Online Appendix: Should platforms be allowed to sell on their own marketplaces?

Andrei Hagiu[1], Tat-How Teh[2], and Julian Wright[3]

A Possibility of reselling S's product

In the main text we have assumed that whenever M sells itself, it sources its own in-house offering directly at zero marginal cost. An alternative arrangement is M has an additional option of obtaining S's product at the wholesale price w.

In what follows, we show that the implications of banning dual mode derived in the baseline model of the main text continue to hold with this alternative arrangement. To distinguish between these two versions of products sold by M's seller component, we use p_m^h to denote M's retail price for the in-house offering and p_m^s to denote its retail price for the product obtained from S.

A.1 Seller mode with reselling of S's product

Obviously, the analysis of the marketplace mode in the main text is unaffected. Let us consider the seller mode where the timing is: (1) S sets the wholesale price w; (2) M chooses whether to resell S's product (in addition to M's in-house offering); and (3) M, S, and fringe sellers compete in retail prices.

For any given wholesale price w set by S at stage 1, there are two possible equilibria in the subgame. In the first equilibrium, M sells its own product only. This equilibrium exists only when $w \geq \Delta$. The second type of equilibrium involves M selling S's product. This equilibrium exists only if $w \leq \Delta$. We can derive the following overall equilibrium:

Proposition A.1 *In the overall equilibrium, S sets the wholesale price $w = \Delta$ and M resells S's product. In the mixed-strategy pricing equilibrium, M sells S's product at price p_m^{s*} distributed according to c.d.f F_m, where*

$$F_m\left(p_m^{s*}\right) = 1 - \frac{\mu(\Delta + c + b - p_m^{s*})}{(1-\mu)\left(p_m^{s*} - b - w\right)} \text{ for } p_m^{s*} \in \left[c + \mu\Delta + (w-c)(1-\mu) + b, c + b + \Delta\right],$$

and set $p_m^{h*} > c + b$ for its in-house product. Meanwhile, S's outside price p_o^* is distributed according to c.d.f F_o, where

$$F_o\left(p_o^*\right) = \begin{cases} 1 - \frac{(\Delta+c-w)\mu+b}{p_o^*-w+b} & \text{for} \quad p_o^* \in [c + \mu\Delta + (w-c)(1-\mu), c+\Delta) \\ 1 & \text{for} \quad p_o^* \geq c + \Delta \end{cases}.$$

Equilibrium profits are $\Pi^{sell} = \left((\Delta + c - w)\mu + b\right)(1-\mu)$ and $\pi^{sell} = \mu\Delta + (w-c)(1-\mu)$.

Proof. The analysis of this model is the same as the one in Section G.3. ∎

Intuitively, it is profitable for S to sell to M in stage 1 as doing so relaxes the subsequent cross-channel competition in stage 2. This is because in the pricing subgame S would partially internalize M's sales.

[1]Boston University Questrom School of Business. E-mail: ahagiu@bu.edu
[2]Department of Economics, National University of Singapore, E-mail: tehtathow@u.nus.edu
[3]Department of Economics, National University of Singapore, E-mail: jwright@nus.edu.sg

A.2 Dual mode with reselling

We have the following timing: (i) M sets the fee τ and S sets the wholesale price w; (ii) after observing τ and w, M chooses whether to resell S's product (in addition to M's in-house offering); and (iii) S decides whether to participate, and then all parties compete in prices.

For each given τ and w, consider stage 3. Suppose M has chosen to resell S's product. If S does not participate, then the subgame unfolds as in the seller mode analyzed in the previous subsection, and S's profit is $\pi = \mu\Delta + (w - c)(1 - \mu)$. If S participates, it can be shown that S's profit is weakly lower than $\mu\Delta + (w - c)(1 - \mu)$ for all τ and w.[4] It follows that S does not participate after knowing that M is reselling its product. Suppose instead M has chosen not to resell S's product, then the subgame unfolds as in the original dual mode of the baseline model in the main text.

In stage 2, if M's chooses to resell S's product, its profit is $\Pi = ((\Delta + c - w)\mu + b)(1 - \mu)$. If it chooses not to resell S's product, its profit is stated in Table 3. Comparing the profit expressions, we can show that M prefers not to resell S's product if and only if

$$\tau \geq (\Delta + c - w)\mu + b \text{ and } \tau \leq b + \mu \min\left\{\Delta, \frac{b+c}{1-\mu}\right\}. \tag{A.1}$$

In this case, the profits are

$$\begin{aligned} \Pi &= \tau(1-\mu) \\ \pi &= \mu\Delta + (1-\mu)\min\left\{\Delta + \frac{b-\tau}{\mu}, \Delta - c\right\}. \end{aligned}$$

Otherwise, M prefers to resell S's product, and the profits are

$$\begin{aligned} \Pi &= (b + (c + \max\{\Delta - w, 0\})\mu)(1-\mu) \\ \pi &= \mu\Delta + (w-c)(1-\mu). \end{aligned}$$

Consider stage 1. By an envelope theorem argument, M's profit is weakly increasing in τ as long as (A.1) holds. Therefore, M's dominant strategy is to set $\tau = b + \mu \min\left\{\Delta, \frac{b+c}{1-\mu}\right\}$. It remains to check S's wholesale pricing decision.

Suppose $\Delta \leq \frac{b+c}{1-\mu}$. When $\tau = b + \mu\Delta$, all $w \geq c$ are profit-equivalent for S because any change in w does not affect M's behaviour at stage 2: M will always choose not to resell S's product. Meanwhile, any $w < c$ is loss-making. Consequently, the overall equilibrium has $\tau = b + \mu\Delta$, $w \geq c$, and M choosing not to resell S's product.

Suppose $\Delta > \frac{b+c}{1-\mu}$. When $\tau = b + \mu\left(\frac{b+c}{1-\mu}\right)$, S can either set $w \leq \Delta + c - \frac{b+c}{1-\mu}$ to induce M to resell S's product, or set $w \geq \Delta + c - \frac{b+c}{1-\mu}$ to induce M not to resell S's product. The maximum profit from the first strategy is $\Delta - b - c$, which is the same as the profit from the second strategy. Consequently, the overall equilibrium has $\tau = b + \mu\left(\frac{b+c}{1-\mu}\right)$, $w \geq \Delta + c - \frac{b+c}{1-\mu}$, and M choosing not to resell S's product.

To summarize, the possibility of reselling does not affect the existing characterization of the dual mode in the main text (Proposition 3). In the overall equilibrium, S's product is sold by S exclusively. Intuitively, reselling is less profitable for M because S determines the term of trade (w). The wholesale price set by S is too high from M's perspective such that M always prefers not to resell S's product.

[4] When M resells S's product, there are only two possible equilibria in the pricing subgame: (i) all inside sales are made by M, so that S earns at most $\mu\Delta + (w-c)(1-\mu)$, or (ii) all inside sales are made by S, so that S earns at most $\mu\Delta + (p_i^* - c - \tau)(1 - \mu)$. For equilibrium (ii) to hold, we must have $p_i^* \leq w + \tau$ because otherwise M can profitably undercuts S (instead of earning through fees). There is no equilibrium with all regular consumers buying directly from S because M is selling S's product in its marketplace.

Comparing the profit expressions, we have

$$\begin{aligned}\Pi^{dual} &= \left(b+\mu\min\left\{\Delta,\frac{b+c}{1-\mu}\right\}\right)(1-\mu)\\ &> (b+c\mu)(1-\mu)=\Pi^{sell}\\ &\geq b(1-\mu)=\Pi^{market}.\end{aligned}$$

This means that banning the dual mode always results in the seller mode (with M reselling S's product):

Proposition A.2 *(Banning dual mode) A ban on the dual mode results in M choosing the seller mode (with M reselling S's product), with Π, CS_{regular}, CS, and W decreasing; CS_{direct} and π increasing.*

Proof. In the seller mode, $p_m^{s*}\in[c\mu+\Delta+b,c+b+\Delta]$. In the dual mode, $p_i^*=\max\{\Delta,c+b+\mu\Delta\}<p_m^{s*}$. Therefore, $CS_{regular}$ is higher in the dual mode. Meanwhile CS_{direct} is higher in the seller mode due to cross-channel competition. Finally, let $0<\eta<1$ denote the probability that regular consumers buy S's product from M in the equilibrium in seller mode. The associated welfare is $W^{sell}=v+\Delta-c+\eta(1-\mu)b<v+\Delta-c+(1-\mu)b=W^{dual}$. As for total consumer surplus:

$$\begin{aligned}CS^{sell} &= W^{sell}-\Pi^{sell}-\pi^{sell}\\ &= v+\Delta-c+\eta(1-\mu)b-(b+c\mu)(1-\mu)-\Delta+c(1-\mu)\\ &= v-c-(1-\eta)(1-\mu)b+(1-\mu)^2\Delta,\end{aligned}$$

which is lower than $CS^{dual}=v-c+(1-\mu)^2\Delta$. ■

B Continuous consumer types

Suppose that we have a continuum of consumer types, and each consumer is indexed by the convenience benefit b obtained from performing transactions through M. We assume $b\in[b_L,b_H]$ is distributed with cdf $G(.)$ and corresponding log-concave density $g(.)$, where $b_L\geq-\infty$ and $b_H\leq\infty$. Everything else is like in the baseline model. We assume the gap b_H-b_L is large enough so that equilibrium prices are always interior.

B.1 Pure marketplace

Recall fringe suppliers always set inside and outside prices at $p_i=c+\tau+\Delta$ and $p_o=c+\Delta$. Consider S's pricing problem after it joins the marketplace. It chooses p_i and p_o to maximize its profit

$$\begin{aligned}&(p_o-c)\,G(p_i-p_o)+(p_I-\tau-c)\,(1-G(p_i-p_o))\\ &\text{subject to } p_i\leq c+\tau+\Delta \text{ and } p_o\leq c+\Delta.\end{aligned}$$

Since S makes sales on both channels, it can always increase profit by raising its prices in both channels (by the same amount) whenever both pricing constraints do not bind. Therefore, at least one pricing constraint must bind. If $p_i^*=c+\tau+\Delta$, then S solves

$$\max_{p_o\leq c+\Delta}\left\{(p_o-c)\,G(p_i^*-p_o)+\Delta\,(1-G(p_i^*-p_o))\right\}.$$

For all $p_o\leq c+\Delta$, an increase in p_o: (i) raises the margin in the outside channel; (ii) shifts demand from the lower-margin outside channel to the higher-margin inside channel (due to the constraint on p_o), and so we must have $p_o^* = c + \Delta$. If we start with $p_o^* = c + \Delta$ instead, then S solves

$$\max_{p_i \leq c+\tau+\Delta} \left\{\Delta G(p_i - p_o^*) + (p_i - \tau - c)\left(1 - G(p_i - p_o^*)\right)\right\},$$

and the same logic as above implies $p_i^* = c + \tau + \Delta$. Therefore, we conclude that $p_i^* = c + \tau + \Delta$ and $p_o^* = c + \Delta$, and S earns profit $\pi^{market} = \Delta$.

On the other hand, if S does not participate, its profit is $\max_{p_o \leq c+\Delta} (p_o - c)\, G(c + \tau + \Delta - p_o) < \pi^{market}$. Therefore, S always participates.

Since the cross-channel utility difference is $p_i^* - p_o^* = \tau$, the number of consumers buying through the platform is $1 - G(\tau)$. Thus, M's profit as a pure marketplace is

$$\Pi^{market} = \max_{\tau} \left\{\tau\left(1 - G(\tau)\right)\right\},$$

so the optimal commission follows the usual monopoly price formula:

$$\tau^m = \frac{1 - G(\tau^m)}{g(\tau^m)}.$$

As in the baseline model with discrete consumer types, the marketplace's profit is independent of Δ. The reason is that the innovative supplier can fully extract the value of its innovation (inside and outside the platform).

B.2 Pure seller

We know S sets its outside price to maximize $(p_o - c)\, G(p_m - p_o + \Delta)$ subject to $p_o \leq c + \Delta$. Meanwhile, M's cost is zero so it maximizes $p_m\left(1 - G(p_m - p_o + \Delta)\right)$. If the constraint on p_o is non-binding, the equilibrium prices are jointly pinned down by:

$$p_o^* = c + \frac{G(p_m^* - p_o^* + \Delta)}{g(p_m^* - p_o^* + \Delta)} \text{ and } p_m^* = \frac{1 - G(p_m^* - p_o^* + \Delta)}{g(p_m^* - p_o^* + \Delta)}.$$

It is useful to denote the equilibrium *cross-channel utility difference* by A, where A is the unique solution to

$$A = \Delta - c + \frac{1 - 2G(A)}{g(A)}. \tag{B.1}$$

Then we have $p_o^* = c + \frac{G(A)}{g(A)}$ and $p_m^* = \frac{1-G(A)}{g(A)}$. The constraint $p_o \leq c + \Delta$ is non-binding if and only if $\frac{G(A)}{g(A)} < \Delta$. If instead $\frac{G(A)}{g(A)} \geq \Delta$, then the equilibrium prices are $p_o^* = c + \Delta$ and $p_m^* = \frac{1-G(B)}{g(B)}$, where B is the unique solution to

$$B = -c + \frac{1 - G(B)}{g(B)}. \tag{B.2}$$

It is useful to note that the log-concavity of g implies $\frac{1-G}{g}$ and $\frac{1-2G}{g}$ are decreasing functions, so

$$A \leq B \iff \Delta \leq \frac{G(A)}{g(A)},$$

where equality holds when $\Delta = \frac{G(A)}{g(A)}$, so that the condition in the right-hand side can be equivalently written as $\Delta \leq \frac{G(B)}{g(B)}$ whenever convenient. Then, the equilibrium profits can be summarized as $\Pi^{sell} = \min\left\{\frac{(1-G(B))^2}{g(B)}, \frac{(1-G(A))^2}{g(A)}\right\}$ and $\pi^{sell} = \min\left\{\Delta G(B), \frac{G(A)^2}{g(A)}\right\}$.

We relegate all cross-mode comparisons to Section B.4.

B.3 Dual mode

There are, in general, two possible types of equilibrium in the pricing subgame: (i) M makes all the inside sales (seller equilibrium), and (ii) S makes all the inside sales (marketplace equilibrium). As opposed to the baseline model with discrete consumer types, the heterogeneity in consumer types implies there is no direct sales equilibrium (i.e. in which no consumers buy through M) because the assumptions on $G(.)$ mean there are always some consumers who buy through M.

B.3.1 Dual mode - seller equilibrium

Consider first the extreme case where τ is sufficiently high so that M always wins the on-platform competition without being constrained by within-channel competition. Suppose S participates on M, and sets its outside price to maximize $(p_o - c) G(p_m - p_o + \Delta)$ subject to $p_o \leq c + \Delta$. Then M solves

$$\max_{p_m} p_m (1 - G(p_m - p_o + \Delta)) \text{ subject to } p_m \leq c - \Delta + \tau.$$

We first rule out any seller equilibrium in which the constraint on p_m is binding. Suppose by contradiction such a seller equilibrium exists. Then in equilibrium we must have $p_m^* = p_i^* - \Delta = c + \tau - \Delta$ and $p_o^* = c + \min\left\{\Delta, \frac{G(c+\tau-p_o^*)}{g(c+\tau-p_o^*)}\right\}$. However, M can profitably deviate from the candidate equilibrium by setting a very high p_m to let S win the inside competition, earning deviation profit

$$\Pi^{dev} = \tau (1 - G(c + \tau - p_o^*)) > \Pi^{eqm} = (c + \tau - \Delta)(1 - G(c + \tau - p_o^*)).$$

Next, suppose in equilibrium the constraint on p_m is non-binding. We can obtain the equilibrium of the simultaneous pricing game:

$$\begin{aligned} p_o^* &= c + \min\left\{\frac{G(p_m^* - p_o^* + \Delta)}{g(p_m^* - p_o^* + \Delta)}, \Delta\right\} \\ p_m^* &= \frac{1 - G(p_m^* - p_o^* + \Delta)}{g(p_m^* - p_o^* + \Delta)}. \end{aligned}$$

This is the same pricing equilibrium as in the seller mode, so the subgame equilibrium can be concisely described as

$$(p_o^*, p_m^*) = \begin{cases} \left(c + \Delta, \frac{1-G(B)}{g(B)}\right) & \text{if } \Delta \leq \frac{G(A)}{g(A)} \\ \left(c + \frac{G(A)}{g(A)}, \frac{1-G(A)}{g(A)}\right) & \text{if } \Delta > \frac{G(A)}{g(A)} \end{cases}, \tag{B.3}$$

and $p_i^* \geq c + \tau$, where A and B are defined in (B.1) and (B.2).

The equilibrium (B.3) is sustainable provided that (i) S has no incentive to undercut, and (ii) M has no incentive to let S win. Condition (i) is equivalent to

$$c + \tau + \Delta \geq \frac{1 - G(\max\{B, A\})}{g(\max\{B, A\})}. \tag{B.4}$$

If τ does not satisfy this condition, then $p_m^* > c - \Delta + \tau$ and so S has an incentive to undercut from (B.3).

Now consider condition (ii) required for the equilibrium (B.3) to exist. If $\Delta > \frac{G(A)}{g(A)}$, then M has no incentive to let S win if and only if $\tau > \bar{\tau}_A$, where $\bar{\tau}_A$ is the largest solution to the following indifference equation that equates M's deviation profit (by letting S win inside with its price $c + \tau$) with M's equilibrium profit:

$$\bar{\tau}_A \left(1 - G\left(\bar{\tau}_A - \frac{G(A)}{g(A)}\right)\right) = \frac{(1 - G(A))^2}{g(A)}. \tag{B.5}$$

Note that $\bar{\tau}_A \geq \Delta - c + \frac{1-G(A)}{g((A)}$,[5] so that $\tau \geq \bar{\tau}_A$ implies (B.4). If $\Delta \leq \frac{G(A)}{g(A)}$ then M has no incentive to let S win if and only if $\tau > \bar{\tau}_B$, where $\bar{\tau}_B$ is the largest solution to the following indifference equation that equates M's deviation profit (by letting S wins inside) with M's equilibrium profit:

$$\bar{\tau}_B \left(1 - G\left(\bar{\tau}_B - \Delta\right)\right) = \frac{\left(1 - G(B)\right)^2}{g(B)}. \tag{B.6}$$

Again, note that $\bar{\tau}_B \geq \Delta - c + \frac{1-G(B)}{g(B)}$,[6] so that $\tau \geq \bar{\tau}_B$ implies (B.4). Meanwhile, it can be easily verified that $\bar{\tau}_B > \bar{\tau}_A$ if and only if $\Delta > \frac{G(A)}{g(A)}$.

To summarize the construction of the seller equilibrium in dual mode:

- If $\Delta > \frac{G(A)}{g(A)}$, the seller equilibrium is sustainable if and only if $\tau > \bar{\tau}_A$. In this case $\Pi^{eqm} = \frac{(1-G(A))^2}{g(A)}$, $\pi^{eqm} = \frac{G(A)^2}{g(A)}$, $(p_o^*, p_m^*) = \left(c + \frac{G(A)}{g(A)}, \frac{1-G(A)}{g(A)}\right)$ and $p_i^* \geq c + \tau$.
- If $\Delta \leq \frac{G(A)}{g(A)}$, the seller equilibrium is sustainable iff $\tau > \bar{\tau}_B$. In this case $\Pi^{eqm} = \frac{(1-G(B))^2}{g(B)}$, $\pi^{eqm} = \Delta G(B)$, $(p_o^*, p_m^*) = \left(c + \Delta, \frac{1-G(B)}{g(B)}\right)$ and $p_i^* \geq c + \tau$.

B.3.2 Dual mode - marketplace equilibrium

Given that in any marketplace equilibrium S makes all sales in both channels, it can always profitably increase both p_i and p_o until one of the following constraints binds: $p_i \leq \min\{p_m^* + \Delta, c + \Delta + \tau\}$ and $p_o \leq c + \Delta$, where p_m^* is some arbitrarily given price set by M. If only the constraint on the outside price binds, then $p_o^* = c + \Delta$ while p_i is interior and solves

$$\begin{aligned} & \max_{p_i \leq \min\{p_m^* + \Delta, c + \Delta + \tau\}} \left\{ (p_o^* - c)\, G(p_i - p_o^*) + (p_i - \tau - c)\left(1 - G(p_i - p_o^*)\right) \right\} \\ = & \max_{p_i \leq \min\{p_m^* + \Delta, c + \Delta + \tau\}} \left\{ \Delta G(p_i - c - \Delta) + (p_i - \tau - c)\left(1 - G(p_i - c - \Delta)\right) \right\}. \end{aligned}$$

The first-order condition implies $p_i = c + \Delta + \tau + \frac{1-G(p_i - c - \Delta)}{g(p_i - c - \Delta)} > c + \Delta + \tau$, violating the constraint on p_i. Therefore the constraint on p_i must bind. For any given p_i^*, p_o solves

$$\max_{p_o \leq c + \Delta} \left\{ (p_o - c)\, G(p_i^* - p_o) + (p_i^* - \tau - c)\left(1 - G(p_i^* - p_o)\right) \right\}.$$

It is useful to define

$$\phi_\tau \equiv \tau - \frac{G\left(\phi_\tau\right)}{g\left(\phi_\tau\right)} \tag{B.7}$$

so that the first-order condition implies

$$p_o^* = \min\left\{ c + \Delta, \frac{G(\phi_\tau)}{g(\phi_\tau)} + p_i^* - \tau \right\}. \tag{B.8}$$

Then, asymmetric Bertrand competition on the marketplace implies $p_i^* = p_m^* + \Delta$ and $p_m^* \in [\max\{0, c + \tau - \Delta\}, \tau]$. Note p_m^* is indeterminate in this range because M makes no sales in equilibrium. We cannot have $p_m^* > \tau$ because in any such equilibrium M would have an incentive to undercut S and make the inside sales, earning a margin strictly greater than τ. Likewise, any $p_m^* < \max\{0, c + \tau - \Delta\}$ means either M or S is playing a dominated strategy in equilibrium.

[5] This follows from the observation that if we substitute $\Delta - c + \frac{1-G(A)}{g((A)}$ for $\bar{\tau}_A$ in the left-hand side of (B.5), then the left-hand side becomes greater than the right-hand side (recall that by definition $A = \Delta - c + \frac{1-2G(A)}{g(A)}$).

[6] This follows from the observation that if we substitute $\Delta - c + \frac{1-G(B)}{g((B)}$ for $\bar{\tau}_B$ in the left-hand side of (B.6), then the left-hand side becomes greater than the right-hand side.

To confirm this is an equilibrium, we need to make sure M does not have an incentive to deviate. M's equilibrium profit is

$$\Pi^{eqm} = \tau(1 - G(p_i^* - p_o^*)) = \tau(1 - G(\max\{p_m^* - c, \phi_\tau\})),$$

which is decreasing in p_m^*. Given we are looking for equilibrium that maximizes M's profit, we must have $p_m^* = \max\{0, c + \tau - \Delta\}$, and so $p_o^* = \min\{c + \Delta, p_m^* + \Delta - \phi_\tau\}$, while $p_i^* = p_m^* + \Delta$. There are four possible equilibrium configurations (ignoring firms' incentive to deviate):

- **Configuration 1:** $p_m^* = 0$, $p_o^* = c + \Delta$. This requires $\tau \leq \min\{\Delta - c, \frac{G(-c)}{g(-c)} - c\}$. $\Pi^{eqm} = \tau(1 - G(-c))$ and $\pi = \Delta - (c + \tau)G(-c)$.
- **Configuration 2:** $p_m^* = 0$, $p_o^* = \Delta - \phi_\tau$. This requires $\tau \in (\frac{G(-c)}{g(-c)} - c, \Delta - c]$. $\Pi^{eqm} = \tau(1 - G(\phi_\tau))$ and $\pi = \Delta - (c + \tau)G(\phi_\tau)$
- **Configuration 3:** $p_m^* = c + \tau - \Delta$, $p_o^* = c + \tau - \phi_\tau$. This requires $\tau \in [\Delta - c, \bar{\tau}_1)$, where

$$\bar{\tau}_1 \equiv \phi_{\bar{\tau}_1} + \Delta \tag{B.9}$$

  is such that $\tau < \bar{\tau}_1 \Leftrightarrow \tau - \phi_\tau < \Delta$. $\Pi^{eqm} = \tau(1 - G(\phi_\tau))$ and $\pi = \Delta G(\phi_\tau)$.
- **Configuration 4:** $p_m^* = c + \tau - \Delta$, $p_o^* = c + \Delta$. This requires $\tau \geq \max\{\Delta - c, \bar{\tau}_1\}$. $\Pi^{eqm} = \tau(1 - G(\tau - \Delta))$ and $\pi = \Delta G(\tau - \Delta)$.

Bertrand competition means S has no incentive to deviate in any of these equilibria. So we simply need to make sure M has no incentive to deviate (by undercutting S) for equilibria with $p_m^* = c + \tau - \Delta > 0$. To do so, we will use the following technical lemma:

Lemma B.1 *$\Delta > \frac{G(-c)}{g(-c)}$ if and only if $\bar{\tau}_1 > \Delta - c$*

Proof. Given $\frac{d\phi_\tau}{d\tau} \in (0, 1)$, we know $\bar{\tau}_1 > \Delta - c$ if and only if $\Delta - c < \phi_{\Delta - c} + \Delta$, or $\phi_{\Delta - c} > c$. Using (B.7), the last condition is equivalent to $\Delta > \frac{G(-c)}{g(-c)}$. ■

Suppose $\Delta \leq \frac{G(-c)}{g(-c)}$, or equivalently, $\bar{\tau}_1 \leq \Delta - c$. This rules out configurations 2 and 3. For all $\tau \leq \Delta - c$, configuration 1 applies, and clearly M cannot profitably undercut S. For $\tau > \Delta - c$, configuration 4 applies, and M's deviation profit is

$$\Pi^{dev} = \max_{p'_m < c + \tau - \Delta} p'_m (1 - G(p'_m - c)).$$

Ignoring the upperbound constraint, the deviation profit is maximized at $p'_m = \frac{1 - G(B)}{g(B)}$. For all $\tau \leq \Delta - c + \frac{1 - G(B)}{g(B)} = B + \Delta$, the upperbound constraint on p'_m binds so $\Pi^{dev} = (\tau - \Delta + c)(1 - G(\tau - \Delta)) < \Pi^{eqm}$. For $\tau > \Delta - c + \frac{1 - G(B)}{g(B)}$, we have $\Pi^{dev} = \frac{(1 - G(B))^2}{g(B)}$, so M has no incentive to deviate if and only if $\tau \leq \bar{\tau}_B$, where

$$\bar{\tau}_B (1 - G(\bar{\tau}_B - \Delta)) = \frac{(1 - G(B))^2}{g(B)}$$

as in (B.6). Let then

$$\tau_B^* \equiv \arg\max\{\tau(1 - G(\tau - \Delta))\}.$$

By definition $\tau_B^* < B + \Delta$ because $B + \Delta = \Delta - c + \frac{1 - G(B)}{g(B)} > \frac{1 - G(B)}{g(B)}$, and so by transitivity, $\tau_B^* < \Delta - c + \frac{1 - G(B)}{g(B)} < \bar{\tau}_B$.

Suppose $\Delta > \frac{G(-c)}{g(-c)}$. For $\tau \leq \Delta - c$, configurations 1 and 2 apply, and clearly M cannot profitably undercut S. Consider $\tau \in [\Delta - c, \bar{\tau}_1)$. From configuration 3, the best deviation profit that M can achieve is

$$\Pi^{dev} = \max_{p'_m < c+\tau-\Delta} \left\{ p'_m \left(1 - G\left(p'_m - c + \phi_\tau - \tau + \Delta\right)\right) \right\}.$$

Ignoring the upperbound constraint, the deviation profit is maximized at $p'_m = \frac{1-G(X_\tau)}{g(X_\tau)}$, where

$$X_\tau \equiv \Delta - c - \tau + \phi_\tau + \frac{1 - G(X_\tau)}{g(X_\tau)}. \tag{B.10}$$

For all $\tau \leq \Delta - c + \frac{1-G(X_\tau)}{g(X_\tau)}$ (or equivalently, $\tau \leq A + \frac{G(A)}{g(A)}$),[7] the upperbound constraint on p'_m binds so $\Pi^{dev} = (\tau - \Delta + c)(1 - G(\phi_\tau)) < \Pi^{eqm}$. For $\tau > \Delta - c + \frac{1-G(X_\tau)}{g(X_\tau)}$ (or equivalently, $\tau > A + \frac{G(A)}{g(A)}$), $\Pi^{dev} = \frac{(1-G(X_\tau))^2}{g(X_\tau)}$, and M has no incentive to deviate if and only if $\tau \leq \bar{\tau}_X$, where

$$\bar{\tau}_X \left(1 - G\left(\phi_{\bar{\tau}_X}\right)\right) = \frac{(1 - G(X_{\bar{\tau}_X}))^2}{g(X_{\bar{\tau}_X})}. \tag{B.11}$$

The existence of $\bar{\tau}_X$ follows from the intermediate value theorem. In what follows, we assume $\tau(1-G(\phi_\tau))$ is quasiconcave.[8] Let

$$\tau_X^* \equiv \arg\max_\tau \tau(1 - G(\phi_\tau)),$$

or $\tau_X^* = \frac{1-G(\phi_{\tau_X^*})}{g(\phi_{\tau_X^*})(d\phi_\tau/d\tau)}$. Finally, for $\tau \geq \bar{\tau}_1$, configuration 4 applies and the analysis follows from the previous paragraph. In particular, the configuration is an equilibrium if and only if $\tau \leq \bar{\tau}_B$.

The following two technical lemmas identify the relative ordering of these cutoffs.

Lemma B.2 *(i)* $\bar{\tau}_X \leq \bar{\tau}_1 \iff \bar{\tau}_B \leq \bar{\tau}_1$*; (ii)* $\bar{\tau}_X \leq \bar{\tau}_1 \implies \bar{\tau}_X \geq \bar{\tau}_B$*; (iii)* $\bar{\tau}_B \geq B + \Delta$*, and* $\bar{\tau}_X \geq A + \frac{G(A)}{g(A)}$*; (iv)* $\tau_X^* \leq \bar{\tau}_1 \implies \tau_B^* \leq \bar{\tau}_1$.

Proof. (i) From definitions, $\bar{\tau}_X \leq \bar{\tau}_1 \iff \bar{\tau}_1 \left(1 - G\left(\phi_{\bar{\tau}_1}\right)\right) \leq \frac{\left(1-G(X_{\bar{\tau}_1})\right)^2}{g(X_{\bar{\tau}_1})} = \frac{(1-G(B))^2}{g(B)}$, where the last equality used $X_{\bar{\tau}_1} = B$, while $\bar{\tau}_X \leq \bar{\tau}_1 \iff \bar{\tau}_1 \left(1 - G\left(\phi_{\bar{\tau}_1}\right)\right) \leq \frac{(1-G(B))^2}{g(B)}$. So $\bar{\tau}_X \leq \bar{\tau}_1 \iff \bar{\tau}_B \leq \bar{\tau}_1$. (ii) To show $\bar{\tau}_X > \bar{\tau}_B$, consider

$$\Gamma(\tau) \equiv \tau(1 - G(c + \tau - p_o^*)) - \max_{p'_m} p'_m \left(1 - G\left(p'_m - p_o^* + \Delta\right)\right).$$

If we denote $p^{dev} = \arg\max_{p'_m} p'_m \left(1 - G\left(p'_m - p_o^* + \Delta\right)\right)$, then by envelope theorem:

$$\frac{d\Gamma(\tau)}{dp_o^*} = \left(\tau - \left(p^{dev} - c\right) \frac{g\left(p^{dev} - p_o^* + \Delta\right)}{g(c + \tau - p_o^*)}\right) g(c + \tau - p_O^*) \geq 0,$$

where we used log-concavity of g and $p^{dev} \leq c + \tau - \Delta$. Given the supposition $\bar{\tau}_X \leq \bar{\tau}_1$, we have $p_o^* = c + \bar{\tau}_X - \phi_{\bar{\tau}_X} \leq c + \Delta$, and so

$$\Gamma(\bar{\tau}_X)_{p_o^* = c + \bar{\tau}_X - \phi_{\bar{\tau}_X}} = 0 \leq \Gamma(\bar{\tau}_X)_{p_o^* = c + \Delta}$$

[7] Specifically, $\tau \leq A + \frac{G(A)}{g(A)} \Leftrightarrow \phi_\tau \leq A \Leftrightarrow X_\tau \leq A$. Therefore, $\tau \leq A + \frac{G(A)}{g(A)} = \Delta - c + \frac{1-G(A)}{g(A)}$ implies $\tau \leq \Delta - c + \frac{1-G(X_\tau)}{g(X_\tau)}$. Likewise, $\tau > A + \frac{G(A)}{g(A)} = \Delta - c + \frac{1-G(A)}{g(A)}$ implies $\tau > \Delta - c + \frac{1-G(X_\tau)}{g(X_\tau)}$. Therefore the two conditions are equivalent.

then RHS is $\frac{(1-G(A))^2}{g(A)}$, LHS is $(\Delta - c + \frac{1-G(A)}{g(A)})(1 - G(A))$. Therefore, for general $\tau > A + \frac{G(A)}{g(A)}$, we have $X_\tau < A$.

[8] A sufficient condition is ϕ_τ being convex, which is satisfied if G is uniform.

Suppose by contradiction $\bar{\tau}_X > \bar{\tau}_B$, then given $\Gamma(\tau)_{p_o^*=c+\Delta}$ is decreasing for $\tau \geq \bar{\tau}_B$ and $\Gamma(\bar{\tau}_B)_{p_o^*=c+\Delta} = 0$, we have $\Gamma(\bar{\tau}_X)_{p_o^*=c+\Delta} < 0$, a contradiction. Therefore, $\bar{\tau}_X \leq \bar{\tau}_B$ must hold. (iii) From the definitions, we know $\bar{\tau}_B \geq \Delta - c + \frac{1-G(B)}{g(B)}$ and $\bar{\tau}_X \geq \Delta - c + \frac{1-G(X)}{g(X)}$. (iv) From definitions $\tau_X^* \leq \bar{\tau}_1 \iff \bar{\tau}_1 > \frac{1-G(\phi_{\bar{\tau}_1})}{g(\phi_{\bar{\tau}_1})(d\phi_\tau/d\tau)}$, and $\tau_B^* \leq \bar{\tau}_1 \iff \bar{\tau}_1 > \frac{1-G(\bar{\tau}_1-\Delta)}{g(\bar{\tau}_1-\Delta)} = \frac{1-G(\phi_{\bar{\tau}_1})}{g(\phi_{\bar{\tau}_1})}$. So $\tau_X^* \leq \bar{\tau}_1 \implies \tau_B^* \leq \bar{\tau}_1$, given that $d\phi_\tau/d\tau \in (0,1)$. ■

B.3.3 Dual mode - overall equilibrium

We can now combine both types of equilibrium to pin down S's participation decision and M's optimization problem in setting τ. Recall that if S does not participate, then the pricing subgame unfolds as if M operated as a pure seller.

Suppose $\Delta \leq \frac{G(-c)}{g(-c)} < \frac{G(A)}{g(A)}$ and S participates. In the pricing subgame, the seller equilibrium exists if and only if $\tau \geq \bar{\tau}_B$. Meanwhile, from the previous subsection, we know the marketplace equilibrium exists if and only if $\tau \leq \bar{\tau}_B$. To summarize the outcome of the post-participation subgame:

Range of τ	$\Pi^{eqm}(\tau)$	$\pi^{eqm}(\tau)$
$\tau \leq \Delta - c$	$\tau(1-G(-c))$	$\Delta - (c+\tau)(1-G(-c))$
$\tau \in [\Delta - c, \bar{\tau}_B]$	$\tau(1-G(\tau-\Delta))$	$\Delta G(\tau-\Delta)$
$\tau \geq \bar{\tau}_B$ (seller eqm)	$\frac{(1-G(B))^2}{g(B)}$	$\Delta G(B)$

If S does not participate, its profit is $\pi^{np} = \Delta G(B)$. For $\tau \leq \Delta - c$, we have $\pi^{eqm} \geq \pi^{np}$ if and only if $\tau(1-G(-c)) \leq \Delta(1-G(B)) - c(1-G(-c))$, implying $\tau(1-G(-c)) \leq (\Delta - c)(1-G(B))$ due to $-c < B$. For $\tau \geq \Delta - c$, we have $\pi^{eqm} \geq \pi^{np}$ if and only if $\tau \geq B + \Delta$. Note that $B + \Delta \leq \bar{\tau}_B$ by Lemma B.2 point (iii), so $\tau \geq B + \Delta$ is feasible. By setting $\tau = B + \Delta$, M achieves the profit $\Pi^{eqm} = (B+\Delta)(1-G(B))$, which is higher than the profit from setting $\tau > \bar{\tau}_B$ or $\tau \leq \Delta - c$ that still ensures participation. Moreover, $\tau_B^* < B + \Delta$ and so M cannot achieve higher profit by setting $\tau \in (B+\Delta, \bar{\tau}_B]$. We conclude $\tau^{dual} = B + \Delta$.

Next, suppose $\frac{G(-c)}{g(-c)} < \Delta \leq \frac{G(A)}{g(A)}$ and S participates. The seller equilibrium exists if and only if $\tau \geq \bar{\tau}_B$. Meanwhile,

$$A + \frac{G(A)}{g(A)} \geq \bar{\tau}_1 \iff \phi_{\bar{\tau}_1} \leq A \iff \Delta \leq \frac{G(A)}{g(A)}.$$

From Lemma B.2 point (iii), we know that $\bar{\tau}_X \geq A + \frac{G(A)}{g(A)}$, and so $\Delta \leq \frac{G(A)}{g(A)} \implies \bar{\tau}_X \geq \bar{\tau}_1 \Leftrightarrow \bar{\tau}_B \leq \bar{\tau}_1$ by Lemma B.2 point (i). Therefore, from the analysis of the marketplace equilibrium for the case $\Delta > \frac{G(-c)}{g(-c)}$, we know that the marketplace equilibrium exists if and only if $\tau \leq \bar{\tau}_B$. To summarize the outcome of the subgame that starts after S's decision to participate:

Range of τ	$\Pi^{eqm}(\tau)$	$\pi^{eqm}(\tau)$
$\tau \leq \frac{G(-c)}{g(-c)} - c$	$\tau(1-G(-c))$	$\Delta - (c+\tau)(1-G(-c))$
$\tau \in \left[\frac{G(-c)}{g(-c)} - c, \Delta - c\right]$	$\tau(1-G(\phi_\tau))$	$\Delta - (c+\tau)(1-G(\phi_\tau))$
$\tau \in [\Delta - c, \bar{\tau}_1)$	$\tau(1-G(\phi_\tau))$	$G(\phi_\tau)^2/g(\phi_\tau)$
$\tau \in [\bar{\tau}_1, \bar{\tau}_B]$	$\tau(1-G(\tau-\Delta))$	$\Delta G(\tau-\Delta)$
$\tau \geq \bar{\tau}_B$ (seller eqm)	$\frac{(1-G(B))^2}{g(B)}$	$\Delta G(B)$

If S does not participate, its profit is $\pi^{np} = \Delta G(B)$. To proceed, first note that the definitions of A, B and $\bar{\tau}_1$ imply

$$\Delta \leq \frac{G(A)}{g(A)} \Leftrightarrow \Delta \leq \frac{G(B)}{g(B)} \Leftrightarrow B + \Delta \geq \bar{\tau}_1.$$

This means that for all $\tau \leq \bar{\tau}_1$, we have $\phi_\tau \leq \phi_{\bar{\tau}_1} = \bar{\tau}_1 - \Delta \leq B$. For $\tau \leq \Delta - c$, similar to the previous paragraph, $\pi^{eqm} \geq \pi^{np}$ only if $\Pi^{eqm}(\tau) \leq (\Delta - c)(1 - G(B))$. For $\tau \in [\Delta - c, \bar{\tau}_1)$, we have $G(\phi_\tau)^2 / g(\phi_\tau) < \Delta G(\phi_\tau) \leq \Delta G(B)$, where the first inequality used $G(\phi_\tau)/g(\phi_\tau) < \Delta$ for all $\tau < \bar{\tau}_1$ given how $\bar{\tau}_1$ is defined. Therefore S does not participate for τ within this region. For $\tau \geq \Delta - c$, we have $\pi^{eqm} \geq \pi^{np}$ if and only if $\tau \geq B + \Delta$, and note $B + \Delta \leq \bar{\tau}_B$ and so $\tau \geq B + \Delta$ is feasible. By setting $\tau = B + \Delta$, M achieves the profit $\Pi^{eqm} = (B+\Delta)(1 - G(B))$, higher than the profit from setting $\tau \notin [\bar{\tau}_1, \bar{\tau}_B]$ (while still ensuring participation). Moreover, we know $\tau_B^* < B + \Delta$ and so M cannot achieve higher profit by setting $\tau \in (B + \Delta, \bar{\tau}_B]$. We conclude $\tau^{dual} = B + \Delta$.

Suppose $\Delta > \frac{G(A)}{g(A)}$. Note that $\Delta > \frac{G(A)}{g(A)} \implies A > -c + \frac{1-G(A)}{g(A)} \implies A > -c$, implying $\Delta > \frac{G(-c)}{g(-c)}$. The seller equilibrium exists if and only if $\tau \geq \bar{\tau}_A$. If $\bar{\tau}_B > \bar{\tau}_1$, then the seller equilibrium exists if and only if $\tau \leq \bar{\tau}_B$, and $\bar{\tau}_B > \bar{\tau}_A$ given $\Delta > \frac{G(A)}{g(A)}$. If $\bar{\tau}_B \leq \bar{\tau}_1$, then the marketplace equilibrium exists if and only if $\tau \leq \bar{\tau}_X$, and note $\bar{\tau}_X > A + \frac{G(A)}{g(A)} > \bar{\tau}_A$ by Lemma B.2 point (iii). There is therefore a parameter region in which both the seller equilibrium and the marketplace equilibrium coexist.

Consider first the subcase of $\Delta > \frac{G(A)}{g(A)}$ with $\bar{\tau}_X > \bar{\tau}_1$. We have

Range of τ	$\Pi^{eqm}(\tau)$	$\pi^{eqm}(\tau)$
$\tau \leq \frac{G(-c)}{g(-c)} - c$	$\tau(1 - G(-c))$	$\Delta - (c + \tau)(1 - G(-c))$
$\tau \in \left[\frac{G(-c)}{g(-c)} - c, \Delta - c\right]$	$\tau(1 - G(\phi_\tau))$	$\Delta - (c + \tau)(1 - G(\phi_\tau))$
$\tau \in [\Delta - c, \bar{\tau}_1)$	$\tau(1 - G(\phi_\tau))$	$G(\phi_\tau)^2 / g(\phi_\tau)$
$\tau \in [\bar{\tau}_1, \bar{\tau}_B]$	$\tau(1 - G(\tau - \Delta))$	$\Delta G(\tau - \Delta)$
$\tau \geq \bar{\tau}_A$ (seller eqm)	$\frac{(1-G(A))^2}{g(A)}$	$G(A)^2/g(A)$.

If S does not participate, its profit is $\pi^{np} = G(A)^2/g(A)$. For $\tau \leq \Delta - c$, we have $\Delta - c < A + \frac{G(A)}{g(A)}$ implies $\phi_\tau < A$ (due to the definition of A), and so $\pi^{eqm} \geq \pi^{np}$ if and only if

$$\Pi^{eqm}(\tau) \leq \Delta - c(1 - G(\phi_\tau)) - \frac{G(A)^2}{g(A)} = (\Delta - c)(1 - G(\phi_\tau)) - \left(\Delta G(\phi_\tau) - \frac{G(A)^2}{g(A)}\right).$$

For $\tau \in [\Delta - c, \bar{\tau}_1)$, we have $\pi^{eqm} = G(\phi_\tau)^2 / g(\phi_\tau)$. Given $\Delta > \frac{G(A)}{g(A)}$, we know $A + \frac{G(A)}{g(A)} \in [\Delta - c, \bar{\tau}_1]$ and so $\tau \geq A + \frac{G(A)}{g(A)}$ is feasible. Notice that for $\tau \geq A + \frac{G(A)}{g(A)}$, we have $\pi^{eqm} = G(\phi_\tau)^2 / g(\phi_\tau)$ increasing and continuous in τ and $\pi^{eqm} = G(\phi_\tau)^2 / g(\phi_\tau)$ when $\tau = A + \frac{G(A)}{g(A)}$. Therefore, $\pi^{eqm} \geq G(A)^2/g(A) = \pi^{np}$. Moreover, by setting $\tau = A + \frac{G(A)}{g(A)}$, M achieves the profit $\Pi^{eqm} = (\Delta - c + \frac{1-G(A)}{g(A)})(1 - G(A))$. For $\tau \in [\bar{\tau}_1, \bar{\tau}_B]$, given $\Delta > \frac{G(A)}{g(A)} \implies \bar{\tau}_1 > B + \Delta > \tau_B^*$, we know that $\Pi^{eqm}(\tau)$ is decreasing for all τ in this range. Therefore, all $\tau \in [\bar{\tau}_1, \bar{\tau}_B]$ is dominated by $\bar{\tau}_1$. We conclude that M either sets $\tau \in \left[A + \frac{G(A)}{g(A)}, \bar{\tau}_1\right]$, or sets $\tau \leq \Delta - c$.

Next consider the subcase of $\Delta > \frac{G(A)}{g(A)}$ with $\bar{\tau}_X \leq \bar{\tau}_1$. We have

Range of τ	$\Pi^{eqm}(\tau)$	$\pi^{eqm}(\tau)$
$\tau \leq \frac{G(-c)}{g(-c)} - c$	$\tau(1 - G(-c))$	$\Delta - (c + \tau)(1 - G(-c))$
$\tau \in \left[\frac{G(-c)}{g(-c)} - c, \Delta - c\right]$	$\tau(1 - G(\phi_\tau))$	$\Delta - (c + \tau)(1 - G(\phi_\tau))$
$\tau \in [\Delta - c, \bar{\tau}_X]$	$\tau(1 - G(\phi_\tau))$	$G(\phi_\tau)^2 / g(\phi_\tau)$
$\tau \geq \bar{\tau}_A$	$\frac{(1-G(A))^2}{g(A)}$	$G(A)^2/g(A)$.

By a very similar analysis to that in the previous paragraph, we can establish that M optimally sets $\tau \in \left[A + \frac{G(A)}{g(A)}, \bar{\tau}_X\right]$, or sets some $\tau \leq \Delta - c$.

To summarize:

Proposition B.1 *In the overall equilibrium, M always sets τ to induce the marketplace equilibrium.*

- *If $\Delta \leq \frac{G(A)}{g(A)}$, M sets $\tau^{dual} = B + \Delta$ and S participates. In equilibrium, $p_m^* = c + \tau^{dual} - \Delta$, $p_i^* = c + \tau^{dual}$ and $p_o^* = c + \Delta$.*
- *If $\Delta > \frac{G(A)}{g(A)}$, either (i) M sets $\tau^{dual} \in \left[A + \frac{G(A)}{g(A)}, \min\{\bar{\tau}_1, \bar{\tau}_X\}\right]$ and S participates, with equilibrium prices $p_m^* = c + \tau^{dual} - \Delta$, $p_i^* = c + \tau^{dual}$ and $p_o^* = c + \tau^{dual} - \phi_{\tau^{dual}}$; or (ii) M sets $\tau^{dual} \leq \Delta - c$, and S participates, with equilibrium prices $p_m^* = 0$, $p_i^* = \Delta$ and $p_o^* = \Delta + \min\{c, -\phi_{\tau^{dual}}\}$.*

To complete the equilibrium characterization, we explore the case $\Delta > \frac{G(A)}{g(A)}$ with numerical simulations. Based on Proposition B.1, M's optimization problem is:

$$\max_{\tau} \Pi^{eqm}(\tau) \text{ subject to } \pi^{eqm}(\tau) \geq \pi^{np},$$

where

Range of τ	$\Pi^{eqm}(\tau)$	$\pi^{eqm}(\tau) - \pi^{np}$
$\tau \leq \Delta - c$	$\tau(1 - G(\max\{\phi_\tau, -c\}))$	$\Delta - (c+\tau)(1 - G(\max\{\phi_\tau, -c\})) - \frac{G(A)^2}{g(A)}$
$\tau \in \left[A + \frac{G(A)}{g(A)}, \min\{\bar{\tau}_1, \bar{\tau}_X\}\right]$	$\tau(1 - G(\phi_\tau))$	$\frac{G(\phi_\tau)^2}{g(\phi_\tau)} - \frac{G(A)^2}{g(A)}$

Intuitively, there are two possible regimes for M's commission: (i) a high τ that makes S earns zero profit from inside sales, but τ is high enough so that sufficiently many consumers purchase from S outside to ensure S participates, and (ii) a sufficiently low commission that makes S earns sufficient profit from inside sales, thereby ensuring its participation.

Example 1 *(Numerical example) Let $G \sim U[-1, 1]$, $c = 0.5$, and $\Delta \in [1.25, 3]$, Here, $\Delta > \frac{G(A)}{g(A)}$ is equivalent to $\Delta \geq 1.25$.*[9]

Figure 1 below plots M's optimal commission τ for any given Δ, and the induced inside and outside prices p_i^* and p_o^*. For Δ that is low enough, we have a "high commission regime" where M optimally sets a high commission at $\tau^{dual} \in \left[A + \frac{G(A)}{g(A)}, \min\{\bar{\tau}_1, \bar{\tau}_X\}\right]$. For this set of parameters, the participation constraint binds in the high commission regime, so $\tau^{dual} = A + \frac{G(A)}{g(A)} = \frac{2(\Delta+1)}{3}$. As Δ becomes higher, this participation constraint becomes tight, and M optimally switches to the "low commission regime" of $\tau^{dual} \leq \Delta - c$. Therefore, for intermediate value of Δ, the participation constraint binds M commission below $\tau^{dual} \leq \arg\max \tau(1 - G(\phi)) = 1.5$. In the low commission regime, the participation constraint relaxes when Δ increases, and so $\tau^{dual} \to 1.5$ as Δ increases.

B.4 Comparisons of the different modes

We first compare between the two pure modes:

Proposition B.2
- *M's profit: $\Pi^{market} > \Pi^{sell}$ if and only if $\Delta > c + \frac{G(\tau^m)}{g(\tau^m)}$.*
- *S's profit: $\pi^{market} > \pi^{sell}$.*
- *Total consumer surplus: $CS^{market} \leq CS^{sell}$, where the inequality is strict if $c > 0$.*
- *Welfare: $W^{market} > W^{sell}$ if and only if $\int_{-\infty}^{\max\{A,B\}} (\Delta - c) dG(b) - \int_{\max\{A,B\}}^{\tau^m} b dG(b) > 0$.*

[9] Generally, if G follows $U[b_L, b_H]$, then $B = \frac{1}{2}(b_H - c)$, $A = \frac{\Delta - c + b_H + b_L}{3}$, $\tau^m = \frac{b_H}{2}$, $\phi_\tau \equiv \frac{\tau + b_L}{2}$, $\bar{\tau}_1 \equiv b_L + 2\Delta$, $\tau_X^* = \frac{b_H}{2} - b_L$, and we have $\Delta > \frac{G(A)}{g(A)}$ if and only if $\Delta > \frac{b_H - c}{2} - b_L$

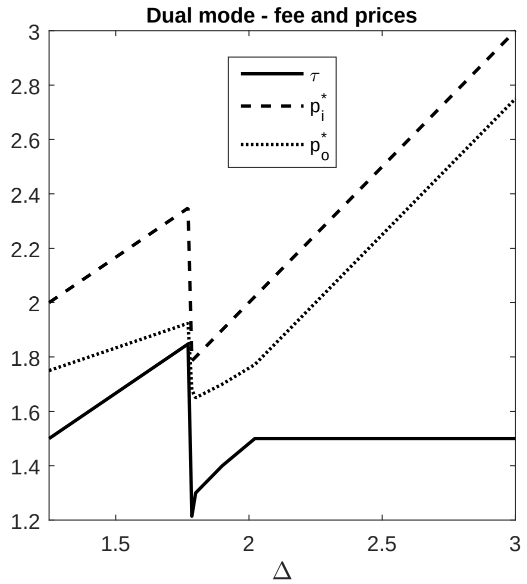

Figure 1: Equilibrium characterization of dual mode when $\Delta > G(A)/g(A)$, assuming $G \sim U[-1,1]$ and $c = 0.5$.

Proof. We know $\Pi^{market} = \frac{(1-G(\tau^m))^2}{g(\tau^m)}$ and $\Pi^{sell} = \frac{(1-G(\max\{A,B\}))^2}{g(\max\{A,B\})}$, so that $\Pi^{market} > \Pi^{sell}$ if and only if $\tau^m < \max\{A,B\}$. The stated condition in the proposition, $\Delta > c + \frac{G(\tau^m)}{g(\tau^m)}$, is equivalent to $\Delta - c + \frac{1-2G(\tau^m)}{g(\tau^m)} > \frac{1-G(\tau^m)}{g(\tau^m)} = \tau^m$, which is then equivalent to $A > \tau^m$ from definition (B.1), implying $\tau^m < \max\{A,B\}$ as required. Next, suppose $\Delta \leq c + \frac{G(\tau^m)}{g(\tau^m)}$, which is equivalent to $A \leq \tau^m$. Moreover, by definition (B.2), we have $B \leq \tau^m$. Thus, $\tau^m \geq \max\{A,B\}$, implying $\Pi^{market} \leq \Pi^{sell}$. Turning to S's profit, we have $\pi^{sell} = \Delta G(B) < \Delta = \pi^{market}$. Next, we can write down consumer surplus in each mode as (after doing some substitutions):

$$CS^{market} = v + \int_{\tau^m}^{\infty} (b - c - \tau^m)\, dG(b) + \int_{-\infty}^{\tau^m} -c dG(b),$$

$$CS^{sell} = \begin{cases} v + \int_B^{\infty} (b - c - B)\, dG(b) + \int_{-\infty}^{B} -c dG(b) & \text{if} \quad \Delta \leq \frac{G(A)}{g(A)} \\ v + \int_A^{\infty} \left(b - \frac{1-G(A)}{g(A)}\right) dG(b) + \int_{-\infty}^{A} \left(\Delta - \frac{G(A)}{g(A)} - c\right) dG(b) & \text{if} \quad \Delta > \frac{G(A)}{g(A)} \end{cases}.$$

If $\Delta \leq \frac{G(A)}{g(A)}$, then $CS^{sell} \geq CS^{market}$ follows from $B \leq \tau^m$. If $\Delta > \frac{G(A)}{g(A)}$, we note $\frac{1-G(A)}{g(A)}$ is decreasing in Δ, and approaches $\frac{1-G(B)}{g(B)} = B + c < c + \tau^m$ when $\Delta \to \frac{G(A)}{g(A)}$. Therefore, $\frac{1-G(A)}{g(A)} < c + \tau^m$ for all $\Delta > \frac{G(A)}{g(A)}$, implying $CS^{sell} > CS^{market}$. Finally, we have

$$W^{market} = v + \int_{\tau^m}^{\infty} (\Delta - c + b)\, dG(b) + \int_{-\infty}^{\tau^m} (\Delta - c)\, dG(b)$$

$$W^{sell} = v + \int_{\max\{A,B\}}^{\infty} b dG(b) + \int_{-\infty}^{\max\{A,B\}} (\Delta - c)\, dG(b)$$

Rearranging, we get

$$W_{market} - W_{sell} = \int_{-\infty}^{\max\{A,B\}} (\Delta - c)\, dG(b) - \int_{\max\{A,B\}}^{\tau^m} b dG(b).$$

■

The condition $\Delta > c + \frac{G(\tau^m)}{g(\tau^m)}$ for M to prefer the marketplace mode over the seller mode obtained here is analogous to the condition $\Delta \geq \frac{c}{1-\mu}$ in the baseline model with discrete consumer types. Thus, M prefers the marketplace mode when Δ is large relative to M's cost efficiency and the mass of consumers preferring to transact through the direct channel (i.e. those with low b), and the seller mode otherwise. The results for S's profit and total consumer surplus are consistent with the baseline model in the main text. The new result is the additional condition for $W^{market} > W^{sell}$. We interpret this condition below, together with the next result.

Compare now the dual mode with the two pure modes. For tractability, we first focus on the case with $\Delta \leq \frac{G(A)}{g(A)}$, in which we have a closed-form solution for M's optimal commission in dual mode. If G follows $U[b_L, b_H]$, the assumption is equivalent to $\Delta \leq \frac{b_H - 2b_L - c}{2}$.

Proposition B.3 *Suppose* $\Delta \leq \frac{G(A)}{g(A)}$*:*

- *M's profit:* $\Pi^{dual} > \Pi^{sell} \geq \Pi^{market}$.
- *S's profit:* $\pi^{market} > \pi^{dual} = \pi^{sell}$.
- *Consumer surplus:* $CS_{dual} = CS_{sell} \geq CS_{market}$*, where the inequality is strict if* $c > 0$.
- *Welfare:* $W^{dual} > W^{sell}$*;* $W^{dual} > W^{market}$ *if and only if*

$$\int_B^{\tau^m} b dG(b) > 0. \tag{B.12}$$

Proof. The supposition $\Delta \leq \frac{G(A)}{g(A)}$ implies $\Pi^{sell} = \frac{(1-G(B))^2}{g(B)} \geq \frac{(1-G(\tau^m))^2}{g(\tau^m)} = \Pi^{market}$. Meanwhile, the fact that in dual mode M strictly prefers choosing a τ that induces the marketplace equilibrium implies that $\Pi^{dual} = \left(\Delta - c + \frac{1-G(B)}{g(B)}\right)(1-G(B)) > \frac{(1-G(B))^2}{g(B)} = \Pi^{sell}$. As for S's profit, we have $\pi^{dual} = \Delta G(B) < \Delta = \pi^{market}$. It is straightforward to verify that $CS_{dual} = CS_{sell}$ given $\Delta \leq \frac{G(A)}{g(A)}$. As for welfare:

$$\begin{aligned} W_{dual} &= v + \int_B^{\infty} (\Delta - c + b) dG(b) + \int_{-\infty}^{B} (\Delta - c) dG(b) \\ &> v + \int_B^{\infty} b dG(b) + \int_{-\infty}^{B} (\Delta - c) dG(b) = W_{sell}. \end{aligned}$$

Finally, $W_{dual} - W_{market} = \int_B^{\tau^m} b dG$. ∎

Notably, with a continuum of consumer types we have $W^{dual} \neq W^{market}$ in general, in contrast to the baseline model. Given that all consumers purchase one unit of S's product regardless of which channel they use, the only welfare difference across these two modes is due to a possible distortion arising from cross-channel price differences. To the extent that S's price is lower in one channel than another, this will induce too many consumers to buy in the channel they do not prefer, potentially forgoing the transaction "benefit" b. Both modes potentially involve distortions. The marketplace involves the inside price being set τ^m higher than the outside price, whereas the dual mode may involve the inside price being set higher or lower than the outside price. If in dual mode the inside price is higher than the outside price,[10] the distortion is lower under the dual mode. However, if in dual mode the inside price is lower than the outside price, then the comparison is ambiguous. Condition (B.12) can then be understood as requiring that the distortion of inducing excessive usage of the marketplace channel in dual mode is more than offset by the under-utilization of the marketplace in the marketplace mode.

[10] A sufficient condition is $G(0)$ must be sufficiently small relative to c, i.e. $c \leq \frac{1-G(0)}{g(0)}$.

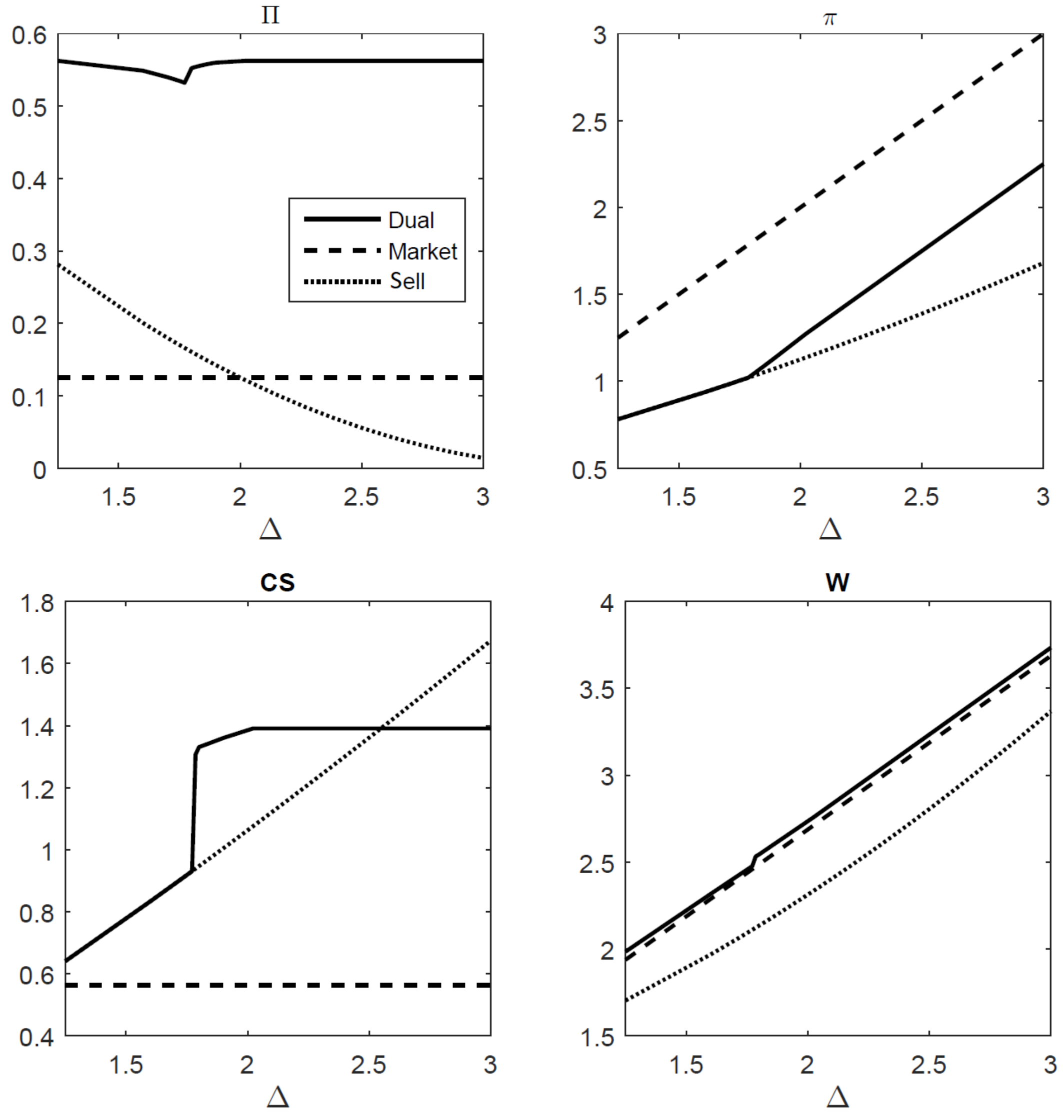

Figure 2: Cross-mode comparisons when $\Delta > G(A)/g(A)$, assuming $G \sim U[-1,1]$ and $c = 0.5$.

To numerically evaluate the case with $\Delta > \frac{G(A)}{g(A)}$, we consider two distinct sets of parameters: (i) $G \sim U[-1,1]$, $c = 0.5$, as in Example 1; and (ii) $G \sim U[-1,0.2]$, $c = 0.5$.[11] Figures 2 - 3 plot M's profit, S's profit, total consumer surplus, and welfare for each set of parameters.

The following observations are in order. First, $\Pi^{dual} > \max\left\{\Pi^{sell}, \Pi^{market}\right\}$, and a ban on the dual mode results in M choosing the seller mode if Δ is small, and choosing the marketplace mode if Δ is high.

Second, $\pi^{market} > \pi^{dual} \geq \pi^{sell}$. The last inequality reflects that S's participation constraint does not necessarily pin down M's commission in the dual mode. For Δ sufficiently large, S achieves a strictly higher profit under the dual mode than under the seller mode, because the benefit from accessing extra consumers (through being hosted) strictly outweighs the loss from having to pay commissions.

Third, the welfare comparisons are generally ambiguous due to the distortions in channel usage as discussed above. For this reason, the welfare effect of banning the dual mode can be ambiguous, in contrast to the baseline model with discrete consumer types.

[11] We also considered a range of other parameter values, obtaining similar qualitative insights. The details and the MATLAB code are available from the authors upon request.

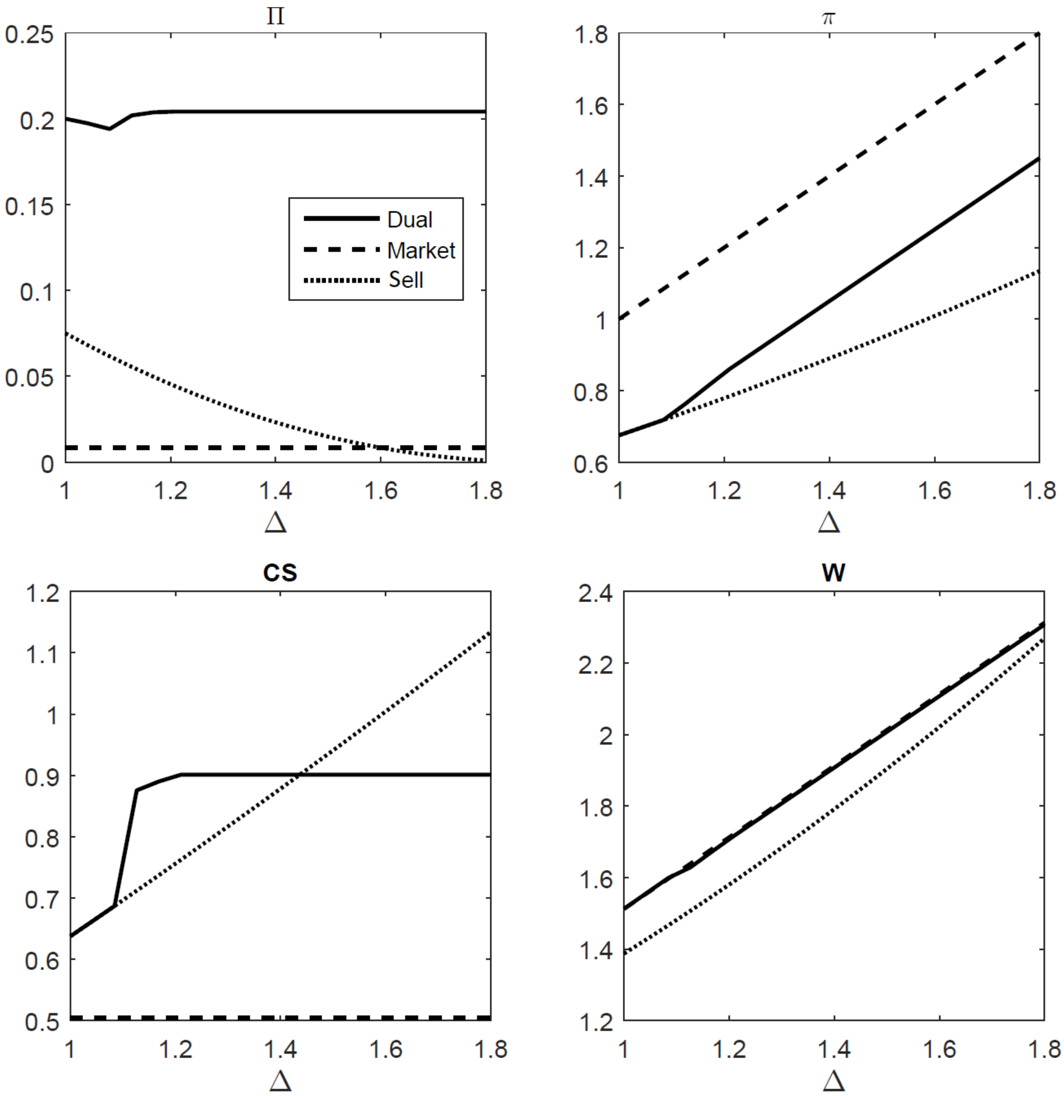

Figure 3: Cross-mode comparisons when $\Delta > G(A)/g(A)$, assuming $G \sim U[-1, 0.2]$ and $c = 0.5$.

Finally, $CS_{dual} > CS_{market}$, while $CS_{dual} \geq CS_{sell}$ if and only if Δ is not too large. Note that in Figure 2, a ban on the dual mode always weakly decreases consumer surplus. This is because in the range of Δ with M switching to the seller mode post-ban, we have $CS_{dual} \geq CS_{sell}$. In contrast, in Figure 3, a ban on the dual mode increases consumer surplus for some intermediate range of Δ, i.e. the range where $\Pi^{sell} > \Pi^{market}$ and $CS_{dual} < CS_{sell}$. This insight is consistent with the baseline model with discrete consumer types: whenever Δ is large enough, the seller mode leads to higher total consumer surplus as it allows for wider dissemination of the innovation surplus. However, if Δ is too high or is too low, M prefers to operate as a marketplace, resulting in lower consumer surplus.

C Commitment to functional separation

A less drastic regulatory alternative to fully banning the dual mode would be to require that M runs the marketplace and the seller divisions independently if it wants to continue adopting the dual mode. These divisions would involve separate teams that are not allowed to communicate or coordinate with each other. In this setting, the dual mode means having separate, competing marketplace and seller divisions, except that M owns both and considers their joint profits. Thus, although in equilibrium it will turn out that the seller division does not make any profits, M may still want to commit to operate it (and cover its fixed costs) if that allows M to extract larger profits from its marketplace division.

To make things clear, we call the new dual mode under which M runs a marketplace and a seller as separate divisions as the "separation mode". In the separation mode, we label the marketplace division as M_0 and the seller division as R. All other assumptions remain the same as in the baseline model: R has a cost of zero, purchases from R or sellers selling through M_0 provide convenience benefits b to regular consumers, and direct consumers never purchase from R or through M_0.

Timing: (1) M_0 sets τ; (2) sellers (including S) simultaneously choose whether to participate; (3) S, R, and fringe sellers set prices simultaneously. Here, there is no reason for R to sell on M_0 given it offers the same benefit directly, has the same underlying cost of zero, and competes for the same regular consumers, with the only difference being that selling on M_0 involves an additional cost of τ. Thus, without loss of generality, we can assume R does not participate on M_0.

To analyze the model, we first derive the equilibrium of the stage 3 subgame, assuming S participates. Similar to the analysis of the dual mode, there are three possible types of equilibria:

- *Marketplace* equilibrium—all regular consumers buy from S through the marketplace.
- *Direct sales* equilibrium—all regular consumers buy from S directly.
- *Seller* equilibrium—regular consumers sometimes buy from R.

Lemma C.1 *(Separation mode, marketplace equilibrium) In any marketplace equilibrium, $p_r^* = 0$, $p_i^* = \Delta$ and $p_o^* = c + \Delta$. The equilibrium exists if and only if $\tau \leq \min\left\{\Delta - c, \frac{b+c\mu}{1-\mu}\right\}$. Equilibrium profits are $\Pi_{M_0} = \tau(1-\mu)$, $\Pi_R = 0$, and $\pi = \mu\Delta + (\Delta - c - \tau)(1-\mu)$.*

Proof. With separation, in any marketplace equilibrium, the competition for regular consumers means S and R necessarily set $p_i^* = \Delta$ and $p_r^* = 0$ (otherwise R has an incentive to undercut), while $p_o^* = c + \Delta$. Clearly, R cannot profitably deviate. To ensure the stated price profile is indeed an equilibrium, we also need to make sure that (i) S is not making losses inside, which requires $\Delta - c - \tau \geq 0$; and (ii) S has no incentive to set a lower p_o to attract regular consumers to the direct channel, which requires

$$\Delta - b - c \leq \mu\Delta + (1-\mu)(\Delta - c - \tau) \iff \tau \leq \frac{b + c\mu}{1-\mu}.$$

Indeed, when $\tau \leq \Delta - c$, the deviation profit that S can attain by setting $p_o = \Delta - b$ to attract all regular consumers to buy directly is $\Delta - b - c$. This is weakly lower than the equilibrium profit if and only if $\tau \leq \frac{b+c\mu}{1-\mu}$. Finally, there is no other marketplace equilibrium given we ruled out all equilibria involving weakly dominated strategies. ■

Lemma C.2 *(Separation mode, direct sales equilibrium)*

- *If $\Delta < \frac{b+c}{1-\mu}$, then there is no direct sales equilibrium.*
- *If $\Delta \geq \frac{b+c}{1-\mu}$, then any price profile satisfying $p_i^* > \Delta$, $p_r^* = 0$ and $p_o^* = \Delta - b$ is a direct sales equilibrium. Direct sales equilibria exist if and only if $\tau \geq \frac{b+c\mu}{1-\mu}$. Equilibrium profits are $\Pi_{M_0} = \Pi_R = 0$ and $\pi = \Delta - b - c$.*

Proof. The proof of Lemma 2 applies. ■

As for the seller equilibrium, the main difference with its counterpart in the dual mode is that R does not have an incentive to sometimes let S win the inside competition, given that it no longer profits from a transaction commission. Therefore, in any possible seller equilibrium, S never makes sales inside.

Lemma C.3 *(Separation mode, seller equilibrium)*

- *If* $\Delta \leq \frac{b+c}{1-\mu}$ *and* $\tau \geq \Delta - c$*, any price profile satisfying* $p_r^* \in [0, \min\{c - \Delta + \tau, c + b - (1-\mu)\Delta\}]$*,* $p_i^* = p_r^* + \Delta$*, and* $p_o^* = c + \Delta$ *is a seller equilibrium. Equilibrium profits are* $\Pi_{M_0} = 0$*,* $\Pi_R = p_r^*(1-\mu)$*, and* $\pi = \mu\Delta$*.*
- *If* $\Delta > \frac{b+c}{1-\mu}$ *or* $\tau < \Delta - c$*, then there is no seller equilibrium.*

Proof. If $\Delta > \frac{b+c}{1-\mu}$ then no seller equilibrium exists. Therefore, we focus on the case $\Delta \leq \frac{b+c}{1-\mu}$ in what follows. Given regular consumers buy from R, we must have $p_r^* \geq 0$, otherwise R would make a loss. We next establish the upper bound for p_r^*. For any given p_r^* such that M sells to all regular consumers, S can profitably undercut by setting p_i slightly below $p_r^* + \Delta$ if and only if $p_r^* > c + \tau - \Delta$. Alternatively, S can undercut by setting p_o slightly below $p_r^* - b + \Delta$ to attract regular consumers to its direct channel, which yields $p_r^* - b + \Delta - c$. This is more profitable than setting $p_o = c + \Delta$ if and only if $p_r^* > c + b - (1-\mu)\Delta$. Thus, any

$$p_r^* \in \Phi_r \equiv [0, \min\{c - \Delta + \tau, c + b - (1-\mu)\Delta\}]$$

with $p_i^* = p_r^* + \Delta$, and $p_o^* = c + \Delta$ can be sustained as a seller equilibrium as long as the set Φ_r is non-empty. And the set is non-empty if and only if $\tau \geq \Delta - c$. By construction, any profile with $p_r^* \notin \Phi_r$ cannot be sustained as a seller equilibrium. ■

The following table summarizes the possible equilibria that can arise if S decides to participate (and given τ), after applying the equilibrium selection rule used in the baseline model. Here, Π_{M_0} and Π_R refer to M_0's and R's profits. For brevity, we do not state equilibrium prices:

- In marketplace equilibria (*ME*), $\Pi_{M_0} = \tau(1-\mu)$, $\Pi_R = 0$, and $\pi = \mu\Delta + (1-\mu)(\Delta - c - \tau)$. The equilibrium exists if and only if $\tau \leq \min\left\{\Delta - c, \frac{b+c\mu}{1-\mu}\right\}$.
- In direct sales equilibria (*DE*), $\Pi_{M_0} = \Pi_R = 0$, and $\pi = \Delta - b - c$. The equilibrium exists if and only if $\Delta \geq \frac{b+c}{1-\mu}$ and $\tau \geq \frac{b+c\mu}{1-\mu}$.
- In seller equilibria (*SE*), $\Pi_{M_0} = 0$, $\Pi_R = p_r^*(1-\mu)$, and $\pi = \mu\Delta$, where

  $$p_r^* \in [0, c - \Delta + \min\{\tau, b + \mu\Delta\}] .$$

  The equilibrium exists if and only if $\Delta \leq \frac{b+c}{1-\mu}$ and $\tau \geq \Delta - c$

$$\begin{array}{c|cc} & \tau \leq \Delta - c & \tau > \Delta - c \\ \hline \Delta \leq \frac{b+c}{1-\mu} & ME & SE \end{array} \quad \text{and} \quad \begin{array}{c|cc} & \tau \leq \frac{b+c\mu}{1-\mu} & \tau > \frac{b+c\mu}{1-\mu} \\ \hline \Delta > \frac{b+c}{1-\mu} & ME & DE \end{array} \tag{C.1}$$

We can then derive the overall equilibrium.

Proposition C.1 *(Separation mode overall equilibrium)* M_0 *sets* $\tau^{sep} = \min\left\{\Delta - c, \frac{b+c\mu}{1-\mu}\right\}$ *and* S *participates. In the resulting marketplace equilibrium,* $p_o^* = c + \Delta$*,* $p_i^* = \Delta$ *and* $p_r^* = 0$*. All regular consumers buy from* S *on* M_0 *and direct consumers buy directly. Equilibrium profits are* $\Pi_{M_0}^{sep} = \tau^{sep}(1-\mu)$*,* $\Pi_R^{sep} = 0$ *and* $\pi^{sep} = \max\{\mu\Delta, \Delta - c - b\}$*. Moreover,* $\Pi_{M_0}^{sep} + \Pi_R^{sep} \leq \Pi^{dual}$*, with strict inequality if* $\Delta < \frac{b+c}{1-\mu}$*.*

Proof. If S does not participate on M_0, it is straightforward to see that Lemma 7 applies. This is because following S's non-participation decision, the pricing problem faced by R in the separation mode is the same as the one faced by M in the dual mode. For any given τ, it follows that S's non-participation profit is $\pi^{np} = \max\{\mu\Delta, \Delta - b - c\}$, and so S always weakly prefers to participate. Then, using the summary above of the possible equilibria that can arise after S decides to participate, we obtain the equilibrium stated in Proposition C.1. For the last part of the proposition, when $\Delta \geq \frac{b+c}{1-\mu}$, we have $\Pi^{sep}_{M_0} + \Pi^{sep}_{R} = (\frac{b+c\mu}{1-\mu})(1-\mu) = \Pi^{dual}$; when $\Delta < \frac{b+c}{1-\mu}$, we have $\Pi^{dual} = (b+\mu\Delta)(1-\mu) > (\Delta - c)(1-\mu) = \Pi^{sep}_{M_0} + \Pi^{sep}_{R}$, where the last inequality is due to $\Delta \leq \frac{b+c}{1-\mu} \iff \Delta - c \leq b + \mu\Delta$. ■

We can now add the separation mode into the comparison of profits and welfare across the various modes:

Proposition C.2 *(Comparisons, with separation mode).*

- *S's profit: $\pi^{market} > \pi^{dual} = \pi^{sep} = \pi^{sell}$.*
- *Welfare: $W^{dual} = W^{market} \geq W^{sell}$, where the inequality is strict if $b > 0$. $W^{dual} \geq W^{sep}$, where the inequality is strict if $F > 0$.*
- *Direct consumers: $CS^{sell}_{\text{direct}} > CS^{dual}_{\text{direct}} = CS^{market}_{\text{direct}} = CS^{sep}_{\text{direct}}$.*
- *Regular consumers: If $\Delta > \frac{b+c}{1-\mu}$, $CS^{sep}_{\text{regular}} = CS^{dual}_{\text{regular}} = CS^{sell}_{\text{regular}} > CS^{market}_{\text{regular}}$; if $\Delta \leq \frac{b+c}{1-\mu}$, $CS^{sep}_{\text{regular}} \geq CS^{dual}_{\text{regular}} > CS^{sell}_{\text{regular}} > CS^{market}_{\text{regular}}$, where the weak inequality is strict if $\Delta < \frac{b+c}{1-\mu}$.*
- *Total consumer surplus: If $\Delta > \frac{b+c}{1-\mu}$, $CS^{sell} > CS^{dual} = CS^{sep} > CS^{market}$; if $\Delta \leq \frac{b+c}{1-\mu}$, $CS^{sep} \geq CS^{dual} > CS^{market}$, where the weak inequality is strict if $\Delta < \frac{b+c}{1-\mu}$, and $CS^{sell} > CS^{market}$.*

Proof. Welfare in the dual mode and the separation mode matches that under the marketplace mode given that in all these settings, regular consumers buy S via M's marketplace. Turning next to consumer surplus, note $CS^{sep}_{direct} = CS^{dual}_{direct} = CS^{market}_{direct}$ given $p^*_o = c + \Delta$ in all three modes, while for regular consumers, $CS^{sep}_{regular} = v + b$. ■

Comparing M's profit in the dual mode and the separation mode, we find $\Pi^{dual} \geq \Pi^{sep}_{M} + \Pi^{sep}_{R}$, with strict inequality if $\Delta < \frac{b+c}{1-\mu}$. On this range of Δ, we have $\tau^{sep} = \Delta - c < b + \mu\Delta = \tau^{dual}$, i.e. the marketplace collects a lower commission in separation mode. To see why, recall that in dual mode, when $\tau > \Delta - c$, we have $p^*_i = c + \tau$ and $p^*_m = c + \tau - \Delta > 0$, and this is an equilibrium because M has no incentive to undercut further given that it is earning its commission. However, in separation mode, $p^*_i = c + \tau$ and $p^*_r = c + \tau - \Delta$ does not constitute an equilibrium because R does not internalize the revenue from the commission and hence it does want to undercut. The competition with R implies a stronger "margin squeeze" on S's inside price, relative to the squeeze in dual mode. Consequently, the marketplace cannot set its commission above $\Delta - c$ because it needs to take into account that S may make a negative margin from inside sales. The lower commission in separation mode reflects the inability of M_0 and R to internalize each other's profit (as compared to the dual mode).

We are now ready to examine the effect of a ban on dual mode. Assume that the separation mode involves some additional fixed cost $F > 0$ for M to set up the two separate divisions (e.g. separate websites, separate teams). As a result, $\Pi^{sep} = \Pi^{sep}_{M_0} + \Pi^{sep}_{R} - F = \tau^{sep}(1-\mu) - F$.

Proposition C.3 *(Ban on dual mode when M can commit to separation mode)*

- *If $F < \mu(b+c)$ and $\Delta \geq \max\left\{\frac{1}{2-\mu}\left(2c + b + \frac{F}{1-\mu}\right), c + b + \frac{F}{1-\mu}\right\}$, then a ban on the dual mode results in M choosing the separation mode, with CS_{regular} and CS increasing, CS_{direct} and π not changing, and Π and W decreasing.*
- *If $\Delta \leq \min\left\{\frac{1}{2-\mu}\left(2c + b + \frac{F}{1-\mu}\right), \frac{c}{1-\mu}\right\}$, then a ban on the dual mode results in M choosing the seller mode, with CS_{regular} decreasing, CS_{direct} increasing, π not changing; Π and W decreasing; and CS decreasing if $\Delta < b + c$ and increasing if $\Delta \geq b + c$.*

- *For all other parameter values, a ban on the dual mode results in M choosing the marketplace mode, with CS_{regular}, CS and Π decreasing; π increasing; and CS_{direct} and W not changing.*

Proof. (Proposition C.3). By inspection, if $\Delta > \frac{b+c}{1-\mu}$, then

$$\begin{aligned} \Pi^{sep} &= (\frac{b+c\mu}{1-\mu})(1-\mu) - F > 0 = \Pi^{sell} \\ \Pi^{sep} &= (\frac{b+c\mu}{1-\mu})(1-\mu) - F \geq b(1-\mu) = \Pi^{sell} \iff F \leq (b+c)\mu. \end{aligned}$$

Suppose instead $\Delta \leq \frac{b+c}{1-\mu}$,

$$\begin{aligned} \Pi^{sep} &= (\Delta - c)(1-\mu) - F \geq b(1-\mu) = \Pi^{market} \iff \Delta \geq c + b + \frac{F}{1-\mu}, \\ \Pi^{sep} &= (\Delta - c)(1-\mu) - F \geq (b + \mu\Delta + c - \Delta)(1-\mu) = \Pi^{sell} \iff \Delta \geq \frac{2c + b + \frac{F}{1-\mu}}{2-\mu} \end{aligned}$$

$$\Pi^{market} \geq \Pi^{sell} \iff \Delta \geq \frac{c}{1-\mu}.$$

If $F < \mu c - b(1-\mu)$, the ordering of these thresholds is: $c + b + \frac{F}{1-\mu} < \frac{2c+b+\frac{F}{1-\mu}}{2-\mu} < \frac{c}{1-\mu} \leq \frac{b+c}{1-\mu}$. If $F \in [\mu c - b(1-\mu), \mu(b+c)]$, the ordering of these thresholds is: $\frac{c}{1-\mu} \leq \frac{2c+b+\frac{F}{1-\mu}}{2-\mu} \leq c + b + \frac{F}{1-\mu} \leq \frac{b+c}{1-\mu}$. If $F > \mu c + \mu b$, the ordering of these thresholds is: $\frac{c}{1-\mu} < \frac{2c+b+\frac{F}{1-\mu}}{2-\mu} < \frac{b+c}{1-\mu} < c + b + \frac{F}{1-\mu}$. Combining the comparisons for these thresholds yields:

- Suppose $F < \mu(b+c) - b$. A ban on dual mode results in separation mode if $\Delta \geq \frac{1}{2-\mu}\left(2c + b + \frac{F}{1-\mu}\right)$; and seller mode if $\Delta \leq \frac{1}{2-\mu}\left(2c + b + \frac{F}{1-\mu}\right)$.
- Suppose $F \in [\mu(b+c) - b, \mu(b+c)]$. A ban on dual mode results in separation mode if $\Delta \geq c + b + \frac{F}{1-\mu}$; marketplace mode if $\Delta \in \left[\frac{c}{1-\mu}, c + b + \frac{F}{1-\mu}\right]$; seller mode if $\Delta \leq \frac{c}{1-\mu}$.
- Suppose $F > \mu(b+c)$. A ban on dual mode results in marketplace mode if $\Delta \geq \frac{c}{1-\mu}$; seller mode if $\Delta \leq \frac{c}{1-\mu}$.

■

Relative to the baseline model, the new possibility in Proposition C.3 is that M can choose the separation mode after the ban on its dual mode, which it will do when F is low and Δ is high. Compared to the dual mode, the separation mode always results in higher consumer surplus. Intuitively, in the separation mode, the stronger margin squeeze leads to an even lower inside price. However, the separation mode is less welfare-efficient due to the fixed cost of having two separate modes.

Comparing the post-intervention outcome here with Proposition 6, it can be shown that the separation mode (whenever it is chosen by M) always leads to a higher post-intervention total consumer surplus, compared to the case where M can only choose between marketplace and seller modes. In this sense, a softer version of breaking up Amazon — by allowing it to operate two independent divisions — is preferable to a complete breakup from consumers' perspective.

Finally, we verify that the post-intervention outcome in the separation mode results in a higher consumer surplus than the post-intervention outcome when the separation mode is unavailable. We focus on $F < \mu(b+c)$ and $\Delta \geq \max\left\{\frac{1}{2-\mu}\left(2c + b + \frac{F}{1-\mu}\right), c + b + \frac{F}{1-\mu}\right\}$. If $\Delta \geq \frac{c}{1-\mu}$, so that the marketplace mode is chosen if the separation mode is unavailable, Proposition C.2 implies $CS^{sep}_{direct} = CS^{market}_{direct}$ and $CS^{sep}_{regular} > CS^{market}_{regular}$, so $CS^{sep} > CS^{market}$. If $\Delta \leq \frac{c}{1-\mu}$, so that the seller mode is chosen if the separation mode is unavailable, we note

$$
\begin{aligned}
CS^{sep} &= (v+b)(1-\mu)+(v-c)\mu \\
&= v-c+(1-\mu)(b+c) \\
&\geq v-c+(1-\mu)^2\Delta \\
&\geq v-c+(1-\mu)^2\Delta-(1-\mu)(1-\eta)(b+c-\Delta) \\
&= CS^{sell},
\end{aligned}
$$

where η is the probability that regular consumers buy from M in the equilibrium.

D Section 4 with alternative comparisons

D.1 Product recommendations and steering only

Consider the alternative version of the model in Section 4 with product recommendations and steering but in which imitation is always prohibited throughout. We assume that the innovation decision is still endogenous, even though the results below remain the same if it is exogenous. In this setting, we re-examine the implications of (i) banning dual mode and (ii) banning steering.

The marketplace and seller modes remain the same as in Section 4. Meanwhile the equilibrium of the dual mode is described in Lemma 9. If dual mode is banned, we know M switches to seller mode. The following results follow from direct comparisons:

- A ban on the dual mode results in M choosing the seller mode, with Π, Δ, and W decreasing; π, CS, $CS_{regular}$, and CS_{direct} remaining unchanged.

If steering is banned, then following the analysis in the proof of Proposition 11 we know M will choose the seller mode if $\min\left\{\frac{(b+c)\mu}{1-\mu},\mu\bar{\Delta}\right\}<c$ and continues in the dual mode if $\min\left\{\frac{(b+c)\mu}{1-\mu},\mu\bar{\Delta}\right\}\geq c$. We therefore get the following result:

- If $\min\left\{\frac{(b+c)\mu}{1-\mu},\mu\Delta\right\}<c$, requiring objective recommendations results in M choosing the seller mode, with Π, Δ, and W decreasing; π, CS, $CS_{regular}$, and CS_{direct} remaining unchanged.
- If $\min\left\{\frac{(b+c)\mu}{1-\mu},\mu\Delta\right\}\geq c$, requiring objective recommendations results in M continuing to choose the dual mode, with Π decreasing; π, CS, and $CS_{regular}$ increasing; Δ, W, and CS_{direct} remaining unchanged.

Comparing these two interventions, we note they lead to different outcomes when $\min\left\{\frac{(b+c)\mu}{1-\mu},\mu\Delta\right\}\geq c$. In particular, $CS_{regular}$, CS, Π, π, Δ, and W are all weakly higher when steering is banned compared to the case of an outright ban on the dual mode. Hence, requiring objective recommendations may be a more targeted remedy in addressing biased recommendations.

D.2 Innovation and product imitation only

Consider the alternative version of the model in Section 4 with innovation and product imitation but in which steering is prohibited. This means that we adopt the same information assumption as in the baseline model, i.e. regular consumers are aware of all available products in the market, regardless of whether S participates on M. We re-examine the implications of (i) banning dual mode and (ii) banning imitation.

The marketplace remains the same as Section 4. S extracts the entire innovation surplus, it optimally chooses $\Delta^{market}=\Delta^H$.

In the seller mode, by Proposition 2, we have

$$\bar{\pi}^{sell}(\Delta) = \begin{cases} \mu\Delta - K(\Delta) & \text{for} \quad \Delta \leq \frac{b+c}{1-\mu} \\ \Delta - b - c - K(\Delta) & \text{for} \quad \Delta > \frac{b+c}{1-\mu} \end{cases}. \tag{D.1}$$

$\bar{\pi}^{sell}$ is continuous but it may not be single-peaked depending on other parameters, and the profit-maximizing Δ can be characterized as:

Lemma D.1 *(Innovation level in the seller mode). Denote*

$$\bar{\Delta} \equiv \frac{\Delta^H - \mu\Delta^L - \left(K(\Delta^H) - K(\Delta^L)\right)}{1-\mu} \in (\Delta^L, \Delta^H).$$

- *Suppose* $\bar{\Delta} > \frac{b+c}{1-\mu}$*: in equilibrium* $\Delta^{sell} = \Delta^H$*. The equilibrium profits are* $\Pi^{sell} = 0$ *and* $\pi^{sell} = \Delta^H - b - c - K\left(\Delta^H\right)$*.*
- *Suppose* $\bar{\Delta} \leq \frac{b+c}{1-\mu}$*: in equilibrium* $\Delta^{sell} = \Delta^L$*. The equilibrium profits are* $\Pi^{sell} = (1-\mu)\left(c + b - (1-\mu)\Delta^L\right)$ *and* $\pi^{sell} = \mu\Delta^L - K\left(\Delta^L\right)$*.*

Proof. For all $\frac{b+c}{1-\mu} \leq \Delta^L$, $\bar{\pi}^{sell}(\Delta)$ has exactly one interior peak point at $\Delta = \Delta^H > \Delta^L \geq \frac{b+c}{1-\mu}$, so S optimally chooses Δ^H. For all $\frac{b+c}{1-\mu} \geq \Delta^H$, $\bar{\pi}^{sell}(\Delta)$ has exactly one interior peak point at $\Delta = \Delta^L < \Delta^H \leq \frac{b+c}{1-\mu}$, so S optimally chooses Δ^L. For $\Delta^L < \frac{b+c}{1-\mu} < \Delta^H$, $\bar{\pi}^{sell}(\Delta)$ has two interior peak points:

$$\max_{\Delta \leq \frac{b+c}{1-\mu}} \bar{\pi}^{sell}(\Delta) = \mu\Delta^L - K(\Delta^L) \quad \text{and} \quad \max_{\Delta > \frac{b+c}{1-\mu}} \bar{\pi}^{sell}(\Delta) = \Delta^H - b - c - K(\Delta^H),$$

where $\bar{\pi}^{sell}\left(\Delta^L\right) > \bar{\pi}^{sell}\left(\Delta^H\right)$ if and only if

$$\frac{b+c}{1-\mu} > \frac{\Delta^H - \mu\Delta^L - \left(K(\Delta^H) - K(\Delta^L)\right)}{1-\mu} \equiv \bar{\Delta}.$$

It is straightforward to verify that $\bar{\Delta} \in \left[\Delta^L, \Delta^H\right]$, using that $\Delta^L = \arg\max\{\mu\Delta - K(\Delta)\}$ and $\Delta^H = \arg\max\{\Delta - K(\Delta)\}$. Note if $\frac{b+c}{1-\mu} = \bar{\Delta}$, then $\bar{\pi}^{sell}\left(\Delta^L\right) = \bar{\pi}^{sell}\left(\Delta^H\right)$ so that S is indifferent, in which case our equilibrium selection rule implies that the equilibrium with innovation Δ^L is selected. ■

Consider the dual mode. We first solve for the pricing in stage 4 assuming product imitation occurs. This is the same subgame considered in Section 4 when steering is banned, and the equilibrium for each given (τ, Δ) is described by Lemma 10, with equilibrium profits $\Pi = (c + \min\{b + \mu\Delta, \tau\})(1-\mu)$ and $\pi = \mu\Delta$. Meanwhile, if product imitation does not occur, the equilibrium in stage 4 is given by Lemmas 1 - 3 in the main text. Comparing across these equilibria, it follows that M always wants to imitate S's product at the beginning of stage 3.

In the presence of product imitation, and given that regular consumers are always aware of S, S does not always find it profitable to join the marketplace. Specifically, for each given Δ, if $\Delta > \frac{b+c}{1-\mu}$ then S's non-participation profit is $\Delta - b - c$, which is higher than its participation profit and so it does not participate (we denote this outcome as *NP*). If instead $\Delta \leq \frac{b+c}{1-\mu}$, S's non-participation profit is $\Delta\mu$, which is the same as its participation profit with imitation. Based on our selection rule, we select the equilibrium in which S breaks the tie in favor of participating. See Section E of the Online Appendix on how this tie-breaking rule can be seen as a limiting case in which the extent of horizontal differentiation between M and S becomes arbitrarily small.

Then, S's expected profit in stage 2 is

$$\bar{\pi}^{dual}(\Delta) = \begin{cases} \mu\Delta - K(\Delta) \ \textit{(RE)} & \text{for} \quad \Delta \leq \frac{b+c}{1-\mu} \\ \Delta - b - c - K(\Delta) \ \textit{(NP)} & \text{for} \quad \Delta > \frac{b+c}{1-\mu} \end{cases}.$$

This function is the same as equation (D.1) so that S's optimal Δ can be characterized similarly to Lemma D.1. Then, the overall equilibrium in dual mode is:

Proposition D.1 *(Dual mode equilibrium with product imitation) M sets $\tau^{dual} = b + \mu\Delta^L$.*

- If $\bar{\Delta} > \frac{b+c}{1-\mu}$, S sets $\Delta = \Delta^H$ and does not participate in stage 2. In stage 3, $p_o^* = \Delta^H - b$, and $p_m^* = 0$, all regular consumers buy from S directly, while $\Pi^{dual} = 0$ and $\pi^{dual} = \Delta^H - b - c - K(\Delta^H)$.
- If $\bar{\Delta} \leq \frac{b+c}{1-\mu}$, S sets $\Delta = \Delta^L$ and participates in stage 2. In stage 3, $p_o^* = c + \Delta^L$ and $p_m^* = p_i^* = c + \tau^{dual}$, all regular consumers buy from M, while $\Pi^{dual} = (1-\mu)\left(b + c + \mu\Delta^L\right)$ and $\pi^{dual} = \mu\Delta^L - K(\Delta^L)$.

Proof. The derivation of S's innovation decision follows from Lemma D.1. Then, given that τ does not influence participation, M sets $\tau^{dual} = b + \mu\Delta^L$ to maximize its profit in the case where S participates (i.e. Lemma 10). The complete equilibrium characterization then follows from Lemma 7 (if S does not participate) and Lemma 10 (if S participates). ■

Notably, if $\bar{\Delta} > \frac{b+c}{1-\mu}$, the possibility of product imitation deters S from participating in the marketplace in case M operates in dual mode. Instead, S sets a high innovation level such that all regular consumers end up buying from it directly, resulting in M earning zero profit in the dual mode. This reflects M's inability to commit to not imitate S's product. In this case, M prefers the marketplace mode. On the other hand, if $\bar{\Delta} \leq \frac{b+c}{1-\mu}$, S is willing to participate under dual mode, and M prefers the dual mode over the other two modes.

Banning the dual mode in the presence of product imitation has the following implications:

- If $\bar{\Delta} > \frac{b+c}{1-\mu}$, a ban on the dual mode has no effect.
- If $\bar{\Delta} \leq \frac{b+c}{1-\mu}$ and $\Delta^L \geq \frac{c}{1-\mu}$, a ban on the dual mode results in M choosing the marketplace mode, with Π, $CS_{regular}$, and CS decreasing; π and Δ increasing; CS_{direct} not changing; and W decreasing if $c > \bar{\Delta} - \Delta^L$, not changing if $c = \bar{\Delta} - \Delta^L$, and increasing if $c < \bar{\Delta} - \Delta^L$.
- If $\bar{\Delta} \leq \frac{b+c}{1-\mu}$ and $\Delta^L < \frac{c}{1-\mu}$, a ban on the dual mode results in M choosing the seller mode, with Π, $CS_{regular}$, and W decreasing; CS_{direct} increasing; π and Δ not changing; and CS decreasing if $\Delta^L < b + c$, not changing if $\Delta^L = b + c$, and increasing if $\Delta^L > b + c$.

Proof. We focus on $\bar{\Delta} \leq \frac{b+c}{1-\mu}$ in what follows. In this case, $\Pi^{dual} = (1-\mu)\left(b + c + \mu\Delta^L\right)$ which is higher than $\Pi^{sell} = (1-\mu)(b + c - (1-\mu)\Delta^L)$ and $\Pi^{market} = (1-\mu)\, b$. Next, $\Delta^{dual} = \Delta^{sell} = \Delta^L < \Delta^H = \Delta^{market}$. For welfare:

$$\begin{aligned} W^{market} &= v + \Delta^H + (1-\mu)\, b - c - K(\Delta^H) \\ W^{dual} &= v + \Delta^L + (1-\mu)\, b - \mu c - K(\Delta^L), \end{aligned}$$

given that M sells to all regular consumers in dual mode. Therefore, $W^{market} > W^{dual}$ if and only if $\bar{\Delta} - \Delta^L \equiv \frac{\Delta^H - K(\Delta^H) - (\Delta^L - K(\Delta^L))}{1-\mu} > c$. Meanwhile $W^{dual} > W^{sell}$ follows from the baseline model (since these two modes have the same Δ). In the dual mode, $p_m^* = c + \tau^{dual} = c + \mu\Delta^L + b$, so when $\Delta^L \geq \frac{c}{1-\mu}$, we have $CS_{regular}^{dual} = v - c + (1-\mu)\,\Delta^L > CS_{regular}^{market} = v - c$. When instead $\Delta^L \leq \frac{c}{1-\mu}$, the distribution support $p_o^* \in [c + \mu\Delta^L, c + \Delta^L]$ in seller mode implies $CS_{regular}^{sell} < v - c + (1-\mu)\,\Delta^L$, so it follows that $CS_{regular}^{dual} > CS_{regular}^{sell}$. Meanwhile $p_o^* = c + \Delta$ in both the marketplace mode and the dual mode, so $CS_{direct}^{dual} = CS_{direct}^{market} = v - c < CS_{direct}^{sell}$. It follows that $CS^{dual} > CS^{market}$. Next,

$$\begin{aligned} CS^{sell} &= W^{sell} - \Pi^{sell} - \pi^{sell} \\ &= v - c + (1-\mu)^2\Delta^L + (1-\mu)(1-\eta)(\Delta^L - b - c), \end{aligned}$$

while

$$CS^{dual} = (1-\mu)CS^{dual}_{regular} + \mu CS^{dual}_{direct} = v - c + (1-\mu)^2\Delta^L.$$

So $CS^{dual} \leq CS^{sell}$ if and only if $\Delta^L - b - c \geq 0$. ■

Surprisingly, even though we allow for the possibility of M to freely imitate S's superior product in dual mode, and we take into account the effect of this through S's choice on how much to innovate, a ban on dual mode is not necessarily good for consumers or welfare. Specifically, note that if $\bar{\Delta} > \frac{b+c}{1-\mu}$, M always prefers the marketplace mode (since in dual mode, S would not participate) and hence the ban has no effect. If $\bar{\Delta} \leq \frac{b+c}{1-\mu}$ and $\Delta^L \leq \frac{c}{1-\mu}$, the ban results in M choosing the seller mode, with qualitative implications that are the same as the second part of Proposition 6 in the main text.

The interesting case occurs when $\bar{\Delta} \leq \frac{b+c}{1-\mu}$ and $\Delta^L \geq \frac{c}{1-\mu}$, whereby the ban results in M choosing the marketplace mode, with qualitative implications similar to the first part of Proposition 6 in the main text except that welfare can increase after the ban. This reflects the trade-off between the innovation incentive and utilizing M's inherent cost advantage, as discussed in the main text.

An alternative policy would be to ban imitation while still allowing M to operate in dual mode:

- If $\bar{\Delta} > \frac{b+c}{1-\mu}$, a ban on imitation results in M switching from the marketplace to the dual mode (i.e. the ban makes the dual mode viable), with Π, $CS_{regular}$, and CS increasing; π decreasing; and CS_{direct}, Δ, and W not changing.
- If $\bar{\Delta} \leq \frac{b+c}{1-\mu}$, a ban on imitation results in M continuing to operate in the dual mode, with Δ, $CS_{regular}$, and CS increasing; π and CS_{direct} not changing; W decreasing if $\bar{\Delta} - \Delta^L < c$, not changing if $\bar{\Delta} - \Delta^L = c$, and increasing if $\bar{\Delta} - \Delta^L > c$; and Π decreasing if $\bar{\Delta} - \Delta^L < \frac{c}{\mu}$, not changing if $\bar{\Delta} - \Delta^L = \frac{c}{\mu}$, and increasing if $\bar{\Delta} - \Delta^L > \frac{c}{\mu}$.

Proof. In this proof, we use superscript $dual\,(I)$ to denote the equilibrium of the dual mode with imitation, and $dual\,(NI)$ to denote the equilibrium of the dual mode without imitation. It is easy to derive the equilibrium in the dual mode without imitation:

- M sets $\tau^{dual(NI)} = b + \mu \min\left\{\frac{b+c}{1-\mu}, \bar{\Delta}\right\}$, S participates and chooses innovation level Δ^H.
- If $\bar{\Delta} \geq \frac{b+c}{1-\mu}$, the equilibrium prices are $p_o^* = c + \Delta^H$, $p_i^* = \Delta^H$, and $p_m^* = 0$.
- If $\bar{\Delta} \leq \frac{b+c}{1-\mu}$, the equilibrium prices are $p_o^* = c + \Delta^H$, $p_i^* = c + \tau^{dual(NI)} + \Delta^H - \bar{\Delta}$, and $p_m^* = c + \tau^{dual} - \bar{\Delta}$.
- All regular consumers buy from S on M and direct consumers buy directly.
- The equilibrium profits are $\Pi^{dual(NI)} = \tau^{dual(NI)}(1-\mu)$ and $\pi^{dual(NI)} = \max\left\{\Delta^H - b - c, \Delta^H - \bar{\Delta}(1-\mu)\right\} - K\left(\Delta^H\right)$.

When $\bar{\Delta} > \frac{b+c}{1-\mu}$, the profit from the dual mode without imitation is $\Pi^{dual(NI)} = b + c\mu \geq b(1-\mu) = \Pi^{market}$. So the ban on imitation means M switches from the marketplace mode to the dual mode without imitation. We have

$$\begin{aligned} CS^{dual(NI)}_{regular} &= v + \Delta^H + b - \Delta^H = v + b > v - c = CS^{market}_{regular} \\ CS^{dual(NI)}_{direct} &= CS^{market}_{direct} = v - c \\ W^{dual(NI)} &= W^{market} = v - c + \Delta^H + (1-\mu)b - K(\Delta^H). \end{aligned}$$

Finally, $\pi^{dual(NI)} = \Delta^H - K(\Delta^H) - b - c < \Delta^H - K(\Delta^H) = \pi^{market}$.

For $\bar{\Delta} \leq \frac{b+c}{1-\mu}$, profit from the dual mode without imitation is $\Pi^{dual(NI)} = (1-\mu)\left(b + \mu\bar{\Delta}\right) > \max\left\{\Pi^{market}, \Pi^{sell}\right\} = \max\left\{(1-\mu)b, (1-\mu)\left(b + c - (1-\mu)\Delta^L\right)\right\}$, where the inequality utilizes $\mu\bar{\Delta} >$

$\mu\Delta^L > c - (1-\mu)\,\Delta^L$. So the ban on imitation means M continues to operate in the dual mode, but without imitation. Note $W^{dual(NI)} = W^{market}$, and so $W^{dual(NI)} > W^{dual(I)}$ if and only if $\bar{\Delta} - \Delta^L \equiv \frac{\Delta^H - K(\Delta^H) - (\Delta^L - K(\Delta^L))}{1-\mu} > c$. . Meanwhile, $CS^{dual(NI)}_{direct} = CS^{dual(I)}_{direct} = v - c$ and $CS^{dual(NI)}_{regular} = v - c + (1-\mu)\,\bar{\Delta} > v - c + (1-\mu)\,\Delta^L = CS^{dual(I)}_{regular}$. Combining both comparisons yields $CS^{dual(NI)} > CS^{dual(I)}$. Next we note that

$$
\begin{aligned}
\pi^{dual(NI)} &= \max\left\{\Delta^H - b - c, \Delta^H - (1-\mu)\bar{\Delta}\right\} - K(\Delta^H) \\
&= \Delta^H - (1-\mu)\bar{\Delta} - K(\Delta^H) \\
&= \mu\Delta^L - K(\Delta^L) = \pi^{dual(I)}.
\end{aligned}
$$

Finally, $\Pi^{dual(I)} = (1-\mu)\left(c + b + \mu\Delta^L\right)$, which is higher than $\Pi^{dual(NI)} = (b + \mu\bar{\Delta})\,(1-\mu)$ if and only if

$$
b + \frac{\mu}{1-\mu}\left(\Delta^H - \Delta^L\mu - \left(K(\Delta^H) - K(\Delta^L)\right)\right) \leq c + b + \mu\Delta^L,
$$

or, equivalently, $\bar{\Delta} - \Delta^L \leq \frac{c}{\mu}$. ■

Comparing these two interventions, we note that banning imitation always results in M operating in dual mode while banning dual mode outright results in M switching to either the marketplace mode or the seller mode. This comparison implies W and $CS_{regular}$ are weakly higher and CS_{direct} is weakly lower when imitation is banned relative to the outcome when the dual mode is banned, suggesting banning imitation under dual mode may be better than banning dual mode altogether.

E Section 4 with imperfect imitation and horizontal differentiation

In this section, we allow product imitation to be imperfect to explore how this affects the results derived in Section 4 of the main text. Suppose after M imitates S's product, its imitation product has value $v + \Delta - tx_i$, where $t \in (0, \Delta^L)$ is a parameter capturing the imperfection in imitation, and x_i is a consumer-specific disutility for the imperfect imitation product, which is identically and independently drawn from uniform distribution over $[0,1]$. Moreover, M observes the realization of x_i before making its product recommendations. Notice that if $t \to 0$ then we recover the model with perfect imitation in Section 4.

If product imitation has not occurred in stage 3, then the stage 4 pricing game unfolds as in Section 4. In particular, M's profit is $\Pi = \tau(1-\mu)$ if $\tau \in [b+c, b+\Delta]$ and $\Pi = (b+c)(1-\mu)$ otherwise.

Next, we solve the stage 4 pricing game in case product imitation has occurred in stage 3. Suppose $\tau \leq b + \Delta$ (later, we will verify that M never has a strict incentive to set $\tau > b + \Delta$). It is easy to see that M recommends S if (i) S's inside product is preferred by regular consumers over the products available in the direct channel (formally, $\Delta + b - p_i \geq \max\{\Delta - p_o, -c\}$) and (ii) the commission is higher than the margin M could earn by trying to sell itself (formally, $p_m \leq \tau$ or $b + \Delta - tx_i - p_m \leq -c$). The recommendation strategy by M means S's pricing never affects S's probability to get recommended as long as $p_i \leq p_o + b$. As such, S optimally increases both prices until $p_i = c + \Delta + b$ and $p_o = c + \Delta$.

Let G denote the cummulative distribution function of uniform distribution, then M's pricing problem in stage 3 is to choose $p_m \leq b + c + \Delta$ to maximize

$$
\Pi(p_m) = \begin{cases} \tau(1-\mu) & \text{for} \quad p_m \leq \tau \\ (p_m - \tau)G\left(\frac{b+\Delta+c-p_m}{t}\right)(1-\mu) + \tau(1-\mu) & \text{for} \quad p_m > \tau \end{cases}.
$$

Using the usual first-order condition and taking into account boundary constraints, the optimal price by

M is

$$p_m^* = \begin{cases} \frac{b+\Delta+c+\tau}{2} & \text{for} \quad \tau > b+\Delta+c-2t \\ b+\Delta+c-t & \text{for} \quad \tau \leq b+\Delta+c-2t \end{cases}.$$

Intuitively, when τ and t are large such that $\tau > b+\Delta+c-2t$ holds, M's pricing is such that S still makes a positive amount of sales in equilibrium. The two firms' profits (ignoring the innovation cost) are

$$\begin{aligned} \Pi &= \frac{1-\mu}{t}\left(\frac{b+\Delta+c-\tau}{2}\right)^2 + \tau(1-\mu) \\ \pi &= (\Delta+b-\tau)\left(1-\frac{b+\Delta+c-\tau}{2t}\right)(1-\mu)+\mu\Delta. \end{aligned}$$

In contrast, if τ and t are small such that $\tau > b+\Delta+c-2t$ holds, it is more profitable for M to make all the sales by itself, and we have an outcome that is analogous to the perfect imitation model in Section 4. Profits are $\Pi = (b+\Delta+c-t)(1-\mu)$ and $\pi = \mu\Delta$. Comparing these profits to the subgame without imitation, it is clear that M prefers to imitate in stage 3.

In stage 2, if S does not participate, all regular consumers are unaware of it and so S's profit is $\mu\Delta$. Therefore, S always weakly prefer to participate as long as $\tau \leq b+\Delta$, and *strictly* so if $\tau \in (b+\Delta+c-2t, b+\Delta)$. Notice that $(b+\Delta+c-2t, b+\Delta)$ is non-empty as long as $t > c/2$. This verifies the claim in the main text that if the extent of differentiation between M's and S's product is not too small then S has a strict incentive to participate. The incentive to participate becomes smaller as t decreases and becomes zero when $t = c/2$. Hence, the assumption of S breaking tie in favor of participation in Section 4 can be seen as a special case of letting $t \to c/2$ from above. To be more precise, we select the tie-breaking outcome as the limit of $t \to c/2$, and then apply this tie-breaking for all $t \leq c/2$, which then includes the case of $t \to 0$ (perfect imitation, as in Section 4).

In what follows, we focus on the case of $t > c/2$ (if $t \leq c/2$, the analysis of the model is the same as in the dual mode of Section 4). This means that S's choice of innovation is

$$\arg\max_{\Delta}\left\{(\Delta+b-\tau)\left(1-\frac{b+\Delta+c-\tau}{2t}\right)(1-\mu)+\mu\Delta-K(\Delta)\right\},$$

i.e. $\Delta^*(\tau)$ solves

$$\left(1-\frac{c}{2t}+\frac{\tau-b-\Delta^*}{t}\right)(1-\mu)+\mu = K'(\Delta^*). \tag{E.1}$$

Notice that $\frac{\partial\Delta^*}{\partial\tau} \in (0, 1-\mu)$ given K is convex.

In stage 1, M optimally sets the highest possible τ subject to the participation constraint of S, i.e. τ^{dual} solves $\tau = b+\Delta^*(\tau)$. For any τ that is higher, S never makes sale on the platform, resulting in weakly lower profit for M (and strictly so if $t > c/2$). Substituting the fixed-point relationship $\tau = b+\Delta^*(\tau)$ into (E.1), the equilibrium innovation in dual mode (with imperfect imitation) is $\Delta^{dual} = \Delta^D$, where Δ^D solves

$$K'(\Delta^D) = 1-\frac{c(1-\mu)}{2t}. \tag{E.2}$$

Clearly, $\Delta^D < \Delta^H$. In addition, $\Delta^D > \Delta^L$ whenever $t > c/2$ and $\Delta^D = \Delta^L$ whenever $t \leq c/2$. To summarize:

Proposition E.1 *(Dual mode equilibrium with imperfect product imitation and steering)*

- *Suppose $t > c/2$. In the overall equilibrium M sets $\tau^{dual} = b+\Delta^D$. S sets Δ^D given by (E.2) and participates. In the pricing subgame, $p_o^* = c+\Delta^D$ and $p_i^* = c+\Delta^D+b$, while $p_m^* = b+\Delta^D+\frac{c}{2}$. M recommends its own product to regular consumers who have $x_i \leq \frac{b+\Delta+c-p_m}{t}$ and recommends S's product to regular consumers who have $x_i > \frac{b+\Delta+c-p_m}{t}$.*

- *Suppose $t < c/2$. In the overall equilibrium M sets $\tau^{dual} = b+\Delta^L$. S sets Δ^L and participates. In the pricing subgame, $p_o^* = c+\Delta^D$ and $p_i^* = c+\Delta^D+b$, while $p_m^* = c+\Delta^D+b-t$. M recommends its own product to all regular consumers.*
  The equilibrium profits are $\Pi^{dual} = \left(\min\left\{\frac{c^2}{4t}, c-t\right\} + b + \Delta^D\right)(1-\mu)$ and $\pi^{dual} = \mu\Delta^D - K(\Delta^D)$.

A few quick remarks are in order. First, the innovation level Δ^D is weakly higher than the innovation level Δ^L when the imitation is perfect, and strictly so when $t > c/2$. Second, M's price leaves each regular consumer with surplus greater than $v-c$ due to the heterogeneity in consumer disutility for M's imperfect imitation product:

$$
\begin{aligned}
CS_{regular} &= v - c + t\int_0^{\frac{c}{2t}} \left[\frac{c}{2t} - x_i\right] dx_i \\
&= v - c + \min\left\{\frac{c^2}{8t}, \frac{t}{2}\right\}.
\end{aligned}
$$

E.1 Policy interventions

The following results are analogous to the policy interventions discussed in Section 4. We first note that the post-ban equilibria for (i) ban on dual mode, (ii) ban on product imitation, and (iii) ban on both product imitation and steering are all unaffected by the possibility of imperfect imitation, meaning that these equilibria are the same as in Section 4. Comparing these outcomes with Proposition E.1 yields the following results.

- A ban on the dual mode results in M choosing the seller mode, with Π, Δ, $CS_{regular}$, CS, and W decreasing; CS_{direct} and π remaining unchanged.
- A ban on imitation results in M continuing in the dual mode, with Δ increasing; π remaining unchanged; CS decreasing; Π increasing if and only if $\min\left\{\frac{c^2}{4t}, c-t\right\} < \bar{\Delta} - \Delta^D$; W increasing if and only if
$$\min\left\{\frac{c^2}{4t}, c-t\right\} < \frac{\Delta^H - K^H - (\Delta^D - K^D)}{1-\mu} - \min\left\{\frac{c^2}{8t}, \frac{t}{2}\right\}. \tag{E.3}$$
- If $\min\left\{\frac{(b+c)\mu}{1-\mu}, \mu\bar{\Delta}\right\} < c$, a ban on both imitation and steering results in the same implications as a ban on the dual mode.
- If $\min\left\{\frac{(b+c)\mu}{1-\mu}, \mu\bar{\Delta}\right\} \geq c$, a ban on both imitation and steering results in M continuing to operate in the dual mode, with Π decreasing; π, Δ, increasing; CS_{direct} remaining unchanged; $CS_{regular}$ and CS increasing if and only if $\min\left\{\frac{c^2}{8t}, \frac{t}{2}\right\} < (1-\mu)\min\left\{\frac{b+c}{1-\mu}, \bar{\Delta}\right\}$; W increasing if and only if (E.3) holds.

These results recover Propositions 8, 9, and 11 if $t \to 0$.

We now consider the equilibrium when only steering is banned. For simplicity, we focus on the case of $\mu \to 0$. It is easy to verify that M always wants to imitate S's product in stage 3. Therefore, it suffices to solve the stage 4 pricing game in case product imitation has occurred in stage 3. There are two possible equilibria in the pricing subgame: (i) $p_i^* \leq b + p_o^*$ so that S's outside channel is inactive in equilibrium and all regular consumers buy on the marketplace; and (ii) $p_i^* > b + p_o^*$ so that S's inside channel is inactive in equilibrium. Given $\mu \to 0$, the first type of equilibrium exists if and only if $\tau \leq b$, and the second type of equilibrium exists if and only if $\tau > b$.

We now derive the first type of equilibrium ($p_i^* \leq b + p_o^*$). A consumer is indifferent between M's and S's product if and only if $tx_i = p_i - p_m$. The profit functions (ignoring the innovation cost) are

$$\begin{aligned}
\hat{\Pi} &= p_m \left(\frac{p_i - p_m}{t}\right) + \tau \left(1 - \frac{p_i - p_m}{t}\right) \\
&= (p_m - \tau)\left(\frac{p_i - p_m}{t}\right) + \tau, \\
\hat{\pi} &= (p_i - c - \tau)\left(1 - \frac{p_i - p_m}{t}\right).
\end{aligned}$$

The equilibrium prices are

$$(p_m^*, p_i^*) = \begin{cases} \left(\frac{c+t}{3} + \tau, \frac{2(c+t)}{3} + \tau\right) & \text{for} \quad t > \frac{c}{2} \\ (c + \tau - t, c + \tau) & \text{for} \quad t \leq \frac{c}{2} \end{cases}. \tag{E.4}$$

Intuitively, when t is sufficiently small, in equilibrium M sells to all regular consumers given its cost advantage. For $t \leq c/2$, the equilibrium profits are $\hat{\Pi} = c - t + \tau$ and $\pi = 0$. For $t > c/2$, the equilibrium profits are $\hat{\Pi} = \frac{1}{t}\left(\frac{c+t}{3}\right)^2 + \tau$ and $\hat{\pi} = \frac{1}{t}\left(\frac{2t-c}{3}\right)^2$. We know S's non-participation profit is zero and so it always weakly prefers to participate, and strictly so if $t > c/2$.

Following the same derivation, we can derive the second type of equilibrium ($p_i^* > b + p_o^*$). The relevant profit functions are

$$\begin{aligned}
\tilde{\Pi} &= p_m \left(\frac{p_o + b - p_m}{t}\right) \\
\tilde{\pi} &= (p_o - c)\left(1 - \frac{p_o + b - p_m}{t}\right).
\end{aligned}$$

The equilibrium prices are

$$(p_m^*, p_o^*) = \begin{cases} \left(\frac{c+b+t}{3}, \frac{2(c+b+t)}{3} - b\right) & \text{for} \quad t > \frac{c+b}{2} \\ \left(\frac{c+b}{2}, c\right) & \text{for} \quad t \leq \frac{c+b}{2} \end{cases}.$$

And equilibrium profit of M is $\tilde{\Pi} = \frac{1}{9t}(c + b + t)^2$ if $t > \frac{c+b}{2}$ and $\tilde{\Pi} = \frac{1}{4t}(c + b)^2$ if $t \leq \frac{c+b}{2}$. In addition, S always weakly prefers to participate, and strictly so if $t > \frac{c+b}{2}$.

Comparing the two types of equilibria, we note (i) in stage 2, S's profit is always decreasing in Δ given it bears the entire innovation cost, and so it always chooses Δ^L; (ii) in stage 1, M always prefers setting $\tau = b$ because the profit from the first type of equilibrium is increasing in τ and higher than the profit from the second type of equilibrium (to see this, notice $\tilde{\Pi} - \hat{\Pi}|_{\tau=b} = 0$ when $b = 0$ and $\tilde{\Pi} - \hat{\Pi}|_{\tau=b}$ is decreasing in b).

In the overall equilibrium of dual mode after steering is banned, M sets $\tau = b$. S sets Δ^L and participates. In the pricing subgame, p_m^* and p_i^* are given by (E.4), and $p_o^* > p_i^* + b$. M's equilibrium profit is $\Pi = \min\left\{\frac{1}{t}\left(\frac{c+t}{3}\right)^2, c - t\right\} + b$. However, this profit is lower than what M could earn by operating as a seller ($b + c$). Hence, the ban always result in M switching to seller mode. Comparing the outcome with Proposition E.1 yields the following result (recall that we have assumed $\mu \to 0$ so $CS = CS_{regular}$):

- A ban on steering results in M choosing the seller mode, with Π, Δ, CS, and W decreasing; and π remaining unchanged.

It should be emphasized that the result of M always choosing the seller mode is an artefact of $\mu \to 0$ and $t > 0$. More generally, if $\mu > 0$, then in the dual mode without steering M is able to set $\tau > b$ while still sustaining the first type of equilibrium (i.e. all regular consumers buy on the platform). When μ is not too small relative to t, a ban on steering would instead result in M continuing to choose the dual mode (with imitation but without steering), as in Section 4 of the main text.

F Competition and endogenous market structure

If both intermediaries operate as pure sellers, then in equilibrium $\Pi_1^* = \Pi_2^* = 0$ by the logic for symmetric Bertrand competition between M_1 and M_2. If both intermediaries operate as pure marketplaces both intermediaries must compete their fees down to zero in order to attract S, implying $\Pi_1^* = \Pi_2^* = 0$ in the overall equilibrium. If exactly one intermediary operates as pure seller and the other intermediary operates one of the other two modes, then the analysis proceeds as in the separation mode in Section C. When the other intermediary operates as the dual mode, the only caveat is that in *SE* of Table C.1, we have $p_r^* = 0$ exactly. Then, the overall equilibrium is described by Proposition C.1. Therefore, there are only two remaining cases to consider: when at least one intermediary operates in the dual mode, while the other intermediary operates as either (1) the dual mode or (2) the marketplace mode.

F.1 Both M_1 and M_2 operate in dual mode

Consider the stage 3 subgame. As in the baseline model, there are three broad types of equilibria:

- marketplace equilibria (all regular consumers purchase from S through one of the marketplaces);
- direct sales equilibria (all regular consumers purchase from S directly);
- seller equilibria (at least one of the intermediaries make a positive amount of sales to regular consumers).

We assume, without loss of generality, $\tau_1 \leq \tau_2$, and whenever regular consumers are indifferent between S's product offered in both marketplaces, they purchase through M_1. All other tie-breaking rules follow from the baseline model. We first solve the equilibrium of the stage 3 subgame, assuming that S participates on both marketplaces. Let p_{m1}^* and p_{m2}^* denote the prices set by M_1 and M_2, and let p_{i1} and p_{i2} be the inside prices set by S when selling through M_1 and M_2.

Lemma F.1 *In any marketplace equilibrium, $p_{m1}^* \geq p_{m2}^* = 0$, $p_{i1}^* = \Delta$, $p_{i2}^* > \Delta$, $p_o^* = c + \Delta$. The equilibrium exists if and only if $\tau_1 \leq \min\left\{\Delta - c, \frac{b+c\mu}{1-\mu}\right\}$. The equilibrium profits are $\Pi_{M_1} = \tau_1(1-\mu)$, $\Pi_{M_2} = 0$, and $\pi = \mu\Delta + (\Delta - c - \tau_1)(1-\mu)$.*

Proof. Suppose $\tau_1 < \tau_2$, S's price must be such that all such purchases are made through M_1, as otherwise it can profitably raise its price at M_2, p_{i2}, to divert consumers to M_1 where the margin is higher. This implies M_2's profit is zero in any such equilibrium, so it necessarily has an incentive to deviate to attract regular consumers with its own offering, as long as $p_{i1}^* > \Delta$. This implies in equilibrium $p_{i1}^* = \Delta$, $p_{i2}^* > \Delta$, $p_o^* = c + \Delta$, and M_2 sets $p_{m2}^* = 0$, and M_1's price is indeterminate and can take any value $p_{m1}^* \geq 0$. Clearly, M_1 and M_2 have no incentive to deviate. To ensure the stated price profile is indeed an equilibrium, it remains to check (i) S is not making losses inside ($\Delta - c - \tau_1 \geq 0$); and (ii) S has no incentive to set a low p_o to attract regular consumer to the direct channel, which requires

$$\Delta - b - c \leq \mu\Delta + (1-\mu)(\Delta - c - \tau_1) \iff \tau_1 \leq \frac{b + c\mu}{1-\mu}.$$

Finally, there is no other marketpalce equilibrium given we ruled out all equilibria involving weakly dominated strategies.

Suppose $\tau_1 = \tau_2 = \tau$. Given the symmetry, S sets the same inside prices across the two marketplaces, so $p_{i1}^* = p_{i2}^* = p_i^*$. The equilibrium profits of M_1 and M_2 are, respectively, $\tau(1-\mu)$ and zero given the tie-breaking rule. This implies M_2's profit is zero in any such equilibrium, so it necessarily has an incentive to deviate to attract regular consumers with its own offering. Thus, the remaining steps follow immediately from the previous paragraph. ■

Lemma F.2 *(Direct sales equilibrium)*

- *If $\Delta < \frac{b+c}{1-\mu}$, then there is no direct sales equilibrium.*
- *If $\Delta \geq \frac{b+c}{1-\mu}$, then any price profile satisfying $p^*_{i1} = p^*_{i2} > \Delta$, $p^*_{m1} = p^*_{m2} = 0$, $p^*_o = \Delta - b$ is a direct sales equilibrium. Direct sales equilibria exist if and only if $\tau_1 \geq \frac{b+c\mu}{1-\mu}$. The equilibrium profits are $\Pi_{M_1} = \Pi_{M_2} = 0$ and $\pi = \Delta - b - c$.*

Proof. The proof of Lemma 2 applies. ■

Lemma F.3 *(Seller equilibrium)*

- *If $\Delta \leq \frac{b+c}{1-\mu}$ and $\tau_1 \geq \Delta - c$, in the seller equilibrium, $p^*_{m1} = p^*_{m2} = 0$, $p^*_{i1} > \Delta$, $p^*_{i2} > \Delta$, and $p^*_o = c + \Delta$ is a seller equilibrium. All regular consumers either buy from M_1 or M_2. The equilibrium profits are $\Pi_{M_1} = \Pi_{M_2} = 0$ and $\pi = \mu\Delta$.*
- *If $\Delta > \frac{b+c}{1-\mu}$ or $\tau_1 < \Delta - c$, there is no seller equilibrium.*

Proof. For $\Delta \leq \frac{b+c}{1-\mu}$ and $\tau_1 \geq \Delta - c$, M_1 and M_2 clearly have no incentive to deviate. S's equilibrium profit is $\Delta\mu$, while its deviation profit is either $\Delta - b - c$ (from setting a low outside price) or $\Delta\mu + (1-\mu)(\Delta - c - \tau_1)$ (from setting a low inside price), both of which are lower than the equilibrium profit. If $\tau_1 < \Delta - c$ or $\Delta > \frac{b+c}{1-\mu}$, then at least one of these two deviation becomes strictly profitable for S, and the equilibrium above does not exist. ■

Combining these lemmas, for $\tau_1 \leq \tau_2$ the equilibria of the stage 3 subgame, conditional on S participating on both marketplaces, can be summarized as:

- In marketplace equilibria (ME), $\Pi_{M_1} = \tau_1(1-\mu)$, $\Pi_{M_2} = 0$, and $\pi = \mu\Delta + (\Delta - c - \tau_1)(1-\mu)$. The equilibrium exists if and only if $\tau_1 \leq \min\left\{\Delta - c, \frac{b+c\mu}{1-\mu}\right\}$.
- In direct sales equilibria (DE), $\Pi_{M_2} = \Pi_{M_1} = 0$, and $\pi = \Delta - b - c$. The equilibrium exists if and only if $\Delta \geq \frac{b+c}{1-\mu}$ and $\tau_1 \geq \frac{b+c\mu}{1-\mu}$
- In seller equilibria (*SE*), $\Pi_{M_1} = \Pi_{M_2} = 0$, and $\pi = \mu\Delta$. The equilibrium exists if and only if $\Delta \leq \frac{b+c}{1-\mu}$ and $\tau_1 \geq \Delta - c$.

	$\tau_1 \leq \Delta - c$	$\tau_1 > \Delta - c$
$\Delta < \frac{b+c}{1-\mu}$	*ME*	*SE*

and

	$\tau_1 \leq \frac{b+c\mu}{1-\mu}$	$\tau_1 > \frac{b+c\mu}{1-\mu}$
$\Delta > \frac{b+c}{1-\mu}$	*ME*	*DE*

(F.1)

If instead S participates only on one of the marketplaces (say, M_1), then the analysis proceeds as if M_2 is operating as a pure seller. The existing results on the separation mode (Section C) then apply. And the categorization in Table F.1 also applies but with slightly different equilibrium profits:

- In marketplace equilibria (ME), $\Pi_{M_1} = \tau(1-\mu)$, $\Pi_{M_2} = 0$, and $\pi = \mu\Delta + (1-\mu)(\Delta - c - \tau)$. The equilibrium exists if and only if $\tau_1 \leq \min\left\{\Delta - c, \frac{b+c\mu}{1-\mu}\right\}$.
- In direct sales equilibria (DE), $\Pi_{M_1} = \Pi_{M_2} = 0$, and $\pi = \Delta - b - c$. The equilibrium exists if and only if $\Delta \geq \frac{b+c}{1-\mu}$ and $\tau_1 \geq \frac{b+c\mu}{1-\mu}$

- In seller equilibria (*SE*), $\Pi_{M_1} = 0$, $\Pi_{M_2} = p_r^* (1-\mu)$, and $\pi = \mu\Delta$, where

$$p_r^* \in [0, \min\{c - \Delta + \tau, c + b - (1-\mu)\Delta\}].$$

  The equilibrium exists if and only if $\Delta \leq \frac{b+c}{1-\mu}$ and $\tau_1 \geq \Delta - c$.

Finally, if S does not participate at all, it competes with two pure sellers. As described in the main text, this results in both platforms setting $p_{m1}^* = p_{m2}^* = 0$, and S either sets $p_o^* = c + \Delta$ and sells only to direct consumers ($\pi = \mu\Delta$), or sets $p_o^* = \Delta - b$ and sells to all consumers ($\pi = \Delta - b - c$).

Comparing these profits, and given that S is free to join both marketplaces and that S breaks ties in favor of participating, we conclude that in stage 2 S participates on both platforms if multihoming is costless. We know that M_2's equilibrium profit (in the pricing subgame) is zero as long as $\tau_1 \leq \tau_2$. Therefore, in stage 1, for each given level of M_1's commission $\tau_1 > 0$, M_2 has an incentive to undercut by setting $\tau_2 < \min\left\{\tau_1, \min\left\{\Delta - c, \frac{b+c\mu}{1-\mu}\right\}\right\}$ in order to induce an marketplace equilibrium with S selling on M_2's marketplace. A symmetric argument implies M_1 has the same incentive to undercut for any given level of $\tau_2 > 0$, so in equilibrium we have $\tau_1 = \tau_2 = 0$. There is no incentive to unilaterally deviate upward from this commission level because such a deviation does not affect the equilibrium of the continuation subgame.

Note if instead multihoming is costly, then S participates only on the cheaper platform (say, M_1). There is no equilibrium with $\tau_1 > \min\left\{\Delta - c, \frac{b+c\mu}{1-\mu}\right\}$ because M_1 earns zero profit in the resulting direct sales or seller equilibria. For $0 < \tau_1 \leq \min\left\{\Delta - c, \frac{b+c\mu}{1-\mu}\right\}$, M_2 has an incentive to undercut to avoid earning zero profit. Again, M_1 and M_2 compete in commissions to attract S's participation, and in equilibrium we continue to have $\tau_1 = \tau_2 = 0$.

F.2 M_1 operates in dual mode and M_2 operates in marketplace mode

We first consider the case of $\tau_1 > \tau_2$. Suppose S participates on both marketplaces. In any equilibrium in which S is making sales through one of the marketplaces, S's price must be such that all such purchases are made through M_2. There is no equilibrium with M_1 facilitating any sales and so the analysis proceeds as if S is not available on M_1 (i.e. M_1 operates as a pure seller). The existing results on the separation mode (Section C) applies. Therefore, in stage 2, S is weakly better off from participating on both marketplaces (if multihoming is costless) or participating only on M_2 (if multihoming is costly).

Suppose instead $\tau_1 \leq \tau_2$. In any equilibrium in which S is making sales through one of the marketplaces, S's price must be such that all such purchases are made through M_1. Therefore, the marketplace by M_2 is irrelevant to the analysis, and the pricing unfolds as in the baseline model in Section 3.3. Therefore, in stage 2, S is weakly better off from participating on both marketplaces (if multihoming is costless) or participating only on M_1 (if multihoming is costly).

We can now consider stage 1. There is no equilibrium with $\tau_2 > \min\left\{\Delta - c, \frac{b+c\mu}{1-\mu}\right\}$ because at such fee levels S never sells through M_2, regardless of τ_1. For $0 < \tau_2 \leq \min\left\{\Delta - c, \frac{b+c\mu}{1-\mu}\right\}$, M_1 has an incentive to undercut to avoid earning zero profit because otherwise S sells to all regular consumers through M_2. Therefore, in equilibrium we must have $\tau_2 = 0$ and $\tau_1 = 0$. There is no incentive to unilaterally deviate upward from this commission level because such a deviation does not affect the equilibrium of the continuation subgame.

F.3 Entry decisions

We are now ready to analyze the entry and mode choice decisions of the platforms. We can summarize both platforms' profits for all possible combinations of modes in the following table, where the first and second entries in each box represent M_1 and M_2's profit without entry cost.

	M_2 marketplace	M_2 seller	M_2 dual
M_1 marketplace	$0, 0$	$\tau^*(1-\mu), 0$	$0, 0$
M_1 seller	$0, \tau^*(1-\mu)$	$0, 0$	$0, \tau^*(1-\mu)$
M_1 dual	$0, 0$	$\tau^*(1-\mu), 0$	$0, 0$

,

where $\tau^* = \min\left\{\Delta - c, \frac{b+c\mu}{1-\mu}\right\} > 0$. Recall that M_2 observes M_1's mode choice before making its decisions. Therefore, M_2 does not have incentive to enter the market as long as M_1 is not operating as a pure seller given entry cost $F > 0$. Anticipating this, M_1 enters the market and operates in dual mode, which is indeed the most profitable mode given that M_2 does not enter.

G Comparison with wholesaler-retailer model

G.1 Third-party products mode

Suppose that in stage 0 M chooses the third-party products mode. In this case, whenever M does not sell S's product, the only alternative available is the fringe suppliers' product, which is priced at c. For any given wholesale price w set by S in stage 1, there are two possible equilibria in the pricing subgame: (i) M sells the fringe suppliers' product in equilibrium; (ii) M sells S's product in equilibrium. In what follows, we denote M's price for S's product as p_m^s and M's price for the fringe product as p_m^f.

The first equilibrium exists only when $w \geq c+\Delta$. To see this, suppose to the contrary that $w < c+\Delta$, and consider any equilibrium in which M sells fringe suppliers' product at some price p_m^f. In this case, M earns a margin of $p_m^f - c$, but it can profitably deviate to selling S's product at $p_m^s = p_m^f + \Delta$, resulting in the same volume of sales, but a strictly higher margin of $p_m^f + \Delta - w$. The following lemma follows from Proposition 2:

Lemma G.1 *(Equilibrium with M selling the fringe suppliers' product) Suppose $w \geq c + \Delta$. In the pricing subgame:*

- *If $\Delta > \frac{b}{1-\mu}$, in the equilibrium, $p_o^* = c + \Delta - b$ and $p_m^{f*} = c$. All regular consumers purchase from S directly. Equilibrium profits are $\Pi = 0$ and $\pi = \Delta - b$.*
- *If $\Delta \leq \frac{b}{1-\mu}$, in the mixed-strategy equilibrium, p_m^{f*} is distributed according to c.d.f F_m^f with support $[c + b - (1-\mu)\Delta, c + b]$, where*

$$F_m^f\left(p_m^{f*}\right) = \frac{1}{1-\mu}\left(1 - \frac{\mu\Delta}{p_m^{f*} - b + \Delta - c}\right) \text{ for } p_m^{f*} \in [c + b - (1-\mu)\Delta, c + b];$$

p_o^* is distributed according to c.d.f F_o with support $[c + \mu\Delta, c + \Delta]$, where

$$F_o\left(p_o^*\right) = \begin{cases} 1 - \left(\frac{c+b-(1-\mu)\Delta}{p_o^*+b-\Delta}\right) & \text{for} \quad p_o^* \in [c+\mu\Delta, c+\Delta) \\ 1 & \text{for} \quad p_o^* \geq c+\Delta \end{cases}.$$

Equilibrium profits are $\Pi = (b - (1-\mu)\Delta)(1-\mu)$ and $\pi = \mu\Delta$.

The second equilibrium involves M selling S's product. First, this equilibrium exists only when $w \leq c + \Delta$. Otherwise, suppose $w > c + \Delta$ and M sells S's product to a positive mass of consumers at some price p_m^s. In this case, M earns a margin of $p_m^s - w$, but it can profitably deviate to selling the fringe suppliers' product at $p_m^f = p_m^s - \Delta$ to make the same amount of sales but at a strictly higher margin $p_m^s - \Delta - c$. Next, we show that in general the equilibrium in which M sells S's product cannot exist in pure strategies.

Lemma G.2 *If $w \neq c+\Delta$, then there is no pure-strategy equilibrium with M selling S's product. If $w = c+\Delta$, there is a pure-strategy equilibrium with M selling S's product to all regular consumers at $p_m^s = c+\Delta+b$ and S selling its product to all direct consumers at $p_o = c+\Delta$. Equilibrium profits are $\Pi = b(1-\mu)$ and $\pi = \Delta$.*

Proof. Suppose such an equilibrium exists. Due to the convenience benefit b, in any pure-strategy equilibrium M sells S's product to all regular consumers, which then implies S must set $p_o = c+\Delta$ to focus on selling to direct consumers. Given p_o, M does best selling S's product at $p_m^s = c+\Delta+b$, which makes all regular consumers indifferent between buying S's product from M and buying from S directly. In this equilibrium, S's profit is $\pi^{eqm} = \mu\Delta + (1-\mu)(w-c)$ given that it earns from supplying M, and M's profit is $\Pi^{eqm} = (c+b+\Delta-w)(1-\mu)$. If $w > c+\Delta$, then M can profitably deviate to selling the fringe suppliers' product at $p_m^f = c+b$ so as to sell to all regular consumers at a strictly higher margin b. If $w < c+\Delta$, then S can deviate by slightly lowering its outside price to undercut M, attracting all consumers and earning $\pi^{dev} = \Delta > \mu\Delta + (1-\mu)(w-c) = \pi^{eqm}$. Finally, if $w = c+\Delta$ and $c = 0$, neither M nor S have an incentive to deviate from the stated equilibrium. ■

When the pure-strategy equilibrium does not exist, we obtain the following mixed-strategy equilibrium.

Lemma G.3 *(Equilibrium with M selling S's product) Suppose $w \leq c+\Delta$. There exists a mixed-strategy equilibrium in which p_m^{s*} is distributed according to c.d.f F_m, where*

$$F_m\left(p_m^{s*}\right) = 1 - \frac{\mu(\Delta+c+b-p_m^{s*})}{(1-\mu)\left(p_m^{s*}-b-w\right)} \text{ for } p_m^{s*} \in [c+\mu\Delta+(w-c)(1-\mu)+b, c+b+\Delta];$$

p_o^* is distributed according to c.d.f F_o, where

$$F_o\left(p_o^*\right) = \begin{cases} 1 - \frac{(\Delta+c-w)\mu+b}{p_o^*-w+b} & \text{for } \; p_o^* \in [c+\mu\Delta+(w-c)(1-\mu), c+\Delta) \\ 1 & \text{for } \; p_o^* \geq c+\Delta \end{cases}.$$

Equilibrium profits are $\Pi = ((\Delta+c-w)\mu+b)(1-\mu)$ and $\pi = \mu\Delta+(w-c)(1-\mu)$.

Proof. We verify the mixed strategy equilibrium stated in the proposition. The cdf F_m is such that S is indifferent between all $p_o^* \in [c+\mu\Delta+(w-c)(1-\mu), c+\Delta]$. When $p_o^* = c+\Delta$, S attracts only direct consumers and obtains profit $\mu\Delta+(w-c)(1-\mu)$. Therefore, the indifference condition is

$$\left(p_o^*-c\right)\left(\mu+(1-\mu)\left(1-F_m\left(p_o^*+b\right)\right)\right) + (w-c)(1-\mu)F_m\left(p_o^*+b\right) = \mu\Delta+(w-c)(1-\mu).$$

Rearranging the above expression, we can get

$$1 - F_m\left(p_o^*+b\right) = \frac{\mu(\Delta-p_o^*+c)}{\left(p_o^*-w\right)(1-\mu)},$$

or after the change of variables $p_m^{s*} = p_o^*+b$,

$$F_m\left(p_m^{s*}\right) = 1 - \frac{\mu(\Delta+c+b-p_m^*)}{(1-\mu)\left(p_m^*-b-w\right)}.$$

Then $F_m\left(c+\mu\Delta+(w-c)(1-\mu)+b\right) = 0$ and $F_m\left(c+b+\Delta\right) = 1$, so the distribution is atomless. Meanwhile, the cdf F_o is such that M is indifferent between all $p_m^{s*} \in [c+\mu\Delta+(w-c)(1-\mu)+b, c+b+\Delta]$.

When $p_m^{s*} = c + \mu\Delta + (w-c)(1-\mu) + b$, M attracts regular consumers with probability one and obtains profit $((\Delta + c - w)\mu + b)(1-\mu)$. Therefore, the indifference condition is

$$(p_m^{s*} - w)(1-\mu)(1 - F_o(p_m^* - b)) = ((\Delta + c - w)\mu + b)(1-\mu).$$

Using the change of variables $p_m^{s*} = p_o^* + b$, we obtain

$$F_o(p_o^*) = 1 - \frac{(\Delta + c - w)\mu + b}{p_o^* - w + b}.$$

Then, $F_o(c + \mu\Delta + (w-c)(1-\mu)) = 0$, and

$$\lim_{p_o^* \to c+\Delta} F_o(p_o^*) = 1 - \frac{(\Delta + c - w)\mu + b}{\Delta + c - w + b} < 1,$$

so F_o has an atom at $p_o^* = c + \Delta$.

Finally, we check that neither player can profitably deviate from the stated mixed strategy equilibrium. For S, any $p_o < c + \mu\Delta + (w-c)(1-\mu)$, even if it attracts all consumers, earns strictly lower profits than that obtained from selling only to direct consumers $(\mu\Delta + (w-c)(1-\mu))$. And any $p_o > c + \Delta$ attracts no consumer due to the existence of fringe sellers. A similar logic applies to rule out M deviating to any $p_m^s \notin [c + \mu\Delta + (w-c)(1-\mu) + b, c + b + \Delta]$. ■

We now consider S's wholesale price decision. For $w \leq c + \Delta$, the equilibrium in the pricing subgame has M selling S's product, so that $\pi(w) = \mu\Delta + (w-c)(1-\mu)$. Within this region, S clearly does best setting the highest wholesale price. For $w > c + \Delta$, the equilibrium in the pricing subgame has M selling the fringe suppliers' product, so that $\pi(w) = \max\{\mu\Delta, \Delta - b\}$ is independent of w. If $w = c + \Delta$, both types of equilibrium exists, in which case we select the equilibrium that maximizes S's profit. Notice that $\mu\Delta + \Delta(1-\mu) > \max\{\mu\Delta, \Delta - b\}$. Hence, we conclude S does best setting $w = c + \Delta$ to induce the pure-strategy equilibrium in which M sells S's product. To summarize,

Proposition G.1 *Suppose M chooses the third-party products mode. In the overall equilibrium: S sets the wholesale price $w = c + \Delta$, M sells S's product to all regular consumers at $p_m^{s*} = c + \Delta + b$, and S sells its product to all direct consumers at $p_o = c + \Delta$. Equilibrium profits are $\Pi^{third-party} = b(1-\mu)$ and $\pi^{third-party} = \Delta$.*

G.2 In-house products mode

Suppose that in stage 0 M chooses the in-house products mode. In what follows, we denote S's direct price as p_o and the price for in-house brand as p_m^h. In this case, the pricing subgame unfolds as if M operates as a pure seller in Section 3.2. Therefore, the equilibrium is described by the following lemma, which follows from Proposition 2:

Lemma G.4 *(Equilibrium with M selling in-house brand only)*

- *If $\Delta > \frac{b+c}{1-\mu}$, there exists a pure-strategy equilibrium in which $p_o^* = \Delta - b$ and $p_m^{h*} = 0$. All regular consumers purchase from S directly. Equilibrium profits are $\Pi^{in-house} = 0$ and $\pi^{in-house} = \Delta - b - c$.*
- *If $\Delta \leq \frac{b+c}{1-\mu}$, there exists a mixed-strategy equilibrium in which p_m^{h*} is distributed according to c.d.f F_m^h with support $[c + b - (1-\mu)\Delta, c + b]$, where*

$$F_m^h(p_m^{h*}) = \frac{1}{1-\mu}\left(1 - \frac{\mu\Delta}{p_m^{h*} - b + \Delta - c}\right) \text{ for } p_m^{h*} \in [c + b - (1-\mu)\Delta, c + b];$$

p_o^* *is distributed according to c.d.f* F_o *with support* $[c+\mu\Delta, c+\Delta]$*, where*

$$F_o\left(p_o^*\right) = \begin{cases} 1-\left(\frac{c+b-(1-\mu)\Delta}{p_o^*+b-\Delta}\right) & for \quad p_o^* \in [c+\mu\Delta, c+\Delta) \\ 1 & for \quad p_o^* \geq c+\Delta \end{cases}.$$

Equilibrium profits are $\Pi^{in-house} = (c+b-(1-\mu)\Delta)(1-\mu)$ *and* $\pi^{in-house} = \mu\Delta$.

G.3 Dual products mode

Suppose that in stage 0 M chooses the dual products mode. For any given wholesale price w set by S at stage 1, there are two possible equilibria.

In the first equilibrium, M sells its in-house brand only, as in Section 3.2. Therefore, the equilibrium is the same as in Lemma G.4. This equilibrium exists only when $w \geq \Delta$. To see this, suppose to the contrary that $w < \Delta$, and consider any equilibrium in which M sells its in-house brand at some price p_m^h. In this case, M earns a margin of p_m^h, but it can profitably deviate to selling S's product at $p_m^s = p_m^h + \Delta$, resulting in the same volume of sales but a strictly higher margin $p_m^h + \Delta - w$.

The second type of equilibrium involves M selling S's product. This equilibrium only exists if $w \leq \Delta$. To see why, suppose $w > \Delta$ and M sells S's product to a positive mass of consumers at some price p_m^s, where p_m^s is drawn from some possibly degenerate distribution. In this case M earns a margin of $p_m^s - w$, but it can profitably deviate to selling its in-house brand at $p_m^h = p_m^s - \Delta$ to make the same amount of sales but at a strictly higher margin.

Lemma G.5 *If* $w \neq \Delta$ *or* $c > 0$*, then there is no pure-strategy equilibrium with M selling S's product. If* $w = \Delta$ *and* $c = 0$*, there is a pure-strategy equilibrium with M selling S's product to all regular consumers at* $p_m = c + \Delta + b$ *and S selling its product to all direct consumers at* $p_o = c + \Delta$.

Proof. Suppose such an equilibrium exists. Due to the convenience benefit b, in any pure-strategy equilibrium M sells S's product to all regular consumers, which then implies S must set $p_o = c+\Delta$ to focus on selling to direct consumers. Given p_o, M does best selling S's product at $p_m^s = c+\Delta+b$ which makes all regular consumers indifferent between buying S's product from M and buying from S directly. In this equilibrium, S's profit is $\pi^{eqm} = \mu\Delta + (1-\mu)(w-c)$ and M's profit is $\Pi^{eqm} = (c+b+\Delta-w)(1-\mu)$. If $w > \Delta$, then M can profitably deviate to selling its in-house brand at $p_m^h = c+b$ to all regular consumers, obtaining a strictly higher margin. If $w \leq \Delta$, then S can deviate by slightly lowering its outside price to undercut M, attracting all consumers and earning $\pi^{dev} = \Delta > \mu\Delta + (1-\mu)(w-c) = \pi^{eqm}$, where the last inequality holds whenever $w \neq \Delta$ or $c > 0$. Finally, if $w = \Delta$ and $c = 0$, neither M nor S have an incentive to deviate from the stated equilibrium. ■

When the pure-strategy equilibrium does not exist, the mixed-strategy equilibrium with M selling S's product in Lemma G.3 applies.

Lemma G.6 *(Equilibrium with M selling S's product) Suppose* $w \leq \Delta$.[12] *There exists a mixed-strategy equilibrium in which* p_m^{s*} *is distributed according to c.d.f* F_m*, where*

$$F_m\left(p_m^{s*}\right) = 1 - \frac{\mu(\Delta+c+b-p_m^{s*})}{(1-\mu)\left(p_m^{s*}-b-w\right)} \text{ for } p_m^{s*} \in [c+\mu\Delta+(w-c)(1-\mu)+b, c+b+\Delta];$$

p_o^* *is distributed according to c.d.f* F_o*, where*

$$F_o\left(p_o^*\right) = \begin{cases} 1-\frac{(\Delta+c-w)\mu+b}{p_o^*-w+b} & for \quad p_o^* \in [c+\mu\Delta+(w-c)(1-\mu), c+\Delta) \\ 1 & for \quad p_o^* \geq c+\Delta \end{cases}.$$

[12]Notice that if $c = 0$ and $w = \Delta$, the mixed-strategy equilibrium collapses to the pure-strategy equilibrium of Lemma G.5 (F_m collapses to a single point at $p_m^* = b + \Delta$ and F_o has all its mass concentrated at $p_o^* = \Delta$).

Equilibrium profits are $\Pi = ((\Delta + c - w)\mu + b)(1-\mu)$ *and* $\pi = \mu\Delta + (w-c)(1-\mu)$.

Proof. Proof of Lemma G.3 applies. ■

We now consider S's wholesale price decision. For $w < \Delta$, the equilibrium in the pricing subgame has M selling S's product, so that $\pi(w) = \mu\Delta + (w-c)(1-\mu)$. Within this region, M clearly does best setting the highest wholesale price. For $w > \Delta$, the equilibrium in the pricing subgame has M selling the in-house brand, so that $\pi(w) = \max\{\mu\Delta, \Delta - b - c\}$ is independent of w. If $w = \Delta$, both types of equilibrium exist, in which case we select the equilibrium that maximizes S's profit given that it can always adjust w by an infinitesimal amount to induce the equilibrium it prefers. Note that $\mu\Delta + (\Delta - c)(1-\mu) > \max\{\mu\Delta, \Delta - b - c\}$. Hence, we conclude S does best by setting $w = \Delta$ to induce the equilibrium in which M sells S's product. To summarize,

Proposition G.2 *Suppose M chooses the dual products mode. In the overall equilibrium: S sets wholesale price $w = \Delta$, the equilibrium of the pricing game is described by Lemma G.6, and equilibrium profits are* $\Pi^{dual} = (c\mu + b)(1-\mu)$ *and* $\pi^{dual} = \Delta - c(1-\mu)$.

G.4 Choice of mode and banning the dual products mode

Comparing across the three modes, it is obvious that M does best choosing the dual-products mode because $\Pi^{dual} = (b + c\mu)(1-\mu) > \max\left\{\Pi^{third-party}, \Pi^{in-house}\right\}$. Whenever the dual-products mode is banned, we have $\Pi^{third-party} \geq \Pi^{in-house}$ if and only if $\Delta \geq \frac{c}{1-\mu}$, where $\Pi^{third-party} = \Pi^{in-house}$ if $\Delta = \frac{c}{1-\mu}$. We now provide the proof of Proposition 13 in the main text.

Proof. (Proposition **13**). The results on profits follow from direct inspections. Consider the surplus and welfare results. Suppose $\Delta \geq \frac{c}{1-\mu}$. Let η^{dual} denote the probability that regular consumers buy S's product from M in the mixed-strategy equilibrium of the dual mode. From Lemma G.6, if $w = \Delta$ then F_o has a mass point at $p_o^* = c + \Delta$, with mass $\frac{c\mu+b}{c+b}$. Therefore, $\eta^{dual} \geq \frac{c\mu+b}{c+b}$. We first note $W^{dual} = v + \Delta + b(1-\mu)\eta^{dual} - c < v + \Delta + b(1-\mu) - c = W^{third-party}$ and $CS^{dual}_{direct} = CS^{third-party}_{direct} = (v-c)\mu$. Meanwhile,

$$\begin{aligned}
CS^{dual}_{regular} &= W^{dual} - \Pi^{dual} - \pi^{dual} - CS^{dual}_{direct} \\
&= v + \Delta + b(1-\mu)\eta^{dual} - c - (b + c\mu)(1-\mu) - \Delta + c(1-\mu) - (v-c)\mu \\
&= (v-c)(1-\mu) - b(1-\mu)(1-\eta^{dual}) + c(1-\mu)^2 \\
&\geq (v-c)(1-\mu) - c(1-\mu)^2 \frac{b}{b+c} + c(1-\mu)^2 \\
&\geq (v-c)(1-\mu) = CS^{third-party}_{regular},
\end{aligned}$$

where the inequalities used $\eta^{dual} \geq \frac{c\mu+b}{c+b}$ and $\frac{b}{b+c} \leq 1$. It follows that $CS^{dual} \geq CS^{third-party}$.

Next suppose $\Delta \leq \frac{c}{1-\mu}$. Let $\eta^{in-house}$ denote the probability that regular consumers buy S's product from M in the mixed-strategy equilibrium of the in-house products mode. We have $W^{in-house} = v + \Delta + (b - \Delta)(1-\mu)\eta^{in-house} - c$. Therefore, $W^{in-house} < W^{dual}$ if and only if $(b-\Delta)\eta^{in-house} < b\eta^{dual}$. We know that

$$\eta^{dual} \geq \frac{c\mu + b}{c+b} > \frac{c\mu + b - c}{c + b - c} = \frac{b - c(1-\mu)}{b} \geq \frac{b - \Delta}{b}.$$

Therefore, $b\eta^{dual} > (b - \Delta) \geq (b-\Delta)\eta^{in-house}$, as required. To show the results on consumer surplus, we note the following two preliminary claims:

Claim 1: p_o^* is higher in the dual mode than in the in-house products mode, in the sense of first-order stochastic dominance.

To prove this claim, substitute $w = \Delta$ in dual mode to derive the distribution of p_o^* as

$$F_o^{dual}(p_o^*)\begin{cases} 1-\frac{c\mu+b}{p_o^*-\Delta+b} & \text{for} \quad p_o^* \in [\Delta+c\mu, c+\Delta) \\ 1 & \text{for} \quad p_o^* \geq c+\Delta \end{cases};$$

while the distribution of p_o^* in the in-house products mode is

$$F_o^{in-house}(p_o^*) = \begin{cases} 1-\left(\frac{c+b-(1-\mu)\Delta}{p_o^*+b-\Delta}\right) & \text{for} \quad p_o^* \in [c+\mu\Delta, c+\Delta) \\ 1 & \text{for} \quad p_o^* \geq c+\Delta \end{cases}.$$

For all $p \in [c+\mu\Delta, \Delta+c\mu]$, we have $F_o^{in-house}(p) \geq 0 = F_o^{dual}(p)$; for all $p_o^* \in [\Delta+c\mu, c+\Delta)$, we have $F_o^{in-house}(p) = 1-\left(\frac{c+b-(1-\mu)\Delta}{p+b-\Delta}\right) > 1-\left(\frac{c+b-(1-\mu)c}{p+b-\Delta}\right) = F_o^{dual}(p)$, given $\Delta > c$; for all $p \geq \Delta+c$, we have $F_o^{in-house}(p) = F_o^{dual}(p) = 1$. We conclude $F_o^{in-house}(p) \geq F_o^{dual}(p)$ for all p.

Claim 2: Define $\tilde{p}_m^s \equiv p_m^{s*} - \Delta$. Then $\tilde{p}_m^s$ in dual mode is higher than p_m^{h*} in the in-house products mode, in the sense of first-order stochastic dominance.

To prove this claim, we substitute $\tilde{p}_m^s \equiv p_m^{s*} - \Delta$ into the distribution function in Lemma G.6 to obtain

$$\tilde{F}_m(\tilde{p}_m^s) = \frac{1}{1-\mu}\left(1-\frac{c\mu}{\tilde{p}_m^s-b}\right) \text{ for } \tilde{p}_m^s \in [c+b-(1-\mu)c, c+b].$$

Compare this with

$$F_m^h(p_m^{h*}) = \frac{1}{1-\mu}\left(1-\frac{\mu\Delta}{p_m^{h*}-b+\Delta-c}\right) \text{ for } p_m^{h*} \in [c+b-(1-\mu)\Delta, c+b].$$

For all $p \in [c+b-(1-\mu)\Delta, c+b-(1-\mu)c]$, we have $F_m^h(p) \geq 0 = \tilde{F}_m(p)$; for all $p \in [c+b-(1-\mu)c, c+b]$, we have $F_m^h(p) \geq \tilde{F}_m(p)$ if and only if

$$1-\frac{\mu\Delta}{p-b+\Delta-c} \geq 1-\frac{c\mu}{p-b},$$

which is equivalent to $p \leq c+b$. We conclude $F_m^h(p) \geq \tilde{F}_m(p)$ for all p.

To show $CS_{direct}^{dual} > CS_{direct}^{in-house}$, it suffices to show that p_o^* is lower in the in-house products mode, which follows directly from Claim 1 above. To show $CS_{regular}^{dual} > CS_{regular}^{in-house}$, it suffices to show that (i) p_o^* is lower in the in-house products mode, and (ii) the quality-adjusted inside price $\tilde{p}_m^s \equiv p_m^{s*} - \Delta$ in dual mode is higher than p_m^{h*} in the in-house products mode. Both (i) and (ii) follow from Claims 1 and 2 above. Given both groups of consumers are better off in the dual mode, we must have $CS^{dual} > CS^{in-house}$. ∎

G.5 Model with M setting wholesale prices

In this section, we consider an alternative formulation of the wholesaler-retailer model from Section 5.3 by assuming that M dictates the wholesale price. We first solve for the overall equilibrium in each of the three modes. For any given wholesale price w, the pricing subgame in each of the three modes is the same as in the model presented in Section 5.3.

In the third-party products mode, M optimally sets w such that S is indifferent between supplying and not supplying M. If S does not supply M, the subgame unfolds as if M exclusively sources from fringe suppliers, in which case S's equilibrium profit is $\max\{\mu\Delta, \Delta-b\}$. If S supplies M, its equilibrium profit is $\pi = \mu\Delta+(w-c)(1-\mu)$ by Lemma G.2-G.3. Therefore, S is indifferent when $\mu\Delta+(w-c)(1-\mu) = \max\{\mu\Delta, \Delta-b\}$, i.e when $w = c+\max\left\{\Delta-\frac{b}{1-\mu}, 0\right\}$. At this w, the pricing equilibrium is in mixed-strategies (Lemma G.3). Equilibrium profits are then as follows:

- If $\Delta \leq \frac{b}{1-\mu}$, then $w^{third-party} = c$, $\Pi^{third-party} = (\Delta\mu+b)(1-\mu)$, and $\pi^{third-party} = \mu\Delta$

- If $\Delta > \frac{b}{1-\mu}$, then $w^{third-party} = c + \Delta - \frac{b}{1-\mu}$, $\Pi^{third-party} = b$, and $\pi^{third-party} = \Delta - b$.

In the in-house products mode, M does not source any third-party products so wholesale prices are irrelevant. The equilibrium is given by Lemma G.4:

- If $\Delta \leq \frac{b+c}{1-\mu}$, then $\Pi^{in-house} = (c + b - (1-\mu)\Delta)(1-\mu)$ and $\pi^{in-house} = \mu\Delta$
- If $\Delta > \frac{b+c}{1-\mu}$, then $\Pi^{in-house} = 0$ and $\pi^{in-house} = \Delta - b - c$

Finally, for the dual-products mode, the analysis is the same as for the third-party products mode, except that whenever S does not supply M the subgame unfolds as if M exclusively sells its in-house product. In this case S's equilibrium profit is $\max\{\mu\Delta, \Delta - c - b\}$. This means M must set a lower wholesale price compared to the third-party products mode. Therefore S is indifferent when $\mu\Delta + (w - c)(1-\mu) = \max\{\mu\Delta, \Delta - c - b\}$, i.e when $w = c + \max\left\{\Delta - \frac{b+c}{1-\mu}, 0\right\}$. At this w, the pricing equilibrium is in mixed-strategies (Lemma G.3). Equilibrium profits are as follows:

- If $\Delta \leq \frac{b+c}{1-\mu}$, then $w^{dual} = c$, $\Pi^{dual} = (\Delta\mu + b)(1-\mu)$, and $\pi^{dual} = \mu\Delta$
- If $\Delta > \frac{b+c}{1-\mu}$, then $w^{dual} = c + \Delta - \frac{b+c}{1-\mu}$, $\Pi^{\text{dual}} = b + c\mu$, and $\pi^{dual} = \Delta - b - c$.

Proposition G.3 below shows the effect of banning the dual mode in this setup. We can again compare Proposition G.3 to the baseline model (Proposition 6). First, we have a different cutoff for switching modes because M sets both the wholesale price and the retail price in the third-party products mode, so that this mode behaves very differently compared to the marketplace mode. Second, whenever the ban on the dual products mode results in M choosing the in-house products mode, consumer surplus always increases. This result is driven by the fact that both the outside and the inside prices are higher in the dual products mode than in the in-house products mode. The outside price is higher because in the dual products mode S partially internalizes the revenue of M's inside sales via its wholesale price, meaning that S would be less aggressive in setting its outside prices. This in turns relaxes the inter-channel competition, allowing M, whose price is not constrained by within-channel competition, to charge a higher inside price than the inside price it charges in the in-house products mode.

Proposition G.3 *(Ban on dual products mode in the wholesaler-retailer model)*

- *If $\Delta \geq \frac{c}{1-\mu} - \frac{\mu b}{(1-\mu)^2}$, a ban on the dual products mode results in M choosing the third-party products mode. If $\Delta \leq \frac{b}{1-\mu}$, the ban does not affect the market outcome. If $\Delta > \frac{b}{1-\mu}$, then Π, CS_{regular}, CS_{direct}, and CS decrease, π increases, and the effect on W is ambiguous.*
- *If $\Delta < \frac{c}{1-\mu} - \frac{\mu b}{(1-\mu)^2}$, a ban on the dual products mode results in M choosing the in-house products mode, with Π and W decreasing, CS_{regular}, CS_{direct} and CS increasing, and π not changing.*

Proof. If $\Delta > \frac{b+c}{1-\mu}$, then $\Pi^{third-party} > \Pi^{in-house}$ obviously. If $\Delta \leq \frac{b}{1-\mu}$, then $\Pi^{third-party} = (\Delta\mu + b)(1-\mu) > (c + b - (1-\mu)\Delta)(1-\mu) = \Pi^{in-house}$. If $\frac{b}{1-\mu} < \Delta \leq \frac{b+c}{1-\mu}$, then

$$\Pi^{third-party} = b \geq (c + b - (1-\mu)\Delta)(1-\mu) = \Pi^{in-house}$$

if and only if $\Delta \geq \frac{c}{1-\mu} - \frac{\mu b}{(1-\mu)^2}$. Combining all cases, we obtain $\Pi^{third-party} \geq \Pi^{in-house}$ if $\Delta \geq \frac{c}{1-\mu} - \frac{\mu b}{(1-\mu)^2}$. Suppose instead $\Delta < \frac{c}{1-\mu} - \frac{\mu b}{(1-\mu)^2}$. This clearly implies $\Delta \leq \frac{b+c}{1-\mu}$, and, given $\Delta > c$, it also implies $\Delta > \frac{b}{1-\mu}$.[13] Therefore, $\Delta \leq \frac{c}{1-\mu} - \frac{\mu b}{(1-\mu)^2}$ is sufficient for $\Pi^{third-party} \leq \Pi^{in-house}$, and we note equality holds only when $\Delta = \frac{c}{1-\mu} - \frac{\mu b}{(1-\mu)^2}$. ■

[13]To see this, note that we have $\frac{c}{1-\mu} - \frac{\mu b}{(1-\mu)^2} \geq \Delta > c$. And $c < \frac{c}{1-\mu} - \frac{\mu b}{(1-\mu)^2} \iff c \geq \frac{b}{1-\mu}$, which implies $\Delta > c \geq \frac{b}{1-\mu}$.

Suppose $\Delta \geq \frac{c}{1-\mu} - \frac{\mu b}{(1-\mu)^2}$, so that M switches to the third-party products mode after the ban. We know that the firms' equilibrium strategies are the same in the third-party products mode and in the dual mode, except that the wholesale price is strictly higher in the third-party products mode when $\Delta > \frac{b}{1-\mu}$. In what follows, we show that both the inside and the outside prices become higher when w is higher, which immediately implies $CS^{dual}_{direct} > CS^{third-party}_{direct}$ and $CS^{dual}_{regular} > CS^{third-party}_{regular}$. Specifically, we want to show p^*_o and p^*_m in Lemma G.3 are increasing in w, in the sense of first-order stochastic dominance. Consider the outside price first: we want to show cdf $F_o(p^*_o)$ is decreasing in w. First note the distribution domain $[c + \mu\Delta + (w - c)(1 - \mu), c + \Delta]$ shifts upwards when w increases, and

$$\frac{d}{dw}\left(1 - \frac{(\Delta + c - w)\mu + b}{p^*_o - w + b}\right) = \frac{(p^*_o - \Delta - c)\,\mu - b(1-\mu)}{(p^*_o - w + b)^2} < 0,$$

where the inequality is due to $p^*_o < c + \Delta$. As for the inside price, a direct inspection reveals $F_m(p^*_m)$ is decreasing in w.

Suppose instead $\Delta < \frac{c}{1-\mu} - \frac{\mu b}{(1-\mu)^2}$, so that M switches to the in-house products mode after the ban. Note this implies $\Delta < \frac{b+c}{1-\mu}$, so $w^{dual} = c$. For welfare, $W^{in-house} = v + \Delta + (b - \Delta)(1 - \mu)\eta^{in-house} - c$. Therefore, $W^{in-house} < W^{dual}$ if and only if $(b - \Delta)\eta^{in-house} < b\eta^{dual}$. We know

$$\eta^{dual} \geq \frac{c\mu + b}{c + b} > \frac{c\mu + b - c}{c + b - c} = \frac{b - c(1-\mu)}{b} \geq \frac{b - \Delta}{b}.$$

Therefore, $b\eta^{dual} > (b - \Delta) \geq (b - \Delta)\eta^{in-house}$, as required. To show the results on consumer surplus, we use the following two preliminary claims:

Claim 1: p^*_o is higher in the dual mode than in the in-house products mode, in the sense of first-order stochastic dominance.

To prove this, substitute $w = c$ in dual mode to derive the distribution of p^*_o as

$$F^{dual}_o = \begin{cases} 1 - \frac{\Delta\mu + b}{p^*_o - c + b} & \text{for} \quad p^*_o \in [c + \mu\Delta, c + \Delta) \\ 1 & \text{for} \quad p^*_o \geq c + \Delta \end{cases},$$

We wish to compare it with

$$F^{in-house}_o(p^*_o) = \begin{cases} 1 - \left(\frac{c + b - (1-\mu)\Delta}{p^*_o + b - \Delta}\right) & \text{for} \quad p^*_o \in [c + \mu\Delta, c + \Delta) \\ 1 & \text{for} \quad p^*_o \geq c + \Delta \end{cases}.$$

We want to show $F^{in-house}_o \geq F^{dual}_o$, or $1 - \frac{c+b-(1-\mu)\Delta}{p^*_o + b - \Delta} \geq 1 - \frac{\Delta\mu + b}{p^*_o - c + b}$, which can be shown to be algebraically equivalent to $p^*_o \geq c + \Delta\mu$, which is indeed true given the domain.

Claim 2: Define $\tilde{p}^s_m \equiv p^{s*}_m - \Delta$. Then $\tilde{p}^s_m$ in dual mode follows the same distribution as p^{h*}_m in the in-house products mode.

To prove this, we substitute $\tilde{p}^s_m \equiv p^{s*}_m - \Delta$ into the distribution function to get

$$F_m(\tilde{p}_m) = \frac{1}{1-\mu}\left(1 - \frac{\mu\Delta}{\tilde{p}_m + \Delta - b - c}\right) \text{ for } p^*_m \in [c + b - (1-\mu)\Delta, c + b],$$

which is exactly the same as $F^h_m\left(p^{h*}_m\right)$.