling to those consumers. The interesting thing about a two-sided platform setting is that the platform's "consumers" consist of the users on both sides of the platform (here both final consumers and the competing firms that want to reach them). The interests of the firm side of the platform may be particularly relevant in a setting where the firms involved are individuals or small businesses. Thus, it is natural to explore how fees should be set when less weight is put on the platform's profit.

To keep things general initially, consider the weighted average of the different surplus components making up total surplus, which can be written as

$$
\begin{aligned}
W^{ts} = {} & w_c \left(\int_{f-(\Delta_s+\Delta_m)}^{\bar{b}} (\phi_M + b - f - \mu_M) dH(b) + \int_{\underline{b}}^{f-(\Delta_s+\Delta_m)} (\phi_D - \mu_D)\, dH(b) \right) \\
& + w_f \left(\int_{f-(\Delta_s+\Delta_m)}^{\bar{b}} \mu_M dH(b) + \int_{\underline{b}}^{f-(\Delta_s+\Delta_m)} \mu_D dH(b) \right) \\
& + w_m \int_{f-(\Delta_s+\Delta_m)}^{\bar{b}} (f - c)\, dH(b),
\end{aligned}
$$

where the terms in the expression are consumer surplus (the first line), firms' total profit (the second line), and platform profit (the third line), and the respective weights satisfy $w_c + w_f + w_m = 1$. After simplifying, the derivative of W^{ts} with respect to f is

$$
\begin{aligned}
& w_f \Delta_m h\left(f - (\Delta_s + \Delta_m)\right) - w_c \left(1 - H\left(f - (\Delta_s + \Delta_m)\right)\right) \\
& + w_m \left(1 - H\left(f - (\Delta_s + \Delta_m)\right) - (f - c)\, h\left(f - (\Delta_s + \Delta_m)\right)\right).
\end{aligned}
$$

We consider several different special cases.

3.1.1 Consumer surplus only

Clearly if $w_f = 0$ and $w_m = 0$, so the planner is only interested in maximizing consumer surplus, then the planner should regulate f as low as is feasible to ensure the platform still operates. We take this to be equal to its marginal cost, although obviously if M has fixed costs to cover, then it should be based on average costs. The only reason to set a higher fee is to get consumers to internalize the profit of firms and/or the platform, which are absent here. Thus, we have:

Proposition 4. (Consumer surplus standard) *The fee that maximizes consumer surplus while ensuring the platform covers its cost is* $f^{cs} = c$.

On the other hand, if the constraint is just that the existence of the platform at least make consumers better off, that is always the case in our setting, reflecting that consumers would choose the direct channel if this was not the case.

3.1.2 Total user surplus

Next suppose $w_m = 0$, so we are only interested in total user surplus (or more generally, some weighted average of consumer plus firm surplus); i.e., we ignore the platform's profit altogether. In this case we find

Proposition 5. (Total user surplus standard) *The fee that maximizes a weighted average of consumer surplus and firms' profit while ensuring the platform covers its cost is either the lowest feasible fee* $f^{us} = c$ *(so that* M *can just cover its costs) or any fee such that* $f^{us} \geq \bar{b} + \Delta_s + \Delta_m$ *(so that no consumers will go to* M*). In case the standard is total user surplus* ($w_c = w_f$)*, the planner prefers the low fee if and only if*

$$\int_{c-(\Delta_s+\Delta_m)}^{\bar{b}} \Delta_m dH(b) < \int_{c-(\Delta_s+\Delta_m)}^{\bar{b}} (\Delta_s + \Delta_m - c + b) dH(b) \tag{10}$$

or $\Delta_m < \Delta_s - c + \bar{b}$ *in the case of linear* H.

The proposition shows that any weighted average of consumer surplus and firm profit has a U-shape, with the planner preferring either the lowest feasible fee, which just allows M to cover its costs, or any fee high enough that even the consumer with the highest possible b has no reason to go to M.

How can total user surplus ever be increasing in f up to the point where consumers stop coming to M? Like in the welfare case, the consumer here does not internalize the additional margin that the firm gets when the consumer transacts directly. This means, from their joint perspective, too many consumers will transact via M, which suggests a positive transaction fee is needed. While the platform's existence always makes firms worse off by reducing their profit margin, this loss becomes smaller as the fee increases as more consumers will choose to trade directly. In contrast, consumers, as a whole, become better off with the platform's existence, but decreasingly so when the platform increases its fee and thus increases the price on the platform. When the platform fee is at a low level, an increase in the fee decreases total user surplus as many consumers use the platform to trade in this case and the loss in consumer surplus outweigh the increase in firm profit. When the platform fee is already high, the opposite holds as few consumers trade via the platform.

In the environment that we consider, maximizing total user surplus is not a balanced policy goal as it leads to an outcome either very bad for firms, i.e., $f = c$, or very bad for consumers, i.e., $f \geq \Delta_m + \Delta_s + \overline{b}$. Since the total user surplus created by the platform is

$$\int_{f-(\Delta_s+\Delta_m)}^{\overline{b}} (\Delta_s + b - f) dH(b),$$

which is U-shaped and equal to zero at $f = \Delta_m + \Delta_s + \overline{b}$, in order that the existence of the platform weakly increases total user surplus f must satisfy

$$f \leq \Delta_s + \widehat{b}(f), \tag{11}$$

where $\widehat{b}(f) = \mathbb{E}[b | b \geq f - (\Delta_s + \Delta_m)]$ is the expected value of b for consumers using the platform given the fee f. If H is strictly log-concave, $\widehat{b}(f)$ is increasing in f but at a rate less than one. Thus, for strictly log-concave H, (11) implies there will be a unique cap for f, denoted f^c, below which total user surplus is higher whenever the platform operates. That is, $f^c = \Delta_s + \widehat{b}(f^c)$, and the existence of the platform increases total user surplus for all $f < f^c$. If we add the reasonable assumption that the platform's existence increases total welfare when its fee is set at marginal cost, so that

$$\Delta_s + \widehat{b}(c) > c, \tag{12}$$

then $c < f^c < \Delta_m + \Delta_s + \overline{b}$. Moreover, (12) implies that total user surplus is higher at $f = c$ than at $f = \Delta_m + \Delta_s + \overline{b}$, so among fees that cover costs, total user surplus is maximized at $f = c$.

Gomes and Mantovani (2022) show that at their welfare-maximizing fee, the existence of the platform never reduces total user surplus. This is not always the case in our setting. To see this, assume b is strictly log-concave GPD distributed on $\left[\underline{b}, \overline{b}\right]$, which implies $\widehat{b}\left(f\right) = \frac{f-(\Delta_s+\Delta_m)+\sigma+\epsilon\underline{b}}{1+\epsilon}$, and so (11) implies the relevant cap is

$$f^c = \underline{b} + \Delta_s + \frac{\sigma - \Delta_m}{\epsilon}.$$

Using this as an extra constraint on the regulated fee implies

$$f^{reg} = \min\left(c + \Delta_m, \underline{b} + \Delta_s + \frac{\sigma - \Delta_m}{\epsilon}\right).$$

In the limit case of the exponential distribution ($\epsilon \to 0$), the extra constraint is not binding when $\sigma > \Delta_m$ (or equivalently $f^* > f^e$), which implies no additional cap is needed beyond f^e to ensure total user surplus is enhanced by the existence of the platform. More generally, for high enough $\epsilon > 0$, the constraint can become binding, in which case the regulated cap should be set below f^e if we want to ensure the platform increases total user surplus.

3.1.3 Weighted average of total surplus

Suppose now $0 < w_m < w_c$, so that we consider the surplus of all parties including the platform, but we put less weight on the platform's profit than consumers. Also assume that H takes the exponential distribution as defined in our GPD example. Then second-order conditions always hold, and

$$f^{ts} = c + \frac{w_f}{w_m}\Delta_m - \sigma\left(\frac{w_c - w_m}{w_m}\right),$$

with $f^* > f^{ts}$ iff

$$w_c\sigma > w_f\Delta_m.$$

There are two cases of particular interest for this exponential case:

- Set $w_c = 1 + \alpha > w_m = w_f = 1 - \alpha$ so weight consumer surplus more than profit (firms' and the platform's). Then provided $\alpha < 1$ so $w_m > 0$ and $w_f > 0$,

$$\begin{aligned} f^{ts} &= c + \Delta_m - \frac{2\alpha}{1-\alpha}\sigma \\ &= f^e - \frac{2\alpha}{1-\alpha}\left(f^* - c\right). \end{aligned}$$

- Set $w_c = w_f = 1+\alpha > w_m = 1-\alpha$ so we weight the surplus of users (consumers and firms) more than the platform's profit. Then provided $\alpha < 1$ so $w_m > 0$,

$$\begin{aligned} f^{ts} &= c + \Delta_m - \frac{2\alpha}{1-\alpha}\left(\sigma - \Delta_m\right) \\ &= f^e - \frac{2\alpha}{1-\alpha}\left(f^* - f^e\right). \end{aligned}$$

These results show that when consumer surplus gets more weight than profits (either of firms or the platform), the weighted-welfare maximizing fee is necessarily less than f^e, while when the surplus of users (consumers and firms) gets more weight than the profit of the platform, this is only true if the platform's unregulated fee f^* exceeds f^e. Moreover, note the informational requirements of these two solutions are no more than for regulating the efficient fee. The planner just has to pick the weights it wants to put on the different types of participants. For example, if the planner puts a weight of one-third on the profit of the platform, but weights consumers and firms equally, then it should cap the platform's fee at the normal efficient fee $c + \Delta_m$ less the difference between the unregulated and regulated fee $f^* - f^e = \sigma - \Delta_m$.

4 Extensions

In this section we consider three important extensions of our baseline model.

4.1 Incomplete pass-through

So far we have assumed that firms fully pass through M's fee into their prices. Here we consider how to adjust the formula for the welfare-maximizing fee when there is incomplete pass-through. Allowing for incomplete pass-through also provides a way to understand why firms may prefer lower rather than higher fees, since they get to keep a higher margin on the platform. Furthermore, with incomplete pass-through, it is no longer obvious that any transactional benefits or lower marginal costs that firms obtain from using the platform can simply be subsumed into the distribution of b, so in this extension we allow for this possibility explicitly.

Specifically, we assume that firms face a positive marginal cost d when selling directly that can be avoided when selling via M. And we assume the equilibrium price on M can be written as $p_M(f)$ and in the direct market is $p_D(d)$, where the respective markups are defined as $\mu_M(f) = p_M(f) - f$ and $\mu_D(d) = p_D(d) - d$, with $p'_M(f) < 1$. Then total welfare is

$$W = \int_{p_M(f)-p_D(d)-\Delta_s}^{\overline{b}} (\phi_M + b - c)\, dH(b) + \int_{\underline{b}}^{p_M(f)-p_D(d)-\Delta_s} (\phi_D - d)\, dH(b).$$

Consistent with our previous assumption that $\mu_M < \mu_D$, we assume that $\mu_M(d) < \mu_D(d)$. Then the FOC from welfare maximization implies

$$-(p_M(f) - \mu_D(d) - c)\, p'_M(f)\, h(p_M(f) - p_D(d) - \Delta_s) = 0.$$

So the planner would set f^w such that

$$p_M(f^w) = c + \mu_D(d).$$

Then by the definition of $\mu_M(f)$,

$$f^w = c + \mu_D(d) - \mu_M(f^w) = c + \Delta_m(f^w).$$

The result suggests the same formula for setting the welfare-maximizing fee in (4) applies. The only difficulty is that in the implementation of f^w, the markup differential now depends on f^w. Fortunately, using (4) with the observed markup is still a conservative way to proceed, since it lies between the efficient fee and the platform's unregulated fee, and provided this exercise is updated over time, it converges to the efficient fee level. Thus, using (13) where Δ_m is the empirically measured markup is a reasonably robust way to regulate the fee in the context of imperfect pass-through of fees.

Proposition 6. (Incomplete pass-through) *When firms have incomplete pass-through on M, the welfare-maximizing fee is*

$$f^w = c + \Delta_m(f^w) \tag{13}$$

with the only difference from (4) being that $\Delta_m(f)$ now depends on f. The fee f^w

can be implemented by iteratively applying (13), where in each period $\Delta_m(f)$ is taken as the previous period's observed markup differential $\Delta_m = p_D - (p_M - f)$, and p_D, p_M and f are the prices and fee set in the previous period.

The incomplete pass-through implies a firm's profit margin on the platform $p_M(f) - f$, decreases in f. So incomplete pass-through can help explain why firms may argue in favor of lower fees. The firms' total profit is

$$\int_{p_M(f)-p_D(d)-\Delta_s}^{\bar{b}} (p_M(f) - f)\, dH(b) + \int_{\underline{b}}^{p_M(f)-p_D(d)-\Delta_s} (p_D(d) - d)\, dH(b).$$

The derivative with respect to f is

$$-\left(p_M(f) - f - (p_D(d) - d)\right) p_M'(f)\, h - (1 - p_M'(f))(1 - H),$$

where h and H are both functions of $p_M(f) - p_D(d) - \Delta_s$, which we suppress for brevity. With incomplete pass-through, the second term is negative. Suppose that, given imperfect pass-through, for $f < d$ that is low enough, we have $p_M(f) - f = p_D(d) - d$. Then clearly for all f at or lower than this level, the firms' total profits are decreasing in f. Moreover, provided f is not too high, so the measure of consumers using the platform $1 - H$ is high, then the second term will continue to dominate, and the firms' total profit will continue to decrease in f. For f high enough, $1 - H$ becomes small, and the first term dominates, so that the firms' total profit will increase in f. However, if the firms collectively push for platform regulation that would require it set very high fees, this may raise anticompetitive concerns and/or be blocked due to the harm to consumers. Given this, firms may be better off lobbying for regulators to lower fees.[9]

4.2 Search and showrooming

An application of the general setting is a standard sequential search model with a continuum of firms (see Wang and Wright, 2020). Here consumers' match values for one unit of firm i's product are drawn iid from the common distribution function G for each consumer and each firm. Consumers search to discover these match values as well as prices. In the absence of showrooming or price parity, firms' equilibrium

[9] In the Online Appendix we provide a specific model in which there is imperfect pass-through, and provide a condition for the firms' total profit to increase as f is lowered below the platform's optimal fee.

prices are $p_M = f + \mu(x_M)$ on M and $p_D = \mu(x_D)$ in the direct market, where $\mu(x) = \frac{1-G(x)}{g(x)}$ is decreasing in x given that G is assumed to have an increasing hazard function, and x is the reservation value of the consumers in their search. Thus, $x_M > x_D$ across channels capturing that searching on M is less costly. Lower search costs on M also imply the gross surplus of searching on M is higher. Specifically, a consumer's net utility from searching (and buying) on M is $\phi(x_M) + b - f - \mu(x_M)$, and her utility from searching (and buying) directly is $\phi(x_D) - \mu(x_D)$, where ϕ is the expected match value which is an increasing function of x. Everything in our general model then applies where $\phi_M = \phi(x_M)$, $\phi_D = \phi(x_D)$, $\mu_M = \mu(x_M)$, $\mu_D = \mu(x_D)$, and note $\Delta_s = \phi(x_M) - \phi(x_D)$ and $\Delta_m = \mu(x_D) - \mu(x_M)$ are both positive. In addition, we need $x_M \geq p_M$ and $x_D \geq p_D$ in equilibrium such that all consumers who actively search will eventually buy.

Now consider the extension where an exogenous fraction of consumers ρ can showroom, switching to buy directly from the firm they find on M. Firms cannot distinguish these consumers though, from those coming directly. For this setting, we interpret b as the joining cost (or benefit) that consumers get from the platform, so even if they switch to buy directly they still incur b. Consumers may or may not know whether they can showroom when deciding which channel to use, although if they don't know this, they still know the probability ρ. We consider both cases in the proof of the following proposition.

Proposition 7. *The welfare-maximizing fee f^w in the presence of showrooming is lower than that in the benchmark (i.e. $f^w < f^e$). An increase in ρ reduces f^w.*

The result is intuitive. Firms take into account that some fraction of consumers can be attracted to switch after searching on M. Since they are competing to attract these consumers and these consumers have more elastic demand, they will optimally lower their direct price to do so. Thus, $p_D < \mu_D$. This reduces the margin difference between the two channels, and so reduces the need to set a high fee to offset that margin difference. However, the efficient pricing formula still applies as $f^e = c + \tilde{\Delta}_m$ with $\tilde{\Delta}_m = p_D - \mu_M < \Delta_m$.

4.3 Advertising

The main role of a marketplace platform is arguably to help consumers discover firms. As such, a platform can help firms avoid spending on costly advertising. In this section, we address how this affects the optimal choice of M's fee by considering a situation where consumers initially have limited information about potential trading partners and firms can use direct advertising or the platform to inform them.

There are n symmetric firms producing horizontally differentiated products. Their listing decision on M follows the baseline setting, and as a result all firms will list on M. Once consumers visit M, they can see all the prices and locations (and so their match values) of the n firms. They then select one firm to buy from (or choose the outside option which gives them zero utility).

Consumers can only know about firms in the direct channel when they receive ads from them. Consumers know the underlying configuration of firms but are unaware of their identity. Firms can advertise their identity (and so existence) to consumers. Formally, each firm $i = 1, 2, ..., n$ chooses an advertising intensity $a_i \in [0, 1]$ such that a consumer learns the identity of firm i with probability a_i. The advertising cost is given by $\frac{c_a(a_i)^2}{2}$ with $c_a > 0$. The ads only reveal the firms' identity but not the firm's price or location (and so the match value).[10] In the end, consumers are assumed to only pick one firm in the direct channel or M to go to. We assume advertising is sufficiently costly.

Assumption 1. *Assume*

$$c_a \geq \frac{\mu_D}{n}. \tag{14}$$

The timing of the model is given as follows.

- M sets its fee f.
- Firms decide whether to list on M, what prices to set on each channel, and their advertising intensity.[11]
- Consumers receive ads and observe the fee level M sets. They do not observe anything else.
- Consumers decide among the different firms they receive ads from and M, which one to visit.
- Upon learning the price and match value of the firm they visit, or all the prices and match values of the firms listed on M if they visit M, consumers decide whether to purchase or not.

[10] This type of advertising technology was used in the previous literature to study the competition for consumers' attention. For details, see Haan and Moraga-González (2011) for example.

[11] Note given each firm prices as a monopolist in the direct channel, it doesn't matter if each firm sets advertising choice first and then pricing or vice-versa, or it sets them together.

We first characterize the equilibrium, and then derive the efficient allocation and make the comparison. Suppose a consumer receives firm i's ads. The question is how many other firms' ads the consumer also receives. Let all other firms choose the symmetric advertising intensity $a \in (0,1)$. The probability that this consumer receives ads from one other firm is $C_{n-1}^{1} a(1-a)^{n-2}$, and in this case the consumer visits firm i with probability $\frac{1}{2}$, and the probability that this consumer receives ads from two other firms is $C_{n-1}^{2} a^2 (1-a)^{n-3}$, and in this case the consumer visits firm i with probability $\frac{1}{3}$, and so on. So the total visits firm i receives is

$$\begin{aligned}
& a_i \left[C_{n-1}^{1} a(1-a)^{n-2} \cdot \frac{1}{2} + C_{n-1}^{2} a^2 (1-a)^{n-3} \frac{1}{3} + ... + C_{n-1}^{n-1} a^{n-1} \frac{1}{n} \right] \\
=& a_i \left[C_n^1 a^1 (1-a)^{n-1} \frac{1}{an} + C_n^2 a^2 (1-a)^{n-2} \frac{1}{an} + ... + C_n^n a^n \frac{1}{an} \right] \\
=& \frac{a_i}{an} \left[-C_n^0 (1-a)^n + C_n^0 (1-a)^n + C_n^1 a^1 (1-a)^{n-1} + C_n^2 a^2 (1-a)^{n-2} + ... + C_n^n a^n \right] \\
=& \frac{a_i}{an} \left[1 - (1-a)^n \right].
\end{aligned}$$

When deciding how much advertising to engage in, the only thing that matters for an individual firm is how that affects its profit from consumers that receive its ad. These are consumers who choose to purchase via the firm directly, which gives it additional profit of

$$\frac{a_i \left(1 - (1-a)^n\right)}{na} \mu_D H \left(f - \Delta_s - \Delta_m\right) - \frac{c_a a_i^2}{2},$$

after taking into account the cost of advertising. The profit it gets on M is unaffected by its choice of a_i. A firm's optimal a_i therefore solves

$$\frac{\left(1 - (1-a)^n\right)}{na} \mu_D H \left(f - \Delta_s - \Delta_m\right) = c_a a, \tag{15}$$

and we have the unique solution for a, denoted a^*.

Lemma 1. *Given that $H\left(f - \Delta_s - \Delta_m\right) \neq 0$, there exists a unique $a^* \in (0,1)$ such that (15) holds.*

An increase in a strictly decreases the left-hand side of (15), which then requires each individual firm decrease its advertising in response. The firms' advertising intensities are strategic substitutes. Moreover, since the left-hand side of (15) is strictly decreasing in a while the right-hand side is strictly increasing in a, an upward shift of the left-hand side must imply an increase in a, thus implying a^* increases in

μ_D. A higher f shifts demand onto the direct channel and leads to an increase in a^*. In particular, totally differentiating (15) we have

$$\frac{da^*}{df} = \frac{(1-(1-a^*)^n)\,\mu_D h\,(f-\Delta_s-\Delta_m)}{2c_a a^* n - n\,(1-a^*)^{n-1}\,\mu_D H\,(f-\Delta_s-\Delta_m)}. \tag{16}$$

The lemmas below characterize the equilibrium individual and aggregate advertising reach when $n \to \infty$.

Lemma 2. *The following equilibrium properties hold at the limit as $n \to \infty$: (i) $a^* \to 0$; (ii) $(1-a^*)^n \to 0$; (iii) $n(1-a^*)^{n-1} \to 0$; (iv) $na^* \to \infty$.*

Lemma 2 implies that in the limit as the number of firms gets large, (i) individual advertising intensity goes to zero; (ii) even though each firm's level of advertising goes to zero, the probability of being uninformed goes to zero, so the reach of advertising is complete: the probability a consumer receives at least one ad goes to one; (iii) moreover, conditional on receiving ads from one firm, the probability of not receiving ads from any other of the $n-1$ firms goes to zero; (iv) and finally, the average number of firms that a consumer can observe goes to infinity. In the proof of Lemma 2, we further show that $a^* \underset{n\to\infty}{\approx} \sqrt{\frac{1}{nx}}$, where $x \equiv \frac{c_a}{\mu_D H(f-\Delta_s-\Delta_m)}$.

Each consumer receives ads from at least one firm with probability $1-(1-a^*)^n$ and does not receive ads from any firm with probability $(1-a^*)^n$. In the former case, consumers choose to go to M if $b \geq f-\Delta_s-\Delta_m$. In the latter case, consumers choose to go to M if $b \geq f+\mu_M-\phi_M$. So the total transaction volume on M, conditional on receiving ads, is

$$T(f) \equiv (1-(1-a^*)^n)\,(1-H\,(f-\Delta_s-\Delta_m)) + (1-a^*)^n\,(1-H\,(f+\mu_M-\phi_M))\,.$$

It is clear $f-\Delta_s-\Delta_m \geq f+\mu_M-\phi_M$ given our assumption that $\phi_D \geq \mu_D$. This implies the total demand faced by M decreases as a^* increases.

M will choose f to maximize

$$f^* = \arg\max_f \{(f-c)T\,(f)\}\,.$$

Using Lemma 2, and provided second-order conditions hold, we can then prove the following limit result.

Proposition 8. (The platform's profit maximizing fee given firms advertise) *In the limit as $n \to \infty$, M's profit maximizing fee $f^* \to \widehat{f}$, where $\widehat{f}$ is the unique solution to (2).*

Thus, for a large number of firms, M's profit-maximizing choice of fee converges to the baseline without any advertising. This is not surprising given that with a large number of firms advertising, the probability consumers are informed of at least one firm goes to one, so M's problem is essentially the same as in the case where firms do not need to advertise.

Total welfare is given by

$$\begin{aligned} W &= (1-(1-a^*)^n)\int_{f-\Delta_s-\Delta_m}^{\bar{b}} (\phi_M + b - c)\, dH(b) \\ &\quad +(1-a^*)^n \int_{f+\mu_M-\phi_M}^{\bar{b}} (\phi_M + b - c)\, dH(b) \\ &\quad +(1-(1-a^*)^n)\phi_D H(f-\Delta_s-\Delta_m) - \frac{(a^*)^2}{2}. \end{aligned}$$

Suppose a planner maximizes the total welfare by choosing f. Provided second-order conditions hold, and the solution is interior, we get

Proposition 9. (The planner's welfare maximizing fee given firms advertise) *In the limit as $n \to \infty$, the welfare-maximizing fee satisfies*

$$f^w \to c + \Delta_m - \frac{\mu_D}{2} < f^e.$$

Proposition 9 implies that the welfare-maximizing fee is now lower, reflecting that there is an additional negative cost of setting a high platform fee that pushes consumers to buy directly: it encourages more duplicative advertising by firms which is socially costly in this setting.

Finally, a direct comparison of the equilibrium fee with the welfare-maximizing fee for large n implies:

Proposition 10. (Comparison of fees) *In the limit as $n \to \infty$, we have*

$$f^* \geq f^w \;\Leftrightarrow \lambda\left(c - \Delta_s - \frac{\mu_D}{2}\right) \;\geq \Delta_m - \frac{\mu_D}{2}. \tag{17}$$

A greater μ_M only decrease the right-hand side of (17), while having no impact on the left-hand side. Meanwhile, the left-hand side increases in μ_D, while the right-hand side, which is equal to $\frac{\mu_D}{2} - \mu_M$, also increases in μ_D. This suggests that an increase in μ_D has a more ambiguous impact on fee comparison compared to the baseline model. It encourages firms to advertise, increasing consumers' awareness of the direct channel and counterbalances the need for a higher fee to improve efficiency.

Above we have treated the cost of advertising as a real cost to society. However, the amount spent on advertising by firms may instead represent, at least partially, a transfer from the firms advertising to publishers and other advertising platforms (e.g. Facebook and Google). If we multiply the cost of advertising in the welfare expression by the discount factor $0 \leq \delta \leq 1$, then following the same steps, we get

$$f^w \to c + \Delta_m - \frac{\delta \mu_D}{2}.$$

This implies in the extreme case that advertising expenditure is a pure transfer (so $\delta = 0$), $f^w = f^e$ as in the baseline setting.

In the Online Appendix we explore the extension of the model in this section in which M also needs to advertise to attract consumers. Arguably, if M is still emerging (and so relies on advertising), then it is less likely to be a candidate for regulation. Nevertheless, we do see some well-established platforms advertising aggressively against firms' own ads (e.g. food delivery platforms and hotel booking platforms). To capture this, we assume M also has to advertise to be discovered, and has access to the same advertising technology. Unfortunately, we cannot get a closed form solution for the welfare-maximizing fee even for sufficiently large n, but we are able to show that as $n \to \infty$, the equilibrium fee characterization remains the same as in the baseline. Moreover, we show that if (17) holds, the equilibrium fee is also higher than the welfare-maximizing fee in the limit.

5 Conclusion

This paper proposes a simple yet flexible framework for studying the regulation of the fee a platform charges to suppliers when transactions can be done both directly between firms and consumers and indirectly via the platform. Taking into account that suppliers have lower markups on the platform than in the direct channel, we find the efficient fee exceeds the platform's marginal cost by the difference in markups across the two channels, which eliminates the otherwise excessive use of the platform by consumers. We also explore the conditions for a monopoly platform's fee to exceed this efficient benchmark, which loosely speaking require that the surplus differential created by the platform is high relative to the markup differential. We extend the framework to accommodate other important market elements such as consumer search and supplier advertising. When consumers need to search for products, the presence of showrooming lowers the efficient fee given excessive use of the platform is less of an issue. The need for suppliers to advertise to attract business in the direct channel is another reason the efficient fee may be lower, so as to drive more transactions to the platform and thereby reduce the extent of duplicative advertising done by suppliers.

There are several other factors beyond those that we explored that could drive a distortion between the platform's optimal fee and the socially optimal fee. Three important ones are (i) if there is a fixed cost for suppliers to participate on the platform, there can be excessive or insufficient entry onto the platform; (ii) elastic demand by consumers would push towards the platform's optimal fee being excessive, but the direction of any such distortion is no longer obvious if we also allow suppliers to make investment decisions that respond to their margins; and (iii) positive network effects between participating consumers and suppliers mean that both the socially optimal and privately optimal choice of fee would tend to be lower. None of these effects therefore necessarily lead to a distortion between the platform's and planner's optimal fee in one direction or the other, but they are interesting (and challenging) extensions to consider for future work on this topic.

In contrast to Gomes and Mantovani (2022), we have focused on the case where price parity does not hold, so firms are free to set different prices across different channels. If firms are actually restricted in their price setting, our formula for the efficient fee does not apply. Indeed, applying it leads to a circularity. With prices the same on both channels, as a platform increases its fee, the margin on the platform would decrease compared to the direct channel, which suggests the efficient fee we characterized should increase. The efficient fee in this case should instead be determined by a framework in which the price parity is taken into account, such as in Gomes and Mantovani. It remains, however, to consider the possibility of partial price parity, where some transactions are subject to price parity and others are not.

Another important direction for future research is to consider how our analysis would be affected by platform competition, so that it can capture the case of two large marketplaces that dominate a particular vertical. To the extent consumers tend to singlehome and suppliers multihome, the competitive bottleneck logic should apply, and our analysis should flow through. Even allowing for one platform to intensify competition between suppliers more than the other, our simple markup rule for regulating platform pricing should still apply at the level of each platform based on the principle that for maximum efficiency, the relative prices consumers face for using different channels ought to only reflect differences in the platforms' costs.

Appendix.

Proof of Proposition 1. Note that the left-hand side (LHS) of (2) strictly increases from 0 to ∞ when f increases from 0 to ∞. The right-hand side (RHS) of (2) weakly decreases in f given λ is weakly decreasing (from the assumed weakly increasing hazard rate). Moreover, it decreases from a value greater than c to c when f increases from 0 to ∞. As a result there is a unique solution to (2) which satisfies the stated condition. ■

Proof of Proposition 3. Compare $\widehat{f}$ in (2) and f^e in (4). Notice that the term $\lambda(f-\Delta_s-\Delta_m)$ in (2) strictly decreases in f unless $H(b)$ is an exponential distribution in which case λ is a constant. Then we must have

$$\begin{aligned} f^e \leq \widehat{f} \quad &\Leftrightarrow \quad c+\lambda(f^e-\Delta_s-\Delta_m) \geq c+\lambda(\widehat{f}-\Delta_s-\Delta_m) = \widehat{f} \geq f^e = c+\Delta_m \\ &\Leftrightarrow \quad \lambda(c-\Delta_s) \geq \Delta_m \end{aligned}$$

Totally differentiate (2), and we get

$$\frac{d\widehat{f}}{d\Delta_m} = \frac{-\lambda'}{1-\lambda'}.$$

Since $\lambda' < 0$, we have $0 \leq \frac{d\widehat{f}}{d\Delta_m} < 1$. ■

Proof of Proposition 5. Differentiating W^{us} with respect to f we get

$$\frac{dW^{us}}{df} = w_f \Delta_m h\left(f-(\Delta_s+\Delta_m)\right) - w_c\left(1-H\left(f-(\Delta_s+\Delta_m)\right)\right).$$

Define f where

$$w_c \lambda\left(f-(\Delta_s+\Delta_m)\right) = w_f \Delta_m$$

as the unique value f^{us}. We have $\frac{dW^{us}}{df} = h\left(f-(\Delta_s+\Delta_m)\right)\left[w_f\Delta_m - w_c\lambda\left(f-(\Delta_s+\Delta_m)\right)\right]$. Recall λ is decreasing in f. Therefore, if $f < f^{us}$, $w_c\lambda\left(f-(\Delta_s+\Delta_m)\right) > w_f\Delta_m$ and $\frac{dW^{us}}{df} < 0$, and vice-versa when $f > f^{us}$. So f^{us} characterizes a minimum. The planner will either want to set f as low as possible (subject to M wanting to operate) or as high as possible so that no consumer would go to M. Comparing W^{us} across the two extreme values of f assuming $w_c = w_f$ we get (10).

Proof of Proposition 6. Let f_0 denote the initial fee before regulation, which if it is unregulated should equal $\widehat{f}$, but we allow to be some other level as well. Then applying the regulation in period 1 would imply $f^1 = c + \Delta_m(f^0)$. Given imperfect pass-through, $\Delta_m(f)$ is increasing in f at a rate less than one. Suppose first that $f^0 > f^w$. This implies $\Delta_m(f^0) > \Delta_m(f^w)$ and so $f^1 = c + \Delta_m(f^0) > c + \Delta_m(f^w)$. Also since $f^w = c + \Delta_m(f^w)$, we must have $f^0 > c + \Delta_m(f^0)$. Thus, using $f^1 = c + \Delta_m(f^0)$ implies the regulated fee would be between f^w and f^0. Alternatively if $f^0 < f^w$, by a parallel argument, using $f^1 = c + \Delta_m(f^0)$ implies the regulated fee would be between f^0 and f^w. Iterating over time, if $f^{t-1} < f^w$, $f^t \in (f^{t-1}, f^w)$, while if $f^{t-1} > f^w$, $f^t \in (f^w, f^{t-1})$, showing that the regulated fee converges towards the efficient fee. ■

Proof of Proposition 7. Since each firm is infinitesimal, they cannot affect the measure of consumers who choose to visit M. So they take the distribution of consumers over the two channels as constant. That is, each firm i maximizes

$$p_D^i\left(\rho(1-H(\cdot))\frac{1-G(x_M-p_D+p_D^i)}{1-G(x_M)}+H(\cdot)\frac{1-G(x_D-p_D+p_D^i)}{1-G(x_D)}\right). \tag{18}$$

We analyze first the case where consumers only know that they can showroom after choosing M, and then the case where they know it before choosing channels.

We implicitly assume that consumers who can showroom indeed want to switch in equilibrium. This requires $p_D \leq f^* + \mu_M$, which we need to verify when solving for f^*. This needs to be true for the planner's solution too as we assume ρ consumers switch to buy directly at the end. Let us call those consumers who can showroom "showrooming consumers" and those cannot "regular consumers".

Consumers, without knowing whether they can switch later (but taking into account the probability they will be able to), choose M if and only if

$$\phi_M - (1-\rho)(f+\mu_M) - \rho p_D + b \geq \phi_D - p_D$$

or equivalent,

$$b \geq -\Delta_s + (1-\rho)(f - \widetilde{\Delta}_m),$$

where $\widetilde{\Delta}_m = p_D - \mu_M$. Notice that p_D is a function of f as $H(\cdot) = H(-\Delta_s + (1-\rho)(f - \widetilde{\Delta}_m))$.

The FOC of firm i, after imposing symmetry, gives

$$p_D = \frac{((1-\rho)H(\cdot)+\rho)\mu_D\mu_M}{(1-H(\cdot))\lambda\mu_D + H(\cdot)\mu_M}$$

Notice that p_D is only implicitly defined here as $H(\cdot)$ contains p_D. It is straightforward to verify that $\mu_M < p_D < \mu_D$.

The social planner chooses f to maximize

$$\int_{-\Delta_s+(1-\rho)(f-\widetilde{\Delta}_m)}^{\bar{b}} (\phi_M + b - (1-\rho)c)\, dH(b) + \int_{\underline{b}}^{-\Delta_s+(1-\rho)(f-\widetilde{\Delta}_m)} \phi_D dH(b).$$

The FOC gives the welfare-maximizing fee level

$$f^w = c + \widetilde{\Delta}_m.$$

Notice that f^w is implicitly defined as $\widetilde{\Delta}_m$ is affected by f, and also f^w is affected by ρ only through $\widetilde{\Delta}_m$. However, we know that $f^w < f^e$ as $\widetilde{\Delta}_M < \Delta_M$ for a given f.

Finally, we need to check that $p_D(f^w) < f^w + \mu_M = c + \widetilde{\Delta}_m + \mu_M = c + p_D(f^w)$, so that consumers indeed want to switch to buy directly. This is true by construction.

Notice that in (18), the weight of the first term in the bracket, $\rho(1 - H(-\Delta_s + (1-\rho)(f^w - \widetilde{\Delta}_m))) = \rho(1 - H(-\Delta_s + (1-\rho)c))$, increases in ρ, while the weight of the second term, $H(-\Delta_s + (1-\rho)c)$, decreases in ρ. The maximization of the first term (multiplied by p_d^i) yields a solution equal to μ_M, while the maximization of the second term (multiplied by p_d^i) yields a solution equal to μ_D. Since $\mu_D \geq \mu_M$, we know that p_D decreases in ρ, and thus $f^w = c + \tilde{\Delta}_m = c + p_D - \mu_M$ decreases in ρ.

Alternatively, suppose those consumers who can freely switch channels know it before they choose which channel to visit. Regular consumers choose M if and only if

$$b \geq f - \Delta_s - \widetilde{\Delta}_m,$$

where $\widetilde{\Delta}_m = p_D - \mu_M$. Showrooming consumers always visit M first and switch to buy directly if $p_M \geq p_D$. Then, $p_M = f + \mu_M$. Assuming all showrooming consumers will eventually switch to buy directly, each firm i chooses p_D^i to maximize

$$p_D^i \left(\rho \frac{1 - G(x_M - p_D + p_D^i)}{1 - G(x_M)} + (1-\rho)H(\cdot) \frac{1 - G(x_D - p_D + p_D^i)}{1 - G(x_D)} \right).$$

The FOC together with symmetry gives

$$p_D = \frac{(\rho + (1-\rho)H(\cdot))\mu_D\mu_M}{\rho\mu_D + (1-\rho)H(\cdot)\mu_M}.$$

It is straightforward to show $\mu_M < p_D < \mu_D$.

The social planner chooses f to maximize

$$\rho\phi_M + (1-\rho)\left(\int_{f-\Delta_s-\widetilde{\Delta}_m}^{\bar{b}} (\phi_M + b - c)dH(b) + \int_{\underline{b}}^{f-\Delta_s-\widetilde{\Delta}_m} \phi_D dH(b)\right),$$

which implies

$$f^e = c + \widetilde{\Delta}_m.$$

Since $p_D < \mu_D$ and therefore $\widetilde{\Delta}_m < \Delta_m$, we know that f^e is lower than the efficient fee in our baseline model. From the expression of p_D, we have $\frac{dp_D}{d\rho}$ is proportional to $\mu_M - \mu_D < 0$. Since $f^w = c + \tilde{\Delta}_m = c + p_D - \mu_M$, f^w decreases in ρ. ■

Proof of Lemma 1. (15) can be rewritten as

$$\frac{(1-(1-a)^n)}{(a)^2} = \frac{nc_a}{\mu_D H(f - \Delta_s - \Delta_m)}. \tag{19}$$

We first show that the LHS of (19) is strictly decreasing in a. The LHS can be written as

$$\exp[\ln[1-(1-a)^n] - \ln[a^2]].$$

Take the derivative of $\ln[1-(1-a)^n] - \ln[a^2]$ with respect to a and we get

$$\frac{(na + 2(1-a))(1-a)^{n-1} - 2}{a(1-(1-a)^n)}.$$

The sign of the derivative is the same as the sign of $L(a) \equiv na + 2(1-a))(1-a)^{n-1} - 2$ for all $a \in (0,1)$. We further have $\lim_{a\to 0} L(a) = 0$, $\lim_{a\to 1} L(a) = -2$, and $L'(a) = -((n-2)a+1)n(1-a)^{n-2} < 0$. So $\ln[1-(1-a)^n] - \ln[a^2]$ strictly decreases in a, and so does the LHS of (19). The LHS of (19) goes to ∞ when $a \to 0$ and 1 when $a \to 1$. By Assumption 1 and $0 \le H(\cdot) \le 1$, we know that the RHS of (19) is a constant greater than 1. We then can conclude there is always a unique $a \in (0,1)$ satisfying (19), provided $H(f - \Delta_s - \Delta_m) \neq 0$. ■

Proof of Lemma 2. Equation (15) can be re-written as

$$1-(1-a)^n - xn(a)^2 = 0,$$

where $x \equiv \frac{c_a}{\mu_D H(f-\Delta_s-\Delta_m)}$. Let $a^*(n) \in [0,1]$ be the solution to this equation for a. We instead work with $y \equiv n(a^*(n))^2 \in [0,n]$ and

$$f(y,n) = 1 - \left(1-\sqrt{\frac{y}{n}}\right)^n - xy.$$

We will also treat n as a continuous variable.

We have $f(0,n)=0$ and $f(n,n)=1-xn$, with the latter being strictly negative given Assumption 1. Routine calculations show that $\frac{\partial^2 f}{\partial y^2} < 0$ for every $y \in (0,n)$ and $\lim_{y\to 0} \frac{\partial f}{\partial y} = \infty$. Hence, there exists a unique $y(n) \in (0,n)$ such that $f(y(n),n) = 0$. This is because, since f is strictly concave in y and $f(y,n)$ is strictly positive for y small, f must cross the horizontal axis from above. Moreover, $\left.\frac{\partial f}{\partial y}\right|_{y=y(n)} < 0$. Clearly, $a^*(n) = \sqrt{\frac{y(n)}{n}}$.

Assume for a contradiction that the function $y(n)$ is not bounded. Then, there exists a sequence $(n^k)_{k\geq 0}$ such that $y(n^k) \underset{k\to\infty}{\longrightarrow} \infty$. This gives the contradiction

$$0 = f(y(n^k), n^k) \leq 1 - xy(n^k) \underset{k\to\infty}{\longrightarrow} -\infty.$$

Hence, there exists $\overline{y} > 0$ such that $y(n) \leq \overline{y}$ for every n. It follows that $\lim_{n\to\infty} a^*(n) = 0$.

Routine calculations show that $\partial f/\partial n$ has the same sign as

$$-\frac{1}{2}\sqrt{\frac{y}{n}} - \left(1-\sqrt{\frac{y}{n}}\right)\ln\left(1-\sqrt{\frac{y}{n}}\right).$$

It follows that $\left.\frac{\partial f}{\partial n}\right|_{y=y(n)}$ has the same sign as

$$\begin{aligned}
-\frac{1}{2}a^*(n) - (1-a^*(n))\ln(1-a^*(n)) &= -\frac{1}{2}a^*(n) - (1-a^*(n))(-a^*(n)) + o(a^*(n)) \\
&= \frac{1}{2}a^*(n) + o(a^*(n)) = \frac{1}{2}a^*(n)\left[1+o(1)\right].
\end{aligned}$$

Therefore, $\frac{\partial f}{\partial n}\Big|_{y=y(n)}$ is strictly positive for n sufficiently high. It follows that

$$y'(n) = -\left.\frac{\partial f/\partial n}{\partial f/\partial y}\right|_{y=y(n)}$$

is strictly positive for n high enough. Therefore, there exists n^0 such that $y(\cdot)$ is strictly increasing on $[n^0, \infty)$. This implies that $\ell \equiv \lim_{n\to\infty} y(n)$ exists and is strictly positive. Moreover, since the function $y(\cdot)$ is bounded, ℓ is finite.

We have

$$\begin{aligned} n\ln(1-a^*(n)) &= n(-a^*(n))\frac{\ln(1-a^*(n))}{-a^*(n)} \\ &= -\sqrt{y(n)n}\frac{\ln(1-a^*(n))}{-a^*(n)} \\ &= \underbrace{-\sqrt{\ell n}}_{\underset{n\to\infty}{\longrightarrow} -\infty}\underbrace{\sqrt{\frac{y(n)}{\ell}}}_{\underset{n\to\infty}{\longrightarrow} 1}\underbrace{\frac{\ln(1-a^*(n))}{-a^*(n)}}_{\underset{n\to\infty}{\longrightarrow} 1} \underset{n\to\infty}{\longrightarrow} -\infty. \end{aligned}$$

It follows that

$$(1-a^*(n))^n = \exp\left[n\ln(1-a^*(n))\right] \underset{n\to\infty}{\longrightarrow} 0.$$

Moreover,

$$n(1-a^*(n))^{n-1} = n\exp[n\ln(1-a^*(n)) - \ln(1-a^*(n))] \underset{n\to\infty}{\longrightarrow} 0.$$

Note also that, for every n,

$$0 = f(y(n), n) = 1 - (1-a^*(n))^n - xy(n) \underset{n\to\infty}{\longrightarrow} 1 - x\ell.$$

Hence, $\ell = \lim_{n\to\infty} y(n) = 1/x$. This implies that

$$a^*(n) \approx \sqrt{\frac{1}{nx}},$$

when n is sufficiently large. Then,

$$na^*(n) \underset{n\to\infty}{\longrightarrow} \infty.$$

We have proved the results in Lemma 2. ■

Proof of Proposition 8. Provided second-order conditions hold and the solution is interior, f^* is characterized by the first-order condition

$$\begin{aligned}&(1-(1-a^*)^n)(1-H(f-\Delta_s-\Delta_m))+(1-a^*)^n(1-H(f+\mu_M-\phi_M))\\+&(f-c)\Bigg(n(1-a^*)^{n-1}(1-H(f-\Delta_s-\Delta_m))\frac{da^*}{df}-(1-(1-a^*)^n)h(f-\Delta_s-\Delta_m)\\-&n(1-a^*)^{n-1}(1-H(f+\mu_M-\phi_M))\frac{da^*}{df}-(1-a^*)^n h(f+\mu_M-\phi_M)\Bigg)=0.\end{aligned}$$

Applying the limits in Lemma 2 to the above, the FOC above implies $\lim_{n\to\infty} f^* \to \widehat{f}$. ■

Proof of Proposition 9. Given our assumptions, f^w is characterized by the first-order condition

$$\begin{aligned}&n(1-a^*)^{n-1}\frac{da^*}{df}\int_{f-\Delta_s-\Delta_m}^{\bar{b}}(\phi_M+b-c)dH(b)\\&-(1-(1-a^*)^n)(\phi_M+f-c-\Delta_s-\Delta_m)h(f-\Delta_s-\Delta_m)\\&-n(1-a^*)^{n-1}\frac{da^*}{df}\int_{f+\mu_M-\phi_M}^{\bar{b}}(\phi_M+b-c)dH(b)\\&-(1-a^*)^n(f-c+\mu_M)h(f+\mu_M-\phi_M)\\&+n(1-a^*)^{n-1}\frac{da^*}{df}\phi_D H(f-\Delta_s-\Delta_m)\\&+(1-(1-a^*)^n)\phi_D h(f-\Delta_s-\Delta_m)-nc_a a^*\frac{da^*}{df}\\=\;&0.\end{aligned}$$

To evaluate this FOC in the limit as $n\to\infty$, we first prove the equilibrium a^* satisfies

$$\frac{da^*}{df}\to 0 \quad\text{and}\quad na^*\frac{da^*}{df}\to\frac{\mu_D h(f-\Delta_s-\Delta_m)}{2c_a}$$

as $n\to\infty$. Based on equation (16) and the limit results in Lemma 2, when $n\to\infty$,

$$\frac{da^*}{df}=\frac{\left(1-(1-a^*)^n\right)\mu_D h\left(f-\Delta_s-\Delta_m\right)}{2c_a a^* n-n\left(1-a^*\right)^{n-1}\mu_D H\left(f-\Delta_s-\Delta_m\right)}\to 0, \tag{20}$$

as $1-(1-a^*)^n\to 1$, $a^*n\to\infty$, and $n(1-a^*)^{n-1}\to 0$. Similarly,

$$na^*\frac{da^*}{df}=\frac{\left(1-(1-a^*)^n\right)\mu_D h\left(f-\Delta_s-\Delta_m\right)}{2c_a-\frac{(1-a^*)^{n-1}}{a^*}\mu_D H\left(f-\Delta_s-\Delta_m\right)}\to\frac{\mu_D h(f-\Delta_s-\Delta_m)}{2c_a},$$

since $\frac{(1-a^*)^{n-1}}{a^*}\approx\sqrt{nx}\exp\left[-\frac{n-1}{\sqrt{nx}}\right]\to 0$, where $x\equiv\frac{c_a}{\mu_D H(f-\Delta_s-\Delta_m)}$ is a constant. Now applying these results together with Lemma 2, the first-order condition becomes

$$\left(-\left(\phi_M + f - c - \Delta_s - \Delta_m\right) + \phi_D - \frac{\mu_D}{2}\right) h(f - \Delta_s - \Delta_m) = 0,$$

from which we obtain the result. ■

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